(d) Derive an expression for the steady-state temperature $T(x, \infty) = T_p$, leaving your result in terms of plate parameters $(M, c_p)$, thermal conditions $(T_u, T_m, h)$, the surface temperature $(T(L, t))$, and the heating time $t_p$.

**Lumped Capacitance Method**

5.5 Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which $T_w = 325$ K and $h = 20$ W/m$^2$·K. Assuming the properties of the steel to be $k = 40$ W/m·K, $\rho = 7800$ kg/m$^3$, and $c = 600$ J/kg·K, estimate the time required for the cooling process.

5.6 Consider the steel balls of Problem 5.5, except now the air temperature increases with time as $T_u(t) = 325 + 0.1875t$ at where $a = 0.1875$ K/h.

(a) Sketch the ball temperature versus time for $0 \leq t \leq 1$ h. Also show the ambient temperature, $T_w$, in your graph. Explain special features of the ball temperature behavior.

(b) Find an expression for the ball temperature as a function of time, $T(t)$, and plot the ball temperature for $0 \leq t \leq 1$ h. Was your sketch correct?

5.7 The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature-time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in diameter, is at 66°C before it is inserted into an airstream having a temperature of 27°C. A thermocouple on the outer surface of the sphere indicates 55°C 69 s after the sphere is inserted in the airstream. Assume, and then justify, that the sphere behaves as a spacewise isothermal object and calculate the heat transfer coefficient.

5.8 A solid steel sphere (AISI 1010), 300 mm in diameter, is coated with a dielectric material layer of thickness 2 mm and thermal conductivity 0.04 W/m·K. The coated sphere is initially at a uniform temperature of 500°C and is suddenly quenched in a large oil bath for which $T_w = 100$°C and $h = 3300$ W/m$^2$·K. Estimate the time required for the coated sphere temperature to reach 140°C. Hint: Neglect the effect of energy storage in the dielectric material, since its thermal capacitance $(\rho c V)$ is small compared to that of the steel sphere.

5.9 The base plate of an iron has a thickness of $L = 7$ mm and is made from an aluminum alloy $(\rho = 2800$ kg/m$^3$, $c = 900$ J/kg·K, $k = 180$ W/m·K, $s = 0.80)$. An electric resistance heater is attached to the inner surface of the plate, while the outer surface is exposed to ambient air and large surroundings at $T_w = T_m = 25$°C. The areas of both the inner and outer surfaces are $A_i = 0.040$ m$^2$. If an approximately uniform heat flux of $q'' = 1.25 \times 10^4$ W/m$^2$ is applied to the inner surface of the base plate and the convection coefficient at the outer surface is $h = 10$ W/m$^2$·K, estimate the time required for the plate to reach a temperature of 135°C. Hint: Numerical integration is suggested in order to solve the problem.

5.10 Carbon steel (AISI 1010) shafts of 0.1-m diameter are heat treated in a gas-fired furnace whose gases are at 1200 K and provide a convection coefficient of 100 W/m$^2$·K. If the shafts enter the furnace at 300 K, how long must they remain in the furnace to achieve a centerline temperature of 800 K?

5.11 A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and encloses alternating layers of the storage material and the flow passage.

Each layer of the storage material is an aluminum slab of width $W = 0.05$ m, which is at an initial temperature of 25°C. Consider conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and the convection coefficient assumed to have constant values of $T_w = 600$°C and $h = 100$ W/m$^2$·K throughout the channel. How long will it take to achieve 75% of the maximum possible
5.20 An electronic device, such as a power transistor mounted on a finned heat sink, can be modeled as a spatially isothermal object with internal heat generation and an external convection resistance.

(a) Consider such a system of mass $M$, specific heat $c$, and surface area $A_s$, which is initially in equilibrium with the environment at $T_w$. Suddenly, the electronic device is energized such that a constant heat generation $E_g$ (W) occurs. Show that the temperature response of the device is

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{RC}\right)$$

where $\theta = T - T(\infty)$ and $T(\infty)$ is the steady-state temperature corresponding to $t \to \infty$; $\theta_i = T_i - T(\infty)$; $T_i$ is initial temperature of device; $R$ is thermal resistance $1/hA_s$; and $C$ is thermal capacitance $MC$.

(b) An electronic device, which generates 60 W of heat, is mounted on an aluminum heat sink weighing 0.31 kg and reaches a temperature of 100°C in ambient air at 20°C under steady-state conditions. If the device is initially at 20°C, what temperature will it reach 5 min after the power is switched on?

