Modeling and Optimization of Crowd Guidance for Building Emergency Evacuation

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Abstract—Building emergency evacuation has long been recognized as an important issue, and crowd guidance is a key to improve egress efficiency and occupant survivability. Most existing methods assume that crowd behaviors are independent of emergency situations and are fully controllable under guidance. This assumption makes it difficult to capture important features such as stampeding or blocking. In this paper, a probabilistic model is developed to characterize how fire propagation affects crowds in stressful conditions and in turn egress times. This enables the predictions of potential blockings, and provides a foundation to optimize crowd guidance. An optimization problem is then formulated to evacuate as many people and as fast as possible while reducing the relevant risks through appropriate guidance of crowds. To solve the problem, observing that groups of crowds are mostly independent of each other except when they compete for passages, a divide-and-conquer approach is developed. After the nonlinear coupling passage capacity constraints are approximately relaxed, individual group subproblems are solved by using stochastic dynamic programming with state reduction and the rollout scheme. Individual groups are then coordinated through the iterative updating of multipliers. Testing results demonstrate that, compared with the method ignoring crowd behaviors, our method evacuate more people and faster.

I. INTRODUCTION

EFFECTIVE building evacuation in case of emergencies such as fires, chemical spills, or spreads of biological agents has long been recognized as an important issue, and effective crowd guidance can improve egress efficiency, occupant survivability, and mitigate or prevent undesirable consequences such as stampeding or blocking. To effectively guide crowds, however, is a challenging issue because emergency events may propagate in uncertain ways and affect the availability of egress paths; egress path capacities may constrain the speed of crowd movement; and crowd could be stressed making their behaviors different from their normal modes. Although good results have recently been obtained on microscopic behaviors of a group such as the social force model of Helbing (e.g., Helbing, 2001; Helbing and Johansson, 2007), there is a major gap between these microscopic models of a group evacuating from rooms and the macroscopic models of crowd flows needed to evacuate from a building. Most existing egress guidance methods assume that crowd behaviors are independent of emergency situations and are fully controllable under guidance. These assumptions make it difficult to capture important features such as stampeding or blocking.

To bridge the gap between microscopic and macroscopic crowd models, a probabilistic graphical model is established in Section 3 of this paper to characterize the nonlinear interactions among the states of the emergency events, crowd stresses, egress capacities, and crowd flow rates. As shown in Figure 1, there are several major components in the model:

- There is a “desired flow rate” for crowd collective behaviors, following the “desired velocity” for individual behaviors of Helbing et al, 2000. If an emergency becomes urgent, people become more impatient, and the desired flow rate increases in a probabilistic manner.
- If the aggregate desired flow rate of a crowd is smaller than the egress capacity of a passage, then the crowd flow rate is equal to the desired flow rate without blocking. However, if the aggregated desired flow rate is larger than the passage capacity, then the crowd flow rate decreases drastically in a probabilistic and nonlinear fashion, resulting in the “faster-is-slower effect” of Helbing et al, 2000, and eventually leading to blocking and stampeding.
- Whether an individual would follow guidance is probabilistic and depends on his/her trust on the guidance provided, and his familiarity with the evacuation paths.

The above crowd behavior model is combined with the egress network and the emergency event dynamics in Section 4 to form an optimization problem with the goal to evacuate as many people and as fast as possible while reducing the
relevant risks through appropriate guidance of the crowds. The egress network is modeled as a room-and-path structure by using a directed graph with egress path capacity constraints. For simplicity of illustration, the emergency event under consideration is fire, and its propagation is described by a state transition model. The probabilistic graphical models, the egress network, emergency event dynamics, and crowd guidance then generate the crowd flow rates.

