Graphical Technique for Modeling Integrating (Non–Self-Regulating) Processes without Steady-State Process Data

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Model-fitting techniques for controller tuning that require the process to be initially at steady state cannot generally be used with integrating (non–self-regulating) processes. To address this issue, a graphical model-fitting technique is detailed and demonstrated for determination of first-order plus dead time integrating model parameters from integrating process response plots. The resulting model parameters can be used directly in a range of tuning correlations designed specifically for integrating processes. The advantage of this technique is that it requires only two periods of constant manipulated and disturbance variables sustained just long enough for the process variable to respond and establish a clear slope. This is an important benefit because integrating processes generally cannot be maintained at an initial steady state as required when using techniques published for self-regulating processes. The result is an industry-friendly method. The method is demonstrated for level control in a pumped tank, a classical challenge in industrial practice. Both a simulation and a bench-scale experimental system are used in the demonstration studies.

Keywords Controller tuning; Graphical model fitting; Integrating processes; Non–self-regulating processes

Introduction

Self-regulating processes seek a natural steady-state operating level in open loop if the manipulated and disturbance variables are held constant for a sufficient period of time. Integrating (or non–self-regulating) processes do not possess this attribute. The graphical model-fitting technique proposed in this work provides a practical method for the identification of integrating process models for use in controller tuning.

The graphical methods for the analysis of self-regulating system dynamics (e.g., Åström and Hägglund, 1995; Oggunnaie and Ray, 1994; Seborg et al., 2004; Smith

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and Corripio, 2006) typically determine the parameters of the first order plus dead time (FOPDT) model as shown in the Laplace and time domains by:

$$\frac{Y(s)}{U(s)} = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1}, \quad \tau_p \frac{dy(t)}{dt} + y(t) = K_P u(t - \theta_p) \quad (1)$$

Similarly, the graphical method presented in this work for the analysis of integrating system dynamics determines the parameters of the first order plus dead time integrating (FOPDT integrating) model as shown by:

$$\frac{Y(s)}{U(s)} = \frac{K^*_p e^{-\theta_p s}}{s}, \quad \frac{dy(t)}{dt} = K^*_P u(t - \theta_P) \quad (2)$$

The integrator gain, $K^*_p$, and dead time, $\theta_P$, obtained from a fit of Equation (2) to dynamic data from an integrating process may then be used to compute controller-tuning parameters using tuning correlations designed for such processes.

Figure 1 illustrates the differences between self-regulating and integrating process behavior in open loop (manual mode). For illustrative purposes, both processes begin with controller output (CO) and process variable (PV) at steady state. For each process, the CO steps away from this initial steady-state value and back again. In the self-regulating case, the PV returns to its initial steady-state value. In the integrating case, the PV steadies at a different steady-state value. If the CO had not returned to its initial value (not shown), the self-regulating process would have reached a new steady state while the integrating process would have continued to drift upward until reaching a physical limitation.

Integrating and self-regulating behavior may also be differentiated using closed-loop process response data, as shown by Figure 2. For the self-regulating process in Figure 2(a), the set point (SP) step leads to this new steady-state PV of 55% and a

![Figure 1. Open-loop behavior of self-regulating (a) and integrating (b) processes.](image-url)
new steady-state CO of 55%. For the integrating process in Figure 2(b), the SP step from 50% to 55% leads to a new steady-state PV of 55% with the CO returning to its initial steady-state value of 50%.

**Graphical FOPDT Integrating Fit**

The proposed graphical method of fitting a FOPDT integrating model to process data requires a data set that includes at least two constant values of controller output (CO), \( u_1 \) and \( u_2 \). Both \( u_1 \) and \( u_2 \) must be held constant long enough such that the slope of the measured process variable (PV) response may be visually identified in the data. There must be enough difference between \( u_1 \) and \( u_2 \) that each produces a distinguishably different slope in the PV response. In contrast to the graphical technique proposed in Åström and Hägglund (1995) for modeling integrating processes, this proposed technique does not require steady-state data. This is advantageous because it is often impractical to obtain steady-state data from non-self-regulating systems.