5.21 Before being injected into a furnace, pulverized coal is preheated by passing it through a cylindrical tube whose surface is maintained at $T_{surf} = 1000°C$. The coal pellets are suspended in an airflow and are known to move with a speed of 3 m/s. If the pellets may be approximated as spheres of 1-mm diameter and it may be assumed that they are heated by radiation transfer from the tube surface, how long must the tube be to heat coal entering at 25°C to a temperature of 600°C? Is the use of the lumped capacitance method justified?

5.22 A metal sphere of diameter $D$, which is at a uniform temperature $T_0$, is suddenly removed from a furnace and suspended from a fine wire in a large room with air at a uniform temperature $T_w$ and the surrounding walls at a temperature $T_{surf}$.

(a) Neglecting heat transfer by radiation, obtain an expression for the time required to cool the sphere to some temperature $T$.

(b) Neglecting heat transfer by convection, obtain an expression for the time required to cool the sphere to the temperature $T$.

(c) How would you go about determining the time required for the sphere to cool to the temperature $T$ if both convection and radiation are of the same order of magnitude?

5.23 As permanent space stations increase in size, there is an attendant increase in the amount of electrical power they dissipate. To keep station compartment temperatures from exceeding prescribed limits, it is necessary to transfer the dissipated heat to space. A novel heat rejection scheme that has been proposed for this purpose is termed a Liquid Droplet Radiator (LDR). The heat is first transferred to a high vacuum oil, which is then injected into outer space as a stream of small droplets. The stream is allowed to traverse a distance $L$, over which it cools by radiating energy to outer space at absolute zero temperature. The droplets are then collected and routed back to the space station.

Consider an anodized aluminum sphere ($\rho = 0.75$) 50 mm in diameter, which is at an initial temperature of $T_i = 800 K$. Both the air and surroundings are at 300 K, and the convection coefficient is 10 W/m²·K. For the conditions of parts (a), (b), and (c), determine the time required for the sphere to cool to 400 K. Plot the corresponding temperature histories. Repeat the calculations for a polished aluminum sphere ($\rho = 0.1$).

5.24 In a material processing experiment conducted aboard the space shuttle, a coated niobium sphere of 10-mm diameter is removed from a furnace at 900°C and cooled to a temperature of 300°C. Although properties of the niobium vary over this temperature range, constant values may be assumed to a reasonable approximation, with $\rho = 8600$ kg/m³, $c = 290$ J/kg · K, and $k = 63$ W/m · K.
(a) If cooling is implemented in a large evacuated chamber whose walls are at 25°C, determine the time required to reach the final temperature if the coating is polished and has an emissivity of $\varepsilon = 0.1$. How long would it take if the coating is oxidized and $\varepsilon = 0.6$?

(b) To reduce the time required for cooling, consideration is given to immersion of the sphere in an inert gas stream for which $T_w = 25°C$ and $h = 200 \frac{W}{m^2 \cdot K}$. Neglecting radiation, what is the time required for cooling?

(c) Considering the effect of both radiation and convection, what is the time required for cooling if $h = 200 \frac{W}{m^2 \cdot K}$ and $\varepsilon = 0.6$? Explore the effect on the cooling time of independently varying $h$ and $\varepsilon$.

5.25 Plasma spray-coating processes are often used to provide surface protection for materials exposed to hostile environments, which induce degradation through factors such as wear, corrosion, or outright thermal failure. Ceramic coatings are commonly used for this purpose. By injecting ceramic powder through the nozzle (anode) of a plasma torch, the particles are entrained by the plasma jet, within which they are then accelerated and heated.

During their time-in-flight, the ceramic particles must be heated to their melting point and experience complete conversion to the liquid state. The coating is formed as the molten droplets impinge (splat) on the substrate material and experience rapid solidification. Consider conditions for which spherical alumina ($Al_2O_3$) particles of diameter $D_p = 50 \mu m$, density $\rho_p = 3970 \frac{kg}{m^3}$, thermal conductivity $k_p = 10.5 \frac{W}{m \cdot K}$, and specific heat $c_p = 1560 \frac{J}{kg \cdot K}$ are injected into an arc plasma, which is at $T_w = 10,000 \text{ K}$ and provides a coefficient of $h = 30,000 \frac{W}{m^2 \cdot K}$ for convective heating of the particles. The melting point and latent heat of fusion of alumina are $T_{mp} = 2318 \text{ K}$ and $h_f = 3577 \frac{kJ}{kg}$, respectively.