The problem thus formulated is a Markov decision problem, and to obtain an optimal solution, computation complexity is a major challenge. To solve the problem in a computationally efficient manner, observe that groups of crowds are mostly independent of each other except when they compete for small passages, and such competitions are described by a set of nonlinear egress capacity constraints. A divide-and-conquer approach is therefore developed in Section 5. After the nonlinear coupling egress capacity constraints are “approximately relaxed” based on the sum of individual group flow rates, individual group subproblems are solved by using stochastic dynamic programming with a state space significantly reduced by simplifying the uncontrollable fire status, and the rollout scheme is further applied to reduce the computation complexity (Bertsekas 2005). Individual groups are then coordinated by the iterative updating of Lagrangian multipliers by using the surrogate subgradient method (Zhao, Luh, and Wang, 1999). Numerical testing demonstrates that compared with results obtained by the traditional model, the guidance obtained by the new model can help prevent or mitigate potential blockings, improve evacuation efficiency, and manage risks.

II. LITERATURE REVIEW

This subsection reviews relevant literature on emergency events, egress systems and crowd behaviors.

A. Emergency Events

For the propagation of emergency events such as fire, many models have been developed, including field models (NIST’s Fire Dynamics Simulator; and McGrattan et al., 2008), zone models (Janssens, 2000), and cellular automaton models (Duarte, 1997; Ohgai et al., 2007). A field model or zone model generate good predictions but are computationally expensive while the cellular model is less accurate, but it is superior in computation complexity. In specific a cellular automaton model divides building areas into cells, each with multiple states (e.g., unburned, burning or burnt, Ohgai et al., 2007). The state transition probabilities for a cell depend on the states of its adjacent cells. The cellular model can also simulate propagation of other emergency events such as smoke or chemical spills in a similar way.

B. Egress Networks

Several types of egress network models have been developed such as fine networks, coarse networks, and continuous networks (Hamacher and Tjandra, 2002; Kuligowski and Peacock, 2005). All these network models can be considered as directed graphs, where each area is represented by a node with a specified capacity and each passage from one area to another is represented by a directed arc with specified capacity. The above egress models serve as a basis for egress performance analysis through simulation or network optimization, etc.

C. Crowd Behaviors

From the modeling perspective, crowd models can be categorized into two classes: in macroscopic and microscopic sense. For the former macroscopic models, a crowd is assumed to consist of identical, unthinking and instruction-following individuals, and is modeled as a continuous and homogeneous mass that behaves like a fluid flowing along corridors with a specified rate (Hoogendoorn, 2005, Burlatsky et al., 2008). These models can be used to optimize occupant movement for evacuation planning (Chalmet et al., 1982). However, stress phenomena have not been incorporated in macroscopic crowd models.

The second class of microscopic models considers individual intelligence, and includes cellular automaton models (Weng et al., 2006; Song et al., 2006), lattice gas model (Li et al., 2008) and social-force model (Helbing, Farkas, and Vicsek, 2000; Helbing and Johansson, 2007). These microscopic models can be used to evaluate the effects of high-level egress polices on occupant movement (Lin et al., 2008). However, only simulation methods are used with these microscopic models, and no optimization is involved.

A well-known microscopic model is the social force model of Helbing et al, 2000. The model considers the stress as an important psychological factor affecting the occupant behaviors, and simulation results show several stress phenomena with blocking captured. One of the major results is the faster-is-slower effect as shown in Figure 2, and it means desiring more speed may slow down the people at a passage, and eventually cause blockings.

III. AN NOVEL MODEL WITH BLOCKINGS CAPTURED

The existing macroscopic models cannot capture crowd stress behaviors and resulting blockings, and this motivates us to develop a novel model in this section, and the novel model will provide a foundation to optimize crowd guidance.

The novel model focuses on the faster-is-slower effect, and capture it by revealing the interdependency or interaction among the crowds, fire and egress networks. Specifically, the crowd movement is motivated by fire emergency, but constrained by the egress capacities. This leads to a conflict: the emergency makes people stressed and impatient and they desire moving faster, but the limited egress capacity cannot
meet their desire. As a result, the conflict makes people compete for the passage, and thus causes disorder, stampeding, and blocking. In a probabilistic manner, as the conflict goes intensive, the probability of blockings increases.

