2.34 Small differences in gas pressures are commonly measured with a micromanometer of the type illustrated in Fig. P2.34. This device consists of two large reservoirs each having a cross-sectional area, $A_1$, which are filled with a liquid having a specific weight, $\gamma_1$, and connected by a U-tube of cross-sectional area, $A_2$, containing a liquid of specific weight, $\gamma_2$. When a differential gas pressure, $p_1 - p_2$, is applied a differential reading, $h$, develops. It is desired to have this reading sufficiently large (so that it can be easily read) for small pressure differentials. Determine the relationship between $h$ and $p_1 - p_2$ when the area ratio $A_1/A_2$ is small, and show that the differential reading, $h$, can be magnified by making the difference in specific weights, $\gamma_2 - \gamma_1$, small. Assume that initially (with $p_1 = p_2$) the fluid levels in the two reservoirs are equal.

![Diagram of micromanometer](image)

When a differential pressure, $p_1 - p_2$, is applied we assume that level in left reservoir drops by a distance, $\Delta h$, and right level rises by $\Delta h$. Thus, the manometer equation becomes

$$p_1 + \gamma_1 (h_1 + h - \Delta h) - \gamma_2 h - \gamma_1 (h_1 + \Delta h) = p_2$$

or

$$p_1 - p_2 = \gamma_2 h - \gamma_1 h + \gamma_1 (2 \Delta h)$$

(1)

Since the liquids in the manometer are incompressible,

$$\Delta h / A_2 = h / A_2 \text{ or } 2 \Delta h = h / A_2$$

and if $h / A_2$ is small then $2 \Delta h \ll h$ and last term in Eq.(1) can be neglected. Thus,

$$p_1 - p_2 = (\gamma_2 - \gamma_1) h$$

or

$$h = \frac{p_1 - p_2}{\gamma_2 - \gamma_1}$$

and large values of $h$ can be obtained for small pressure differentials if $\gamma_2 - \gamma_1$ is small.
2.36 Determine the elevation difference, $\Delta h$, between the water levels in the two open tanks shown in Fig. P2.36.

\[
p_1 - \gamma_{H_2O} h + (5G) \gamma_{H_2O} (0.4\text{m}) + \gamma_{H_2O} (h - 0.4\text{m}) + \gamma_{H_2O} (\Delta h) = p_2
\]

Since $p_1 = p_2 = 0$

\[
\Delta h = 0.4\text{m} - (0.9)(0.4\text{m}) = 0.040\text{m}
\]

2.37 For the configuration shown in Fig. P2.37 what must be the value of the specific weight of the unknown fluid? Express your answer in lb/ft$^3$.

Let $\gamma$ be specific weight of unknown fluid. Then,

\[
\gamma_{H_2O} \left[ \frac{(5.5-1.4)}{1.2} \text{ ft} \right] - \gamma \left[ \frac{(3.3-1.4)}{1.2} \text{ ft} \right] - \gamma_{H_2O} \left[ \frac{(4.9-3.3)}{1.2} \text{ ft} \right] = 0
\]

and

\[
\gamma = \frac{\gamma_{H_2O} \left[ (5.5-1.4) - (4.9-3.3) \right] \text{ in.}}{(3.3-1.4) \text{ in.}} = \left( 62.4 \frac{\text{lb}}{\text{ft}^3} \right) \left( \frac{4.1 - 1.6}{1.9} \right)
\]

\[
= 82.1 \frac{\text{lb}}{\text{ft}^3}
\]
2.45 Determine the new differential reading along the inclined leg of the mercury manometer of Fig. P2.45, if the pressure in pipe A is decreased 10 kPa and the pressure in pipe B remains unchanged. The fluid in A has a specific gravity of 0.9 and the fluid in B is water.

For the initial configuration:

\[ P_A + \gamma_A (0.1) + \gamma_Hg (0.05 \sin 30^\circ) - \gamma_{H2O} (0.08) = P_B \]  
\[ (1) \]

Where all lengths are in m. When \( P_A \) decreases left column moves up a distance \( a \), and right column moves down a distance \( a \), as shown in figure. For the final configuration:

\[ P_A' + \gamma_A (0.1 - a \sin 30^\circ) + \gamma_Hg (a \sin 30^\circ + 0.05 \sin 30^\circ + a) - \gamma_{H2O} (0.08 + a) = P_B \]
\[ (2) \]

Where \( P_A' \) is the new pressure in pipe A.