(a) Neglecting radiation, obtain an expression for the time-in-flight, $t_{f-r}$, required to heat a particle from its initial temperature $T_i$ to its melting point $T_{mp}$ and, once at the melting point, for the particle to experience complete melting. Evaluate $t_{f-r}$ for $T_i = 300 \text{ K}$ and the prescribed heating conditions.

(b) Assuming alumina to have an emissivity of $\varepsilon_p = 0.4$ and the particles to exchange radiation with large surroundings at $T_{sr} = 300 \text{ K}$, assess the validity of neglecting radiation.

5.26 Thin film coatings characterized by high resistance to abrasion and fracture may be formed by using microscale composite particles in a plasma spraying process. A spherical particle typically consists of a ceramic core, such as tungsten carbide (WC), and a metallic shell, such as cobalt (Co). The ceramic provides the thin film coating with its desired hardness at elevated temperatures, while the metal serves to coalesce the particles on the coated surface and to inhibit crack formation. In the plasma spraying process, the particles are injected into a plasma gas jet that heats them to a temperature above the melting point of the metallic casing and melts the casing before the particles impact the surface.

Consider spherical particles comprised of a WC core of diameter $D_i = 16 \mu m$, which is encased in a Co shell of outer diameter $D_o = 20 \mu m$. If the particles flow in a plasma gas at $T_w = 10,000 \text{ K}$ and the coefficient associated with convection from the gas to the particles is $h = 20,000 \frac{W}{m^2 \cdot K}$, how long does it take to heat the particles from an initial temperature of $T_i = 300 \text{ K}$ to the melting point of cobalt, $T_{mp} = 1770 \text{ K}$? The density and specific heat of WC (the core of the particle) are $\rho_{wc} = 16,000 \frac{kg}{m^3}$ and $c_{pw} = 300 \frac{J}{kg \cdot K}$, while the corresponding values for Co (the outer shell) are $\rho_{co} = 8900 \frac{kg}{m^3}$ and $c_{pc} = 750 \frac{J}{kg \cdot K}$. Once having reached the melting point, how much additional time is required to completely melt the cobalt if its latent heat of fusion is $h_f = 2.59 \times 10^7 \frac{J}{kg}$? You may use the lumped capacitance method of analysis and neglect radiation exchange between the particle and its surroundings.

5.27 A chip that is of length $L = 5 \text{ mm}$ on a side and thickness $t = 1 \text{ mm}$ is encased in a ceramic substrate, and its exposed surface is convectively cooled by a dielectric liquid for which $h = 150 \frac{W}{m^2 \cdot K}$ and $T_w = 20°C$. 