Above interdependency and interaction among the crowds, fire and egress networks can be formalized by a probabilistic graphical model as shown in Figure 3, where each node of the graphical model denotes a key factor such as fire status or path capacities, and the factors are connected by conditional probability distributions to establish the required relationship.

**B. Impatience Issue**

The crowd impatience reflects the time-pressure derived from the state of emergency events. Generally, if an emergency becomes urgent (e.g., fire propagates to people's locations), people become more impatient and desire moving out faster. (Chertkoff and Kushigian, 1999; F. Ozel, 2001). Here the crowd impatience can be measured by the desired flow rate $lq_e^{d}$, and $lq_e^{d}$ increases as crowd impatience goes up. To achieve above causality, the following probability distribution is given as an example.

The individual’s impatience is given by probability $p_{imp}$ and it means the probability that an individual cannot wait and desire to move out through a path immediately. Specifically, $p_{imp}$ depends on the fire status, i.e.,

$$p_{imp} = \begin{cases} p_{imp}^H & \text{if fire reaches } v_1 \text{ or } v_2 , \\ p_{imp}^M & \text{if fire reaches adjacencies of } v_1 \text{ or } v_2 , \\ p_{imp}^L & \text{otherwise.} \end{cases}$$

Then the collective impatience forms a desired flow rate $lq_e^{d}$, and it is binomial distributed. The probability distribution is given as an example. Given a guidance $u_e$, we assume that each individual will follow the guidance with probability $p_u$, and $p_u$ is given by

$$p_u = \begin{cases} p_u^H & \text{if } u_e \text{ guides people to a familiar path,} \\ p_u^L & \text{otherwise.} \end{cases}$$

Let $lw_e^d$ denote the number of people who will follow the guidance $u_e$ and want to move out through path e during a time unit. Here the concept of the desired flow rate is the counterpart of the desired velocity of Helbing et al, 2000. It is a macroscopic concept on crowd collective behaviors while the desired velocity captures the individual behavior in the microscopic sense.

**C. Crowd Response to Guidance**

Whether an individual would follow guidance is probabilistic and depends on his/her trust on the guidance provided and his familiarity with the evacuation paths. To achieve above relation, the following probability distribution is given as an example. Given a guidance $u_e$, we assume that each individual will follow the guidance with probability $p_u$, and $p_u$ is given by

$$p_u = \begin{cases} p_u^H & \text{if } u_e \text{ guides people to a familiar path,} \\ p_u^L & \text{otherwise.} \end{cases}$$

Then let $lw_e^d$ denote the number of people who will follow the guidance $u_e$ and want to move out through path e. Then $lw_e^d$ is binomial distributed.

Combine above three conditional probability distribution together, and it constructs an overall relationship from the input of the graphical model, i.e., the guidance $u_e$, to the output of the model, i.e. the crowd flow rate $q_e$. Given the conditional variables include path capacity $c_e$ and fire status $s^f$. Then the overall probabilistic relation is given by

$$Pr(q_e \mid u_e) = \sum_{s^f \cdot q_e^d} \sum_{w_e} Pr(q_e \mid q_e^d, c_e) Pr(q_e^d \mid w_e, s^f) Pr(w_e \mid u_e) Pr(s^f).$$

One point to be emphasized is that $u_e$, $w_e$, $q_e^d$ and $q_e$ are all vectors with magnitudes and directions. Since all are in the same direction, the given conditional probability distributions can be specified without concerning the direction. However, to embed the graphical model in an egress network in the next section, direction is still specified in above equations.

The probabilistic graphical model incorporates two
insightful psychological factors: impatience and trust, where impatience is the cause of blocking events, and trust reflects how crowds respond to guidance. An individual’s trust in guidance evolves with how effective the guidance has been and affects one’s egress direction. The high pressure derived from the state of emergency events causes people to become impatient, and it is the cause the faster-is-slower effect.