Subtract Eq. (2) from Eq. (1) to obtain:

\[ P_A - P_A' + \gamma_A (a \sin 30^\circ) - \gamma_Hg (a \sin 30^\circ + 1) + \gamma_{H2O} (a) = 0 \]

Thus,

\[ a = \frac{- (P_A - P_A')}{\gamma_A \sin 30^\circ - \gamma_Hg (a \sin 30^\circ + 1) + \gamma_{H2O}} \]

For \( P_A - P_A' = 10 \) kPa

\[ a = \frac{-10 \text{ kN/m}^2}{(0.9)(9.81 \text{ kN/m}^2)(0.5) - (133 \text{ kN/m}^2)(0.5 + 1) + 9.86 \text{ kN/m}^2} \]

\[ = 0.0540 \text{ m} \]

New differential reading, \( \Delta h \), measured along inclined tube is equal to

\[ \Delta h = \frac{a}{\sin 30^\circ} + 0.05 + a \]
\[ = \frac{0.0540 \text{ m}}{0.5} + 0.05 + a + 0.0540 \text{ m} = 0.212 \text{ m} \]

2-39
2.68 The massless, 4-ft-wide gate shown in Fig. P.2.68 pivots about the frictionless hinge O. It is held in place by the 2000 lb counterweight, W. Determine the water depth, \( h \).

\[
F_R = \gamma h_c A \quad \text{where} \quad h_c = \frac{h}{2}
\]

Thus,

\[
F_R = \gamma h_c \frac{h}{2} \left( h \times b \right)
\]

\[
= \gamma h_c \frac{h^2}{2} \left( \frac{4}{ft} \right)
\]

To locate \( R \),

\[
y_R = \frac{\sum M_o}{2} = \frac{1}{2} \left( \frac{4}{ft} \right) \left( \frac{h}{2} \right)^2 + \frac{h}{2}
\]

\[
= \frac{2}{3} h
\]

For equilibrium,

\[
\sum M_o = 0
\]

\[
F_R d = W (3 \text{ ft}) \quad \text{where} \quad d = h - y_R = \frac{h}{3}
\]

so that

\[
\frac{h}{3} = \frac{(2000 \text{ lb})(3 \text{ ft})}{\gamma h_c \frac{h^2}{2} \left( \frac{4}{ft} \right)}
\]

Thus,

\[
h = \frac{3(2000 \text{ lb})(3 \text{ ft})}{(62.4 \text{ lb/ft}^3) \frac{h^2}{2} \left( \frac{4}{ft} \right)}
\]

\[
h = 5.24 \text{ ft}
\]
2.70 An open tank has a vertical partition and on one side contains gasoline with a density \( \rho = 700 \text{ kg/m}^3 \) at a depth of 4 m, as shown in Fig. P2.70. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, \( h \), will the gate start to open?

\[
F_{R_g} = \gamma_g h_g A_g
\]
where \( \gamma_g \) refers to gasoline.

\[
F_{R_g} = \left(700 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left(2 \text{ m}\right)\left(4 \text{ m} \times 2 \text{ m}\right)
= 110 \times 10^3 \text{ N} = 110 \text{ kN}
\]

\[
F_{R_w} = \gamma_w h_w A_w
\]
where \( \gamma_w \) refers to water.

\[
F_{R_w} = \left(9.80 \times 10^3 \text{ N/m}^3\right)\left(h_2/2\right)\left(2 \text{ m} \times h_2\right)
\]
where \( h_2 \) is depth of water.

\[
F_{R_w} = (9.80 \times 10^3) h_2^2
\]

For equilibrium,

\[
\sum M_H = 0
\]
so that

\[
F_{R_w} l_w = F_{R_g} l_g \quad \text{with} \quad l_w = \frac{h}{3} \quad \text{and} \quad l_g = \frac{4}{3} \text{ m}
\]

Thus,

\[
(9.80 \times 10^3)(h_2^2)(\frac{h}{3}) = (110 \times 10^3 \text{ N})(\frac{4}{3} \text{ m})
\]
and

\[
h = \frac{3.55 \text{ m}}{}
\]
which is the limiting value for \( h \).