\[ \text{(a) } \text{Neglecting radiation, obtain an expression for the} \]
\[ \text{time-in-flight, } t_{f-r}, \text{ required to heat a particle from its initial} \]
\[ \text{temperature } T_i \text{ to its melting point } T_{mp} \text{ and, once at the melting} \]
\[ \text{point, for the particle to experience complete melting. Evaluate } t_{f-r} \text{ for } \]
\[ T_i = 300 \text{ K} \text{ and the prescribed heating conditions.} \]
\[ \text{(b) Assuming alumina to have an emissivity of } \varepsilon_p = 0.4 \text{ and the} \]
\[ \text{particles to exchange radiation with large surroundings at } T_{sr} = 300 \text{ K}, \text{assess the validity of neglecting radiation.} \]
\[ \text{5.26 Thin film coatings characterized by high resistance to} \]
\[ \text{abrasion and fracture may be formed by using microscale composite particles in a} \]
\[ \text{plasma spraying process. A spherical particle typically consists of a ceramic} \]
\[ \text{core, such as tungsten carbide (WC), and a metallic shell, such as cobalt (Co). The} \]
\[ \text{ceramic provides the thin film coating with its desired hardness at elevated} \]
\[ \text{temperatures, while the metal serves to coalesce the particles on the coated} \]
\[ \text{surface and to inhibit crack formation. In the plasma spraying process, the} \]
\[ \text{particles are injected into a plasma gas jet that heats them to a temperature} \]
\[ \text{above the melting point of the metallic casing and melts the casing before the} \]
\[ \text{particles impact the surface.} \]
\[ \text{Consider spherical particles comprised of a WC core of diameter } D_i = 16 \mu m, \text{which is encased in a Co} \]
\[ \text{shell of outer diameter } D_o = 20 \mu m. \text{ If the particles flow in a plasma gas at } T_w = 10,000 \text{ K} \text{ and the} \]
\[ \text{coefficient associated with convection from the gas to the particles is } h = 20,000 \frac{W}{m^2 \cdot K}, \text{ how long does it take} \]
\[ \text{to heat the particles from an initial temperature of } T_i = 300 \text{ K} \text{ to the melting point of cobalt, } T_{mp} = 1770 \text{ K}? \text{ The density and specific heat of WC (the core of the} \]
\[ \text{particle) are } \rho_{wc} = 16,000 \frac{kg}{m^3} \text{ and } c_{pw} = 300 \frac{J}{kg \cdot K}, \text{ while the corresponding values for Co (the outer} \]
\[ \text{shell) are } \rho_{co} = 8900 \frac{kg}{m^3} \text{ and } c_{pc} = 750 \frac{J}{kg \cdot K}. \text{ Once having} \]
\[ \text{reached the melting point, how much additional time is required to completely melt the cobalt if its} \]
\[ \text{latent heat of fusion is } h_f = 2.59 \times 10^7 \frac{J}{kg}? \text{ You may use the lumped capacitance method of analysis and neglect} \]
\[ \text{radiation exchange between the particle and its surroundings.} \]
\[ \text{5.27 A chip that is of length } L = 5 \text{ mm on a side and thickness } t = 1 \text{ mm is encased in a ceramic} \]
\[ \text{substrate, and its exposed surface is convectively cooled by a dielectric liquid for which } h = 150 \frac{W}{m^2 \cdot K} \text{ and } T_w = 20°C.} \]
of the beef are the same as ice, and assume 3% of the oven power \((P = 1 \text{ kW total})\) is absorbed in the food.

(b) After all the ice is converted to liquid, determine how long it will take to heat the beef to \(T_f = 80^\circ\text{C}\) if 95% of the oven power is absorbed in the food. Assume the properties of the beef are the same as liquid water.

(c) When thawing food in microwave ovens, one may observe that some of the food may still be frozen while other parts of the food are overcooked. Explain why this occurs. Explain why most microwave ovens have thaw cycles that are associated with very low oven powers.

One-Dimensional Conduction: The Plane Wall

5.34 Consider the series solution, Equation 5.39, for the plane wall with convection. Calculate midplane \((x = 0)\) and surface \((x = 1)\) temperatures \(T_0\) for \(P_0 = 0.1\) and 1, using \(Bi = 0.1, \ 1, \text{ and } 10\). Consider only the first four eigenvalues. Based on these results discuss the validity of the approximate solutions, Equations 5.40 and 5.41.

5.35 Consider the one-dimensional wall shown in the sketch, which is initially at a uniform temperature \(T_0\) and is suddenly subjected to the convection boundary condition with a fluid at \(T_{\infty}\).

![Sketch of a one-dimensional wall with convection boundary condition.](image)

For a particular wall, case 1, the temperature at \(x = L_1\) after \(t_1 = 100 \text{ s}\) is \(T_1(L_1, t_1) = 315^\circ\text{C}\). Another wall, case 2, has different thickness and thermal conditions as shown below.

<table>
<thead>
<tr>
<th>(L) (m)</th>
<th>(\alpha) ((\text{m}^2/\text{s}))</th>
<th>(k) ((\text{W/m} \cdot \text{K}))</th>
<th>(T_0) ((^\circ\text{C}))</th>
<th>(T_{\infty}) ((^\circ\text{C}))</th>
<th>(h) ((\text{W/m}^2 \cdot \text{K}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>15 \times 10^{-6}</td>
<td>50</td>
<td>300</td>
<td>400</td>
<td>100</td>
</tr>
<tr>
<td>0.40</td>
<td>25 \times 10^{-6}</td>
<td>100</td>
<td>30</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>

How long will it take for the second wall to reach \(28.5^\circ\text{C}\) at the position \(x = L_2\)? Use as the basis for analysis, the dimensionless functional dependence for the transient temperature distribution expressed in Equation 5.38.