At a higher level, the probabilistic graphical model above can be characterized by the energy balance concept (Wang and Luh, under preparation), where the state of the emergency drives the desired flow rate and therefore the “Stress Energy.” This stress energy is “balanced” between the “Static Energy” captured by the crowd density (affected by the path capacity) and the “Dynamic Energy” represented by the flow rate. Such a macroscopic model thus provides a theoretical foundation for linking crowd stress behaviors to the situation, enables us to predict potential blockings in emergency evacuation, and provides a foundation to optimize crowd guidance.

IV. AN OVERALL OPTIMIZATION FORMULATION

The previous section develops a macroscopic crowd model with blocking captured, and it will be incorporated in the traditional network-flow model to capture crowd movement in an egress network. Combine the network-flow model with a fire propagation dynamics, and a system dynamics is obtained, by which an optimization problem is formulated with the goal to evacuate as many people and as fast as possible while reducing the relevant risks.

A. Traditional Network-Flow Model

An egress network is a room-and-path structure where the crowd moves and fire may propagate (Kuligowski and Peacock, 2005; Chalmet et al., 1982). It can be modeled by a directed graph \( DG = (V, E) \), where \( v \in V \) denotes an area; and \( e \in E \) denotes a paths connecting areas. The directed graph means that a default direction is specified in an egress network, and this is consistent with the so-called egress since the egress facility should be directed to exits.

Generally, the directed graph can be represented by

\[
 b(v,e) = \begin{cases} 
 1 & \text{if } e \text{ points into } v, \\
 -1 & \text{if } e \text{ points out of } v, \\
 0 & \text{otherwise.}
\end{cases}
\]  

Take the room-and-path structure illustrated in Figure 2 for example, and it gives \( b(v_1, e_1) = -1 \) and \( b(v_2, e_1) = 1 \).

In evacuation crowd moves in an egress network, and it is modeled by a network-flow model (Kuligowski and Peacock, 2005). Let \( q_e(t) \) denote the crowd flow rate on path \( e \). The magnitude \( \| q_e(t) \| \) is the number of people passing path \( e \) during the time interval \( [t, t+\Delta t] \), and the direction is

\[
 \text{sign}[q_e(t)] = \begin{cases} 
 +1 & \text{if crowd moves along with the default direction of path } e, \\
 -1 & \text{if crowd moves oppositely.}
\end{cases}
\]

Let \( x_v(t) \) denote the number of people in room \( v \) at time \( t \). As people moves in and out, \( x_v(t) \) is updated by

\[
 x_v(t+1) = x_v(t) + \sum_{e \in E} b(v,e)q_e(t).
\]  

Put (9) in a vector form and crowd flow equation is

\[
 x(t+1) = x(t) + Bq(t) .
\]  

Equation (9) and (10) presents the traditional network-flow model. In existing optimization literature the flow rate \( q(t) \) is usually the decision variable (Chalmet et al., 1982; Xiong et al., 2007). This implies that we can exactly control how many people will move in or out through a path. Clearly, this is not consistent with reality.

B. Fire Propagation

Fire could propagate in an egress network, and the probabilistic graphical model requires fire information to be incorporated. The information can be obtained by simulating a fire model. From the perspective of computation this paper uses a cellular automaton model, where a cell is an area of an egress network and the cell state is the fire status, i.e.,

\[
 s^f_v(t) = \begin{cases} 
 1 & \text{if room } v \text{ is on fire at time } t, \\
 0 & \text{otherwise.}
\end{cases}
\]  

Then the overall fire state at time \( t \) is

\[
 s^f(t) = (s^f_1(t), s^f_2(t), \ldots, s^f_{|V|}(t))^T .
\]

The transition of cell states is governed by conditional probabilities that fire propagates from one area to its adjacent areas, i.e.,

\[
 \Pr(s^f_v(t+1) = 1) = 1 - \prod_{v'} (1 - p^F_{vv'} \Pr(s^f_v(t) = 1)) ,
\]

where \( v' \) denotes an area and \( p^F_{vv'} \) is given by

\[
 p^F_{vv'} = \Pr(s^f_v(t+1) = 1 | s^f_{v'}(t) = 1) .
\]

The cellular automaton above formulates a stochastic process. Given initial fire state, the model can be used to predict how fire propagates in a given time horizon.

