Exhaustive stability analysis in a consensus system with time delay and irregular topologies

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A consensus problem and its stability are studied for a group of agents with second-order dynamics and communication delays. The communication topologies are taken as irregular but always connected and undirected. The delays are assumed to be quasi-static and the same for all the interagent channels. A decentralised, PD-like control structure is proposed to create a consensus in the position and velocity of the agents. We present an interesting factorisation feature for the characteristic equation of the system which simplifies the stability analysis considerably from a prohibitively large dimensional problem to a manageable small scale. It facilitates a rare stability picture in the space of the control parameters and the delay, utilising a paradigm named cluster treatment of characteristic roots (CTCR). The influence of the individual factors on the absolute and relative stability of the system is studied. This leads to the introduction of two novel concepts: the most exigent eigenvalue, which refers to the one that defines the delay stability margin of the system, and the most critical eigenvalue, which is the one that dictates the consensus speed of the system. It is observed that the most exigent eigenvalue is not always the most critical, and this feature may be used as a design tool for the control logic. Case studies and simulations results are presented to verify these concepts.

Keywords: consensus; multi-agent systems; time delay systems; CTCR

1. Introduction

In recent years, distributed (decentralised) coordination of systems with multiple agents has been studied in many investigations. This interest is mainly due to the broad spectrum of applications of such systems in many areas, e.g. unmanned search and rescue operations. Among many aspects of the problem, the consensus generation is one of the most widely studied topics. The general objective is to drive all the agents of the group in a way such that they will reach a common value in some variable of interest that may or may not be related to the motion of the agents.

The work of Olfati Saber and Murray (2004) is one of the earlier studies published, which introduces the consensus problem for multi agent coordination. They focus on agents with first-order dynamics and considering different communication topologies among the agents. These topologies are all in the connected class. They propose a protocol for average consensus, which is the control objective that drives the agents to the average value of the group’s initial conditions. Under the simplifying features of the first-order governing dynamics, they also study the behaviour of their protocol when communication delays are present, keeping the topology fixed. Several other researchers (Lin et al. 2008; Lin and Yingmin 2009; Lin and Jia 2009; Sun and Wang 2009a, b; Peng and Yang 2009) have performed further extensions on this earlier work, proposing different protocols for zero consensus of agents that are driven by second-order dynamics. In these studies, the consensus is reached for the target position and the velocity of the agents at zero. They also include variations on the topologies and communication delays. For the analysis of the stability in the delay space, the previous works use conservative methods, based on linear matrix inequalities (LMIs). For instance, Lin et al. (2008), Lin and Jia (2009) and Lin and Yingmin (2009) present a maximum bound in the time delay for which the system remains stable. None of the earlier works, however, presents an exact and exhaustive determination of the stability boundaries of the system in the time delay space, which is the key contribution of the cluster treatment of characteristic roots (CTCR) paradigm (Olgac and Sipahi 2002, 2005, 2006; Ergenc et al. 2007) used in this article.

Recently the authors proposed and analysed a new control strategy for consensus over a group of agents with second-order dynamics which are also operating under a time-delayed communication structure (Cepeda-Gomez and Olgac 2010a, b, c, 2011). The highlight of those reports is the introduction of an

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