5.36 Referring to the semiconductor processing tool of Problem 5.13, it is desired at some point in the manufacturing cycle to cool the chuck, which is made of aluminum alloy 2024. The proposed cooling scheme passes air at \(20^\circ\text{C}\) between the air-supply head and the chuck surface.

![Diagram of a semiconductor processing tool.](image)

(a) If the chuck is initially at a uniform temperature of \(100^\circ\text{C}\), calculate the time required for its lower surface to reach \(25^\circ\text{C}\), assuming a uniform convection coefficient of \(30 \text{ W/m}^2 \cdot \text{K}\) at the head–chuck interface.

(b) Generate a plot of the time-to-cool as a function of the convection coefficient for the range \(10 \leq h \leq 2000 \text{ W/m}^2 \cdot \text{K}\). If the lower limit represents a free convection condition without any head present, comment on the effectiveness of the head design as a method for cooling the chuck.

5.37 Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheating stage for a 100-mm-thick steel plate \((\rho = 7830 \text{ kg/m}^3, c = 550 \text{ J/kg} \cdot \text{K}, k = 48 \text{ W/m} \cdot \text{K})\), which is initially at a uniform temperature of \(T_0 = 200^\circ\text{C}\) and is to be heated to a minimum temperature of \(550^\circ\text{C}\). Heating is effected in a gas-fired furnace, where products of combustion at \(T_m = 800^\circ\text{C}\) maintain a convection coefficient of \(h = 250 \text{ W/m}^2 \cdot \text{K}\) on both surfaces of the plate. How long should the plate be left in the furnace?

5.38 Consider the heavily insulated pipe of Example 5.4, which is suddenly subjected to the flow of hot oil. Use the Transient Conduction, Plane Wall model of IHT to obtain the following solutions.

(a) Calculate the temperature of the inner and outer surfaces of the pipe, the heat flux at the inner surface, and the energy transferred to the wall after 8 min. Compare your results with those obtained in the example.

(b) At what time will the outer surface temperature of the pipe, \(T(0, t)\), reach \(25^\circ\text{C}\)?
To accelerate the heating process, it is recommended that the steam flow be made sufficiently vigorous to maintain the tire surfaces at 200°C throughout the process. Compute and plot the midplane and surface temperatures for this case, as well as for the conditions of part (a).

Copper-coated, epoxy-filled fiberglass circuit boards are treated by heating a stack of them under high pressure as shown in the sketch. The purpose of the pressing-heating operation is to cure the epoxy that bonds the fiberglass sheets, imparting stiffness to the boards. The stack, referred to as a book, is comprised of 10 boards and 11 pressing plates, which prevent epoxy from flowing between the boards and impart a smooth finish to the cured boards. In order to perform simplified thermal analyses, it is reasonable to approximate the book as having an effective thermal conductivity (k) and an effective thermal capacitance (ρc). Calculate the effective properties if each of the boards and plates has a thickness of 2.36 mm and the following thermophysical properties: board (b) ρb = 1000 kg/m³, cb = 1500 J/kg · K, kb = 0.30 W/m · K; plate (p) ρp = 8000 kg/m³, cp = 480 J/kg · K, kp = 12 W/m · K.

Circuit boards are treated by heating a stack of them under high pressure as illustrated in Problem 5.45. The platen at the top and bottom of the stack are maintained at a uniform temperature by a circulating fluid. The purpose of the pressing-heating operation is to cure the epoxy, which bonds the fiberglass sheets, and impart stiffness to the boards. The cure condition is achieved when the epoxy has been maintained at or above 170°C for at least 5 min. The effective thermophysical properties of the stack or book (boards and metal pressing plates) are k = 0.613 W/m · K and ρc = 2.73 × 10⁶ J/m³ · K.

5.47 A plastic coating is applied to wood panels by first depositing molten polymer on a panel and then cooling the surface of the polymer by subjecting it to air flow at 25°C. As first approximations, the heat of reaction associated with solidification of the polymer may be neglected and the polymer/wood interface may be assumed to be adiabatic.

If the thickness of the coating is L = 2 mm and it has an initial uniform temperature of T1 = 200°C, how long will it take for the surface to achieve a safe-to-touch temperature of 42°C if the convection coefficient is h = 200 W/m² · K? What is the corresponding value of the interface temperature? The thermal conductivity and diffusivity of the plastic are k = 0.25 W/m · K and α = 1.20 × 10⁻⁷ m²/s, respectively.