As shown in Figure 3, this technique is illustrated using simulated data for liquid level in a pumped tank, a classical example of an integrating process. To maintain the level (PV), the controller manipulates the tank discharge flow by adjusting a throttling valve (CO) at the discharge of a constant pressure pump.

Figure 4 depicts data from the pumped tank simulation with two values of sustained constant CO at 80% and 66%. Observe that the CO remains at 80% long enough to identify two points at (19 min, 4.2 m level) and (22 min, 3.5 m level) for slope computation. Similarly, the points (29 min, 3.1 m level) and (34 min, 3.6 m level) are identified in the response to the step change to 66% CO. Note also that the steps to 80% CO and 66% CO occur at 16.5 min and 24.0 min, respectively. However, the PV begins to respond to the steps at 17.2 min and 25.5 min, respectively.
The FOPDT integrating model of Equation 2 describes the process behavior at each value of constant controller output \( u_1 \) and \( u_2 \) as:

\[
slope_1 = \left. \frac{dy(t)}{dt} \right|_1 = K_p^+ u(t - \theta_P) = K_p^+ u_1
\]  

and

\[
slope_2 = \left. \frac{dy(t)}{dt} \right|_2 = K_p^+ u(t - \theta_P) = K_p^+ u_2
\]  

Figure 3. Pumped tank simulation.

Figure 4. Open-loop response of pumped tank simulation.
Subtracting Equation (3) from Equation (4) and solving for $K_P$ yields:

$$K_P = \frac{\frac{d \xi}{dt} \bigg|_{t_2} - \frac{d \xi}{dt} \bigg|_{t_1}}{u_2 - u_1} = \frac{\text{slope}_2 - \text{slope}_1}{u_2 - u_1}$$  

(5)

The dead time is approximated by averaging the difference in time between the step change in CO and the beginning of the PV response:

$$\theta_P = \frac{\theta_{P_1} + \theta_{P_2}}{2}$$  

(6)

Equations (7)-(10) apply the data obtained from Figure 4 to Equations (3)-(6), respectively:

$$\text{slope}_1 = \left. \frac{dy(t)}{dt} \right|_{t_1} = \frac{\Delta PV_1}{\Delta t_1} = \frac{3.5 - 4.2}{22 - 19} = -0.23 \text{ m/min}$$  

(7)

$$\text{slope}_2 = \left. \frac{dy(t)}{dt} \right|_{t_2} = \frac{\Delta PV_2}{\Delta t_2} = \frac{3.6 - 3.1}{34 - 29} = 0.10 \text{ m/min}$$  

(8)

$$K_P = \frac{\text{slope}_2 - \text{slope}_1}{u_2 - u_1} = \frac{0.10 - (-0.23)}{66 - 80} = -0.024 \text{ %CO · min}$$  

(9)

$$\theta_P = \frac{1}{2} \left[ (17.2 \text{ min} - 16.5 \text{ min}) + (25.5 \text{ min} - 24.0 \text{ min}) \right] = 1.1 \text{ min}$$  

(10)

The FOPDT integrating model is obtained by applying Equations (9) and (10) to the time domain form of Equation (2):

$$\frac{dy(t)}{dt} = \left[ -0.024 \text{ %CO · min} \right] u(t - 1.1 \text{ min})$$  

(11)

Figure 5 demonstrates the accuracy of the model shown in Equation (11) in fitting the process data shown in Figure 4. The R-squared value of 0.998 and sum of squared errors of 0.854 both indicate a good fit.

The graphical method is applicable to noisy data as long as the slopes are distinguishable and the process moves out of the noise band fast enough to accurately estimate the dead time. In fact, this manual technique allows the user to apply
engineering knowledge and judgment in estimating the slope and dead time through the noise.