C. System Dynamic for Optimization

Incorporate the probabilistic graphical model in the network-flow equation (9), and this yields a panoramic view of the building-wide evacuation with blockings captured, and the crowd movement is characterized by

\[
 x_v(t+1) = x_v(t) + \sum_{e \in E} b(v,e)q_e(u_e(t), s^f(t)) ,
\]

where the guidance \( u_e(t) \) is

\[
 u_e(t) = \begin{cases} 
 +1 & \text{if equal to the default direction of path } e, \\
 -1 & \text{if opposite to the default direction of path } e, \\
 0 & \text{if path } e \text{ should not be used.}
\end{cases}
\]

Put (15) in a vector form and it yields

\[
 x(t+1) = x(t) + B \cdot q(u(t), s^f(t)) .
\]

where the crowd flow rate \( q \) is a function of \( u(t) \) and \( s^f(t) \).

Consequently, a system dynamics is obtained by combining crowd movement component and fire propagation component together in an egress network. Here \( x(t), s^f(t) \) is the system state and the guidance \( u(t) \) is the system input. Furthermore, simplify (17) and the evolution of \( x(t) \) is rewritten by

\[
 x(t+1) = f(x(t), s^f(t), u(t)) .
\]

Similarly, the evolution of \( s^f(t) \) is rewritten by

\[
 s^f(t+1) = h(s^f(t)) .
\]

The resulting process is a Markov process. Then an optimization problem is formulated with the following
D. Constraints

The constraints on path capacities are imbedded in the graphical model as shown previously, and the area capacity is not considered in this paper because existing work shows the bottleneck of evacuation process lies in the path capacity (Chertok and Kushigian, 1999).

Another set of constraints lies in crowd guidance, that is, the guidance should never lead crowd to an area which is on fire currently or will catch fire in the near future. Specifically, this paper formulates the constraints by

\[ u_k(t) \neq 1, \text{ if } \Pr(s_{x_k}(t) = 1) > \beta \text{ or } \Pr(s_{x_k}(t+1) = 1) > \beta \, . \]

\[ u_k(t) \neq -1, \text{ if } \Pr(s_{x_k}(t) = 1) > \beta \text{ or } \Pr(s_{x_k}(t+1) = 1) > \beta \, . \]  

Here \( \beta \) is a given threshold, and the symbols used are given in Figure 2. In our problem, since the fire-fighting effort is not considered, the probability that an area is on fire will increase as time goes by, and this implies (20) can be simplified by

\[ u_k(t) \neq 1, \text{ if } \Pr(s_{x_k}(t+1) = 1) > \beta \, . \]

\[ u_k(t) \neq -1, \text{ if } \Pr(s_{x_k}(t+1) = 1) > \beta \, . \]  

E. The Objective Function

The goal of crowd guidance is to evacuate as many people as possible while reducing the relevant risks. This translates to maximizing a weighted sum of the expected number of total people evacuated, expected cumulative number of people evacuated, and the negative values of the corresponding semi-variances considered as a risk measure.