5.46 One-Dimensional Conduction: The Long Cylinder

5.48 A long rod of 60-mm diameter and thermophysical properties ρ = 8000 kg/m³, c = 500 J/kg · K, and k = 30 W/m · K is initially at a uniform temperature and is heated in a forced convection furnace maintained at 750 K. The convection coefficient is estimated to be 1000 W/m² · K.

(a) What is the centerline temperature of the rod when the surface temperature is 550 K?

(b) In a heat-treating process, the centerline temperature of the rod must be increased from T1 = 300 K to T = 500 K. Compute and plot the centerline temperature histories for h = 100, 500, and 1000 W/m² · K. In each case the calculation may be terminated when T = 500 K.

5.49 A long cylinder of 30-mm diameter, initially at a uniform temperature of 1000 K, is suddenly quenched in a large, constant-temperature oil bath at 350 K. The cylinder properties are k = 1.7 W/m · K, c = 1600 J/kg · K, and ρ = 400 kg/m³, while the convection coefficient is 50 W/m² · K.
\( n \) Problems

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\[ n \text{ Problems} \]

...that the walls must not exceed 325°C and 25°C, respectively, after 30 min of heating, will the requirements be met?

5.80 It is well known that, although two materials are at the same temperature, one may feel cooler to the touch than the other. Consider thick plates of copper and glass, each at an initial temperature of 300 K. Assuming your finger to be at an initial temperature of 310 K and having thermophysical properties of \( \rho = 1000 \text{ kg/m}^3, c = 4180 \text{ J/kg} \cdot \text{K}, \) and \( k = 0.625 \text{ W/m} \cdot \text{K} \), determine whether the copper or the glass will feel cooler to the touch.

5.81 Two stainless steel plates (\( \rho = 8000 \text{ kg/m}^3, c = 500 \text{ J/kg} \cdot \text{K}, k = 15 \text{ W/m} \cdot \text{K} \)), each 20 mm thick and insulated on one surface, are initially at 400 and 300 K when they are pressed together at their uninsulated surfaces. What is the temperature of the insulated surface of the hot plate after 1 min has elapsed?

5.82 Special coatings are often formed by depositing thin layers of a molten material on a solid substrate. Solidification begins at the substrate surface and proceeds until the thickness \( S \) of the solid layer becomes equal to the thickness \( \delta \) of the deposit.

\[ \text{Deposit, } \rho, h_f, \alpha_s; \text{ Substrate, } k_s, \alpha_s \]

(a) Consider conditions for which molten material at its fusion temperature \( T_f \) is deposited on a large substrate that is at an initial uniform temperature \( T_i \). With \( S = 0 \) and \( t = 0 \), develop an expression for estimating the time \( t_f \) required to completely solidify the deposit if it remains at \( T_f \) throughout the solidification process. Express your result in terms of the substrate thermal conductivity and thermal diffusivity (\( k_s, \alpha_s \)), the density and latent heat of fusion of the substrate (\( \rho, h_f \)), the deposit thickness \( \delta \), and the relevant temperatures (\( T_f, T_i \)).

(b) The plasma spray deposition process of Problem 5.25 is used to apply a thin (\( \delta = 2 \text{ mm} \)) alumina coating on a thick tungsten substrate. The substrate has a uniform initial temperature of \( T_i = 300 \text{ K} \), and its thermal conductivity and thermal diffusivity may be approximated as \( k_s = 120 \text{ W/m} \cdot \text{K} \) and \( \alpha_s = 4.0 \times 10^{-5} \text{ m}^2/\text{s} \), respectively. The density and latent heat of fusion of the alumina are \( \rho = 3970 \text{ kg/m}^3 \) and \( h_f = 3577 \text{ kJ/kg} \), respectively, and the alumina solidifies at its fusion temperature (\( T_f = 2318 \text{ K} \)). Assuming that the molten layer is instantaneously deposited on the substrate, estimate the time required to reach solidify.

5.83 When a molten metal is cast in a mold that is a poor conductor, the dominant resistance to heat flow is within the mold wall. Consider conditions for which a liquid metal is solidifying in a thick-walled mold of thermal conductivity \( k_m \) and thermal diffusivity \( \alpha_m \). The density and latent heat of fusion of the metal are designated as \( \rho \) and \( h_f \), respectively, and in both its molten and solid states, the thermal conductivity of the metal is much greater than that of the mold.