Figure 6 demonstrates these techniques for this simulated process with significant measurement noise added. Figure 6 also demonstrates the applicability of this technique when the two slopes are in the same direction (both positive in this case). In this case, the dead time is estimated from the response to a single change in CO. The beginning of the response to the change in CO is indicated by the point where the PV slope noticeably changes. Despite the noise, the integrator gain and dead time estimates of \(-0.024\, \text{m}^{2}/\text{g} \cdot \text{min}\) and 1.0 min, respectively, agree with those estimated from Figure 4 and calculated in Equations (9) and (10).

**Controller Design Using FOPDT Integrating Fit**

The difference between integrating and self-regulating processes requires different controller design strategies. Specifically, they require different tuning correlations.

The proportional integral derivative (PID) controller is the most commonly used controller in industry. While the PID controller has various forms, this work focuses upon the ideal form shown by Equation (12):

\[
    u(t) = u_{\text{bias}} + K_C \left[ e(t) + \frac{1}{\tau_I} \int e(t) \, dt + \tau_D \frac{de(t)}{dt} \right] \tag{12}
\]

Table I lists internal model control (IMC) PID tuning correlations specifically derived for FOPDT Integrating processes (Chien and Fruehauf, 1990; Fruehauf et al., 1994; Rivera et al., 1986; Smith and Corripio, 2006). Tyreus and Luyben

<table>
<thead>
<tr>
<th>Table I. IMC tuning correlations for FOPDT integrating processes</th>
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<tbody>
<tr>
<td>Controller gain, (K_C)</td>
</tr>
<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>PI</td>
</tr>
<tr>
<td>PID</td>
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<td></td>
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</table>

... (cont.)
established a reasonable closed-loop time constant, $\tau_C$:

$$\tau_C = 0.4\sqrt{10}$$

(13)

Various tuning correlations exist for FOPDT integrating processes in addition to the IMC-PID tuning correlations presented in Table I (Arbogast et al., 2005; Chidambaram and Padma Sree, 2003; Lee et al., 2000; Luyben, 2001; O'Dwyer, 2006; Rice and Cooper, 2002; Tyreus and Luyben, 1992; Wang and Cai, 2002). Advanced structures such as cascade control, feed forward with feedback trim, Smith predictor, and dynamic matrix control (DMC) have special conditions or modifications that have to be taken into consideration to yield acceptable control when implemented on integrating processes (Chien et al., 2002; Dougherty and Cooper, 2003; Gupta, 1998; Hang et al., 2003; Kwak et al., 2001; Rice and Cooper, 2003, 2004). The proposed graphical model-fitting technique produces a FOPDT integrating model applicable to various controller tuning correlations, as demonstrated for the IMC-PI tuning correlations.

**Consequences of Plant-Model Mismatch**

Controller design strategies and popular tuning correlations proven for self-regulating processes may yield poor and even unstable performance if applied to integrating processes (Rice and Cooper, 2002). These consequences are illustrated through simulation of the second order plus dead time integrating (SOPDT integrating) process defined in Equation (14):

$$G_P(s) = \frac{K_p e^{-\theta_p s}}{s(\tau_p s + 1)} = \frac{0.004 e^{-0.3 s}}{s(15 s + 1)}$$

(14)

Figures 7(a) and 7(b) show simulated data for the process defined in Equation (14) fit to FOPDT integrating and FOPDT (self-regulating) models, respectively. Based upon the R-squared values of 0.977 and 0.997, respectively, both models appear to fit the process data well.

As shown by the apparent quality of the fit shown in Figure 7(b), practitioners accustomed to fitting dynamic data to FOPDT (self-regulating) models could easily mistake a non-self-regulating process for a self-regulating process. However, the very large process time constant, $\tau_p$, of 530 s indicates that the self-regulating model fit is suspect because only 200 s of data were collected.