Given time horizon \([0, T]\), to evacuate as many people as possible, the objective function should maximize the total number of people evacuated at terminal time \(T\). In the probabilistic sense this requires to maximize

\[ R_1 = \sum_{v \in \text{out}} E[x_v(T)] - c_{\text{avg}} \sum_{v \in \text{out}} \text{var}_{\text{sem}}[x_v(T)]. \]  

In (22) the semi-variances works when \( x_v(T) \) is less than the mean. Intuitively, the semi-variance measures the risk when the number of people at exits is less than the expectation i.e.,

\[ \text{var}_{\text{sem}}[x_v(T)] = E[g(x_v(T) - E[x_v(T)])]. \]  

where function \( g(\cdot) \) is

\[ g(n) = \begin{cases} n & \text{if } n < 0, \\ 0 & \text{otherwise}. \end{cases} \]

To evacuate people as fast as possible the cumulative number of people evacuated is to be maximized, and this corresponds to maximizing

\[ R_2 = \sum_{i=0}^{T-1} \sum_{v \in \text{out}} E[x_v(t)] - c_{\text{avg}} \sum_{i=0}^{T-1} \sum_{v \in \text{out}} \text{var}_{\text{sem}}[x_v(t)]. \]

Finally, the objective function is summarized by

Maximize \( J \) with \( J = c_{\text{avg}} R_1 + R_2 \).

With the objective function in a time-wise additive form, the optimization problem is formulated as a Markov decision problem, and the computation complexity is challenging.

V. SOLUTION METHODOLOGY

To solve the Markov decision problem as formulated previously a divide-and-conquer approach is used, that is, dividing crowd in groups, optimizing guidance to each group and coordinating evacuation processes among multiple groups. Here the multiple groups are coordinated within the Lagrangian relaxation framework, and an individual group subproblem is solved by stochastic dynamic programming with state reduction and the rollout scheme.

A. Lagrangian Relaxation

In our problem crowd are grouped based on their initial locations and different groups are assumed to receive different guidance (Pelechano and Badler, 2006). The original constraint on path \( e \) is \( E[q_e] \leq c_e \), and based on the relation between \( E[l q_e | q_e^d] \) and \( l q_e^d \) as shown in Figure 4, it gives \( E[l q_e^d] = E[\psi_e(l q_e^d)] \), where function \( \psi_e \) is given by (1) and (2).

As a result, the original constraint can be rewritten by \( E[\psi_e(l q_e^d)] \leq c_e \). According to Section 2, it is easily checked that the total desired flow rate is equal to the sum of group desired flow rate, i.e., \( l q_e^d = l q_e^{d(1)} + l q_e^{d(2)} \), then it follows

\[ E[\psi_e(l q_e^{d(1)}) + l q_e^{d(2)}] < c_e \, . \]  

However, above constraint cannot be relaxed directly due to the nonlinear coupling among the groups, i.e.,

\[ \psi_e(l q_e^{d(1)}) + \psi_e(l q_e^{d(2)}) \neq \psi_e(l q_e^{d(1)}) + \psi_e(l q_e^{d(2)}) \, . \]  

Then an approximation method is used by confining the nonlinear function \( \psi(\cdot) \) in the linearity segment, and this yields an approximate constraint which can be relaxed,

\[ E[l q_e^{d(1)}] + l q_e^{d(2)} < \psi^{-1}(c_e) \, . \]  

Given \( n \) groups, then equation (29) can rewritten by

\[ \sum_{i=1}^{n} E[l q_e^{d(i)}] \leq \psi^{-1}(c_e) \, . \]  

Equation (30) approximately relaxes the nonlinear capacity constraints based on the sum of individual desired flow rate, and Lagrangian relaxation can be used with

\[ \text{max } L, \text{ with } L = c_{\text{avg}} \sum_{v \in \text{out}} \sum_{i=0}^{T-1} E[x_v^{i}(T)] - c_{\text{risk}} \sum_{v \in \text{out}} \text{var}_{\text{sem}}[x_v^{i}(T)] \]

\[ + \sum_{i=0}^{T-1} \sum_{v \in \text{out}} \sum_{i=0}^{T-1} [E[x_v^{i}(t)] - c_{\text{risk}} \sum_{i=0}^{T-1} \text{var}_{\text{sem}}[x_v^{i}(t)] \]

\[ - \sum_{i=0}^{T-1} \sum_{v \in \text{out}} \sum_{i=0}^{T-1} \sum_{v \in \text{out}} [E[l q_e^{d(i)}(t)] - \psi^{-1}(c_e)] \, . \]

