Introduction to Discrete Systems

CSE 2500 - Fall 2014

In-class examination (Tuesday September 30th, 2014) – 20% of final grade

First Name: ___________________________      Last Name: _________________________

PeopleSoft ID: ________________________________________________________________

Exam duration: 75 minutes

This examination has 3 questions. Answer all 3 questions. This examination has 7 pages. (The 8th page is blank for your worksheet.)

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<th>Part a</th>
<th>Part b</th>
<th>Part c</th>
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Question 1

Part a (10 points)
Which of the following sets are equal? (R = the set of all real numbers. Z = the set of all integers. Z⁺ = the set of all integers that are > 0. )

A = \{0, 1, 2, 3\}
B = \{x ∈ R \mid -1 ≤ x < 4\}
C = \{x ∈ R \mid -1 < x < 4\}
D = \{x ∈ Z \mid -1 < x < 4\}
E = \{x ∈ Z⁺ \mid -1 < x < 4\}

Part b (10 points)
Write the converse, inverse, and contrapositive of the statement “ p ∧ ¬r → q ”. Please simply your statements.
Part c (10 points)

Use a truth table to determine whether the following argument form is valid or invalid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. (Hint: an argument form is valid if and only if the conclusion is true for every critical line.)

\[
p \rightarrow q \lor \sim r \\
q \rightarrow p \land r \\
\therefore p \rightarrow r
\]
Question 2

Part a (10 points)
Express the statement “If a person is male and is a parent, then this person is someone’s father” as a logical expression involving predicates, quantifiers with a domain consisting of all people, and logical connectives.

Part b (10 points)
Write the negation for the above statement in part a. Please give both the informal statement as well as the formal statement with predicates and quantifiers.
Part c (10 points)

Use the laws of logical equivalence in Theorem 2.1.1 to verify the following logical equivalence. Supply a reason for each step.

\[ \sim (p \lor \sim q) \lor (p \land q) \equiv q \]

**Hint:** The following logical equivalence laws are used:

- Communicative laws: \( p \land q \equiv q \land p \), \( p \lor q \equiv q \lor p \)
- De Morgan’s laws: \( \sim (p \lor q) \equiv \sim p \land \sim q \)
- Distributive laws: \( p \land (q \lor r) \equiv (p \land q) \lor (p \land r) \)
- Negation laws: \( p \lor \sim p \equiv t \)
- Identity laws: \( p \land t \equiv p \)
**Question 3**

**Part a (10 points)**
Is the following argument valid or invalid? Justify your answer.

No good car is cheap.
A Ferrari is not cheap.
Therefore, a Ferrari is a good car.

**Part b (10 points)**
Is the following argument valid or invalid? Justify your answer.

Every nonzero real number has a reciprocal.
The number 8 is a nonzero real number.
Therefore, the number 8 has a reciprocal.

**Part c (10 points)**
Write a new statement by interchanging the quantifiers $\forall$ and $\exists$, and state which statement is true: the given statement, the version with interchanged quantifiers, neither, or both.

$\forall x \in R, \exists y \in R,$ such that $x < y.$
Part d (10 points)

On an island, there are only two types of people: knights who always tell the truth and knaves who always lie. You visit the island and meet two natives who speak to you as follows:

A says: B is a knight.
B says: A and I are of opposite type.

What are A and B?

Hint: Let us reason as follows: (please try to fill in the blank underlined lines. Each blank line is created 1 point.)

Suppose A is a knight

\[ \therefore \text{What A says is true} \quad \text{(by definition of knight)} \]

\[ \text{What A said} \]

\[ \therefore \text{What B says is true} \quad \text{(by definition of knight)} \]

\[ \text{What B said} \]

The supposition is false \quad \text{(by the contradiction rule)}

\[ \text{Negation of supposition} \]

\[ \text{Negation of what A said} \]

\[ \text{Negation of what B said} \]

The answer is that A is a _____________________, and B is a _____________________.