Just before the start of solidification (\( S = 0 \)), the mold, wall is everywhere at an initial uniform temperature \( T_i \), and the molten metal is everywhere at its fusion (melting) point temperature of \( T_f \). Following the start of solidification, there is conduction heat transfer into the mold wall and the thickness of the solidified metal, \( S \), increases with time \( t \).

(a) Sketch the one-dimensional temperature distribution, \( T(x) \), in the mold wall and the metal at \( t = 0 \) and at two subsequent times during the solidification. Clearly indicate any underlying assumptions.

(b) Obtain a relation for the variation of the solid layer thickness \( S \) with time \( t \), expressing your result in terms of appropriate parameters of the system.

5.84 Joints of high quality can be formed by friction welding. Consider the friction welding of two 40-mm-diameter Inconel rods. The bottom rod is stationary, while the top rod is forced into a back-and-forth linear motion characterized by an instantaneous horizontal displacement, \( d(t) = a \cos(\omega t) \) where \( a = 2 \text{ mm} \) and \( \omega = 1000 \text{ rad/s} \). The coefficient of sliding friction between the two pieces is \( \mu = 0.3 \). Determine the compressive force that must be applied in order to heat the joint to the Inconel melting point within \( t = 3 \text{ s} \), starting from an initial temperature of 20°C. \( \text{HINT:} \) The frequency of the motion and resulting heat rate are very high. The temperature response can be approximated as if the heating rate were constant in time, equal to its average value.
Problems

Boundary Layer Profiles

6.1 In flow over a surface, velocity and temperature profiles are of the forms

\[ u(y) = Ay + By^2 - Cy^3 \quad \text{and} \quad T(y) = D + Ey + Fy^2 - Gv^3 \]

where the coefficients A through G are constants. Obtain expressions for the friction coefficient \( C_f \) and the convection coefficient \( h \) in terms of \( u_w, T_m \) and appropriate profile coefficients and fluid properties.

6.2 Water at a temperature of \( T_w = 25^\circ C \) flows over one of the surfaces of a steel wall (AISI 1010) whose temperature is \( T_s,1 = 40^\circ C \). The wall is 0.35 m thick, and its other surface temperature is \( T_s,2 = 100^\circ C \). For steady-state conditions what is the convection coefficient associated with the water flow? What is the temperature gradient in the wall and in the water that is in contact with the wall? Sketch the temperature distribution in the wall and in the adjoining water.

6.3 In a particular application involving airflow over a heated surface, the boundary layer temperature distribution may be approximated as

\[ \frac{T - T_s}{T_w - T_s} = 1 - \exp\left(-P_f \frac{u_w y}{v}\right) \]

where \( y \) is the distance normal to the surface and the Prandtl number, \( Pr = c_p \mu k = 0.7 \), is a dimensionless fluid property. If \( T_w = 400 \text{ K} \), \( T_s = 300 \text{ K} \), and \( u_w/v = 5000 \text{ m}^{-1} \), what is the surface heat flux?

Heat Transfer Coefficients

6.4 For laminar flow over a flat plate, the local heat transfer coefficient \( h_t \) is known to vary as \( x^{-1/2} \), where \( x \) is the distance from the leading edge \((x = 0)\) of the plate. What is the ratio of the average coefficient between the leading edge and some location \( x \) on the plate to the local coefficient at \( x \)?

6.5 For laminar free convection from a heated vertical surface, the local convection coefficient may be expressed as \( h_r = C x^{1/4} \), where \( h_r \) is the coefficient at a distance \( x \) from the leading edge of the surface and the quantity \( C \), which depends on the fluid properties, is independent of \( x \). Obtain an expression for the ratio \( h_r/h_{av} \), where \( h_{av} \) is the average coefficient between the leading edge \((x = 0)\) and the \( x \) location. Sketch the variation of \( h_r \) and \( h_{av} \) with \( x \).

6.6 A circular, hot gas jet at \( T_w \) is directed normal to a circular plate that has radius \( r_e \) and is maintained at a uniform temperature \( T_r \). Gas flow over the plate is axisymmetric, causing the local convection coefficient to have a radial dependence of the form \( h_r(r) = a + br^n \), where \( a, b, \) and \( n \) are constants. Determine the rate of heat transfer to the plate, expressing your result in terms of \( T_w, T_r, r_e, a, b, \) and \( n \).

6.7 Parallel flow of atmospheric air over a flat plate of length \( L = 3 \text{ m} \) is disrupted by an array of stationary rods placed in the flow path over the plate.