Table II presents two cases. In Case 1, the parameters from the FOPDT integrating model shown in Figure 7(a) are used with the IMC-PI tuning correlation for integrating processes shown in Table I. In Case 2, the parameters from the FOPDT (self-regulating) model shown in Figure 7(b) are used with the IMC-PI tuning correlation for self-regulating processes.

Figure 8 illustrates the closed-loop responses using the tuning parameters listed in Table II. For the first set of SP steps from 50% to 55% and back to 50%, the controller is tuned using the proper correlation for an integrating process (Case 1). For each step, the process settles at the new SP in about 500 s. For the next SP step from 50% to 55%, the controller is tuned as self-regulating (Case 2). For this step, the process takes about 1500 s to reach its peak PV and longer to settle at the new SP. Therefore, mistakenly modeling and tuning the process as self-regulating
results in extremely poor controller performance compared to properly modeling and tuning the process as integrating.

Table II and Figure 8 illustrate the importance of understanding the integrating or self-regulating nature of the process in modeling the process and selecting the appropriate controller tuning correlation. However, understanding the nature of the process also informs one’s intuition about the effect of changing the controller gain, $K_C$.

Table II. Consequences of identifying an integrating process as a self-regulating process in controller tuning

<table>
<thead>
<tr>
<th>Case</th>
<th>Tuning correlation</th>
<th>Model parameters</th>
<th>Tuning parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Integrating IMC-PI</td>
<td>$K_p = 0.0039$</td>
<td>$K_C = 6.70$ CO/PV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tau_f = 117$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_p = 16$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Self-regulating IMC-PI</td>
<td>$K_p = 2.2$ PV/CO</td>
<td>$K_C = 0.44$ CO/PV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\tau_f = 530$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\theta_p = 16$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7. Fitting actual process with FOPDT integrating (a) and FOPDT (self-regulating) (b) models.
Figure 8. Closed-loop responses when modeling and tuning for integrating (left) and self-regulating (right) behavior.

Figure 9 demonstrates the effect of changing only $K_C$ on the closed-loop response of integrating and self-regulating processes. The integrating process in Figure 9 is the pumped tank simulation modeled by Equation (11). The self-regulating process in Figure 9 has the form of Equation (1) with the following parameters: $K_p = 1\%PV/\%CO$, $\tau_p = 10$ sec, and $\theta_p = 2$ s. The middle plots show the base case responses of the integrating and self-regulating processes for $\tau_C$ values of 3.5 min and 6.9 s, respectively. $K_C$ is a quarter of the base case value for the left-hand plots and four times the base case value for the right-hand plots.

For the self-regulating process shown at the bottom of Figure 9, decreasing $K_C$ produces a more sluggish but less oscillatory response, while increasing $K_C$ produces a more aggressive and oscillatory response. Therefore, a common convention among practitioners is to reduce oscillation by decreasing $K_C$.

For the integrating process shown at the top of Figure 9, the response grows more oscillatory when $K_C$ deviates, either increasing or decreasing, from the base case.

Figure 9. Effect of increasing controller gain on closed-loop response for integrating (top) and self-regulating (bottom) processes.
This is opposite to the convention (learned through experience with self-regulating processes) that decreasing $K_C$ reduces oscillation. For an integrating process, decreasing $K_C$ could actually increase the amount of oscillation. Therefore, conventions for self-regulating processes may not be applicable for non-self-regulating processes.