By collecting all the terms related to individual group \((i)\), a subproblem with group \((i)\) is obtained by

\[ \text{max } L^{(i)}, \text{ with } L^{(i)} = c_{\text{avg}} \sum_{v \in \text{out}} E[x_v^{i}(T)] - c_{\text{risk}} \sum_{v \in \text{out}} \text{var}_{\text{sem}}[x_v^{i}(T)] \]

\[ + \sum_{i=0}^{T-1} \sum_{v \in \text{out}} [E[x_v^{i}(t)] - c_{\text{risk}} \sum_{i=0}^{T-1} \text{var}_{\text{sem}}[x_v^{i}(t)] \]

\[ - \sum_{i=0}^{T-1} \sum_{v \in \text{out}} \sum_{i=0}^{T-1} [E[l q_e^{d(i)}(t)] - \psi^{-1}(c_e)] \, . \]
Then individual groups are coordinated by the iterative updating of Lagrangian multipliers $\lambda(t,e)$ by using the surrogate subgradient method (Zhao, Luh, and Wang, 1999).

### B. Simplification for Fire State Components

The subproblem of an individual group is maximizing a time-wise additive objective function, and it can be solved by stochastic dynamic programming. Here the subsystem state includes two components: crowd components $x(t)$ and fire components $s(t)$. Since the fire state components is uncontrollable and the $L^{(t)}$ only depends on crowd state component, it is possible to simplify fire state components to reduce the state space for computation (Bertsekas, 2005). Here one could view $s(t)$ as a disturbance rather than a state. The difference is that $s(t)$ can be observed before optimizing $u(t)$ while the common disturbance occurs after $u(t)$ is applied. In our problem, define

$$\tilde{L}^{(t)}(x^{(i)}(t)) = E_{\tilde{F}}\left\{ L^{(t)}(x^{(i)}(t), s^F(t)) \mid x^{(i)}(t) \right\}.$$  \hspace{1cm} (33)

Then the dynamic programming equation used is

$$\tilde{L}^{(t)}(x^{(i)}(t)) = g_i(x^{(i)}(t)) +
E_{\tilde{F}} \max_{u^{(i)}(t)} \left\{ \tilde{L}^{(t)}(x^{(i)}(t+1)) \mid x^{(i)}(t), s^F(t), u^{(i)}(t) \right\},$$  \hspace{1cm} (34)

where $g_i(x^{(i)}(t))$ denotes the stage-wise reward, and it exclusively depends on the crowd state component $x^{(i)}(t)$ since (22) and (25). Equation (34) implies that the reward-to-go can be computed with a significantly reduced state space.

### C. Rollout Scheme

To solve the subproblem in a computationally efficient manner, a rollout scheme is used with dynamic programming, and a heuristic policy is required to approximate the optimal decision and the reward-to-go (Bertsekas, 2005). In our evacuation problem the heuristic policy is using the route with minimum evacuation time, and the evacuation time is approximately calculated by

$$\text{Estimated evacuation time} = \frac{\text{Distance to an Exit}}{\text{Crowd Flow Rate}}$$  \hspace{1cm} (35)

The “distance to an exit” is measured by the locations of the group and the exit, and the “crowd flow rate” is calculated by the probabilistic graphical model with blocking concerned.