**Experimental Pumped Tank System**

This pumped tank experiment provides a physical complement to the pumped tank simulation discussed previously. The piping and instrument diagram (P&ID) for the experiment is illustrated in Figure 10. The storage tank and the main tank are comprised of a 1.5 in. (3.8 cm) internal diameter clear pipe with a height of 19 in. (48 cm) and have a total volume of 550 mL. Liquid is pumped from the storage tank to the main tank via two disturbance pumps labeled Pump101 and Pump102. The flow rates of liquid leaving the disturbance pumps are measured by turbine-based flow meters labeled FT101 and FT102 and controlled by flow controllers FC101 and FC102, respectively. A series of two gravity drained tanks (GDT series) with 1/8 in. (0.3 cm) drainage holes introduces an apparent delay into the disturbance measured by FT101. The level in the main tank is measured using a pressure transducer, LT103, located at the bottom of the tank. A level controller, LC103, manipulates the flow rate from the main tank to the storage tank through Pump103.

Figure 11 shows the response of the main tank level to open-loop manipulations in the LC103 controller output to Pump103 from 25% to 30% to 20% and back to 25%. The FC101 and FC102 controller outputs both remain constant at 50%. Note that the sustained constant values of LC103 CO at 30% and 20% allow use of the graphical model-fitting technique with Equations (3)-(6). The FOPDT integrating

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**Figure 10.** Piping and instrumentation diagram (P&ID) of the pumped tank bench-top experimental system.
model fitting this open-loop response is:
\[
G_p(s) = \frac{(-0.019 \text{ in/SEC}) e^{-4.0 \text{sec}s}}{s}
\]  

(15)

The PI controller is tuned by applying the model fit shown by Equation (15) to the IMC-PI tuning correlation in Table I. Figure 12 shows the closed-loop responses with two IMC-tuned PI controllers. The first response applies the closed-loop time constant, \( \tau_C \), of 12.6 s as specified by Equation (13). The second response applies a larger, more conservative \( \tau_C \) of 25 s. The response for a smaller, more aggressive \( \tau_C \) is not shown because the dramatic controller action becomes constrained by the limits of 0% and 100% and the response is therefore not representative.
Despite the large amount of noise in the level measurement, both controller tunings perform well in tracking the level SP changes. The controller tuned according to Equation (13) produces a response that reaches the new SP value faster than the more conservative controller does. The more conservative controller produces less overshoot of the final SP of 10 in. with slightly less variation in the controller output. These responses demonstrate the utility of $\tau_c$ in applying model-based IMC tuning correlations to achieve desired closed-loop performance.

Conclusions

The graphical model-fitting technique demonstrated may be applied to integrating process response plots like those shown by Figures 4 and 6 to determine FOPDT integrating model parameters. This technique requires two periods of constant manipulated and disturbance variables sustained long enough for the process variable to respond and establish a clear slope. As demonstrated, this technique does not require the measured process variable ever to be at steady state. It is important to apply the FOPDT integrating model parameters determined using this technique to tuning correlations specifically designed for integrating processes, like those shown in Table I.

Table II and Figure 8 demonstrate the consequences of fitting a self-regulating model to an integrating process. This may result in a very large reset time producing a more sluggish response. Figure 9 shows that an integrating process may oscillate increasingly with decreasing controller gain. This counters the convention learned through experience with self-regulating processes. Thus, identification and understanding of integrating processes is critical to achieve acceptable control performance.

Acknowledgment

The authors wish to acknowledge John McIlwain for his insight into applying graphical model-fitting techniques to integrating processes.

Nomenclature

- $e(t)$: set point error $(r(t) - y(t))$
- $K_C$: controller gain $(\text{in } [\frac{y}{u}])$
- $K_P$: process gain $(\text{in } [\frac{y}{u}])$
- $K_P^*$: integrator gain $(\text{in } [\frac{u}{a \cdot \text{time}}])$
- $r(t)$: set point (SP)
- $u(t), U(s)$: controller output (CO) in time (t) and Laplace (s) domains
- $y(t), Y(s)$: measured process variable (PV) in time (t) and Laplace (s) domains
- $\theta_P$: process dead time (in [time])
- $\tau_C$: closed-loop time constant (in [time])
- $\tau_D$: derivative time (in [time])
- $\tau_I$: reset time (in [time])
- $\tau_P$: process time constant (in [time])
References