As a result, given current subsystem state $(x(t), s(t))$ deterministically, the optimal guidance after time $t$ can be approximated by the heuristic policy, and the guidance $u^{(t)}(t)$ can be optimized by

$$L^{(t)}(x^{(i)}(t), s^F(t)) = g_i(x^{(i)}(t)) +
\max_{u^{(i)}(t)} \left\{ \tilde{L}^{(t)}(x^{(i)}(t+1)) \mid x^{(i)}(t), s^F(t), u^{(i)}(t) \right\},$$  \hspace{1cm} (36)

where $\tilde{L}^{(t)}(x^{(i)}(t))$ denotes the approximate reward-to-go, and it is calculated by implementing the decisions derived from the heuristics after time $t$. Moreover, it is calculated with the reduced state space by simplifying the fire state components.

### VI. NUMERICAL RESULTS AND INSIGHTS

The above method has been implemented in Matlab and run on an Intel Core™2 Duo CPU with 2G memory for scenarios with fire as the emergency event. An example is tested where 50 people are evacuated in a structure of 6 rooms, a lobby area, 9 paths and 2 exits as depicted in Figure 5. The lobby area is further divided into 3 subareas for locating the people and fire status. The capacity of Exit 2 is small, with 10 persons per time unit, and capacity of Exit 1 is relatively large, with 30 persons per time unit. Two groups of people are to be evacuated: Group 1 consists of 30 people locating in Office3 (O3) initially; and Group 2 consists of 20 people locating in Lab2 (L2) initially. The fire propagates from Lab3 (L3). Figure 6 shows the initial state of the system.

The crowd guidance is optimized in 20 sequential time points and for each time point 5 time steps are look ahead to implement the approach as introduced in Section V.

Figure 6 shows the optimal paths at the initial time. These paths are obtained because the probabilistic graphical model foresee a potential blocking for the path from Office 3 to Exit 2. Thus Exit 2 is not used and people are guided to use Exit 1. At time 5, the guidance is updated as shown in Figure 7 due to fire propagating to the lounge area affecting the availability of the previous paths as shown in Figure 6. The average computation time is 2.948 seconds for decision-making at each time point.

If the nonlinear crowd behaviors are not considered while other conditions remain the same, the guidance at the initial time is shown in Figure 8 where Group 1 is guided to use Exit 2. However, this would lead to blocking as the crowd try to rush through the narrow passage of Exit 2, resulting in a much slower egress.
To compare the evacuation process optimally guided by our new model and by the traditional model, simulation is implemented based on our new model with blocking concerned. The average objective functions of 25 Monte Carlo simulation runs are summarized in Figure 9, showing the superiority of the new model and the method.

The new model can foresee the blocking at Exit 2 while the traditional model cannot. As a result, the guidance optimized by the traditional model use Exit 2 to evacuate Group 2 initially, but Group 2 cannot pass there as expected, and this delays the evacuation process.

In our example crowd guidance is optimized to prevent or mitigate potential blocking. Therefore, blocking issue is highlighted while the risk management is not mainly concerned. In other examples the risk management could be well concerned with uncertainty derived from the propagating fire or crowd trust on guidance. Here risks can be measured by the semi-variance terms in (22) and (25), representing the uncertainty of the evacuation process.

VII. CONCLUSION

This paper investigates the crowd guidance problem in building emergency evacuation, and two aspects of work are involved: modeling and optimization.

In the modeling aspect a probabilistic graphical model is developed to bridge the gap between microscopic models and macroscopic models and capture the characteristics of crowd behaviors in emergencies. Then a novel model is obtained to characterize the holistic evacuation process, including crowd movement and fire propagation in an egress network. A Markov decision problem is accordingly formulated.

Then the guidance is optimized by using stochastic dynamic programming with the rollout scheme within the Lagrangian relaxation framework. Compared with results obtained by the traditional model, the guidance obtained by the new model can help prevent or mitigate potential blockings, improve evacuation efficiency, and manage risks.

To evaluate and improve our existing work, the guidance obtained by our approach can be incorporated in crowd simulation, and this can be a part of our future work.

REFERENCES