![Diagram of airflow over a flat plate with rods]

Laboratory measurements of the local convection coefficient at the surface of the plate are made for a prescribed value of \( V \) and \( T_r > T_w \). The results are correlated by an expression of the form \( h_r = 0.7 + 13.6x - 3.4x^2 \), where \( h_r \) has units of \( \text{W/m}^2 \cdot \text{K} \) and \( x \) is in meters. Evaluate the average convection coefficient \( h_{av} \) for the entire plate and the ratio \( h_{av}/h_L \) at the trailing edge.

6.8 Air at a free stream temperature of \( T_w = 20^\circ C \) is in parallel flow over a flat plate of length \( L = 5 \text{ m} \) and temperature \( T_r = 90^\circ C \). However, obstacles placed in the flow intensify mixing with increasing distance \( x \) from the leading edge, and the spatial variation of temperatures...
The surface area of the blade may be assumed to be directly proportional to its characteristic length.

6.20 Experimental measurements of the convection heat transfer coefficient for a square bar in cross flow yielded the following values:

\[
\begin{align*}
\bar{h}_1 &= 50 \text{ W/m}^2 \cdot \text{K} \\
\bar{h}_2 &= 40 \text{ W/m}^2 \cdot \text{K}
\end{align*}
\]

when \( V_1 = 20 \text{ m/s} \)

\( \bar{h}_2 = 40 \text{ W/m}^2 \cdot \text{K} \)

when \( V_2 = 15 \text{ m/s} \)

\[
L = 0.5 \text{ m}
\]

Assume that the functional form of the Nusselt number is \( \bar{Nu} = C \text{Re}^n \text{Pr}^m \), where \( C, m, \) and \( n \) are constants.

(a) What will be the convection heat transfer coefficient for a similar bar with \( L = 1 \text{ m} \) when \( V = 15 \text{ m/s} \)?

(b) What will be the convection heat transfer coefficient for a similar bar with \( L = 1 \text{ m} \) when \( V = 30 \text{ m/s} \)?

(c) Would your results be the same if the side of the bar, rather than its diagonal, were used as the characteristic length?

6.21 Experimental results for heat transfer over a flat plate with an extremely rough surface were found to be correlated by an expression of the form

\[
\bar{Nu}_x = 0.04 \text{Re}^{0.8} \text{Pr}^{0.4} \]

where \( \bar{Nu}_x \) is the local value of the Nusselt number at a position \( x \) measured from the leading edge of the plate. Obtain an expression for the ratio of the average heat transfer coefficient \( \bar{h}_x \) to the local coefficient \( h_x \).

6.22 Consider conditions for which a fluid with a free stream velocity of \( V = 1 \text{ m/s} \) flows over a surface with a characteristic length of \( L = 1 \text{ m} \), providing an average convection heat transfer coefficient of \( \bar{h} = 100 \text{ W/m}^2 \cdot \text{K} \). Calculate the dimensionless parameters \( \bar{Nu}_x, \text{Re}_x, \text{Pr} \), and \( \bar{f}_x \) for the following fluids: air, engine oil, mercury, and water. Assume the fluids to be at 300 K.

6.23 For flow over a flat plate of length \( L \), the local heat transfer coefficient \( h_x \) is known to vary as \( x^{-0.25} \), where \( x \) is the distance from the leading edge of the plate. What is the ratio of the average Nusselt number for the entire plate (\( \bar{Nu}_L \)) to the local Nusselt number at \( x = L \) (\( \bar{Nu}_L \))?

6.24 For laminar boundary layer flow over a flat plate with air at 20°C and 1 atm, the thermal boundary layer thickness \( \delta \), is approximately 13% larger than the velocity boundary layer thickness \( \delta_v \). Determine the ratio \( \delta_v / \delta \), if the fluid is ethylene glycol under the same flow conditions.

6.25 Sketch the variation of the velocity and thermal boundary layer thicknesses with distance from the leading edge of a flat plate for the laminar flow of air, water, engine oil, and mercury. For each case assume a mean fluid temperature of 300 K.

6.26 Forced air at \( T_a = 25°C \) and \( V = 10 \text{ m/s} \) is used to cool electronic elements on a circuit board. One such element is a chip, 4 mm by 4 mm, located 120 mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

\[
\bar{Nu}_x = 0.04 \text{Re}^{0.8} \text{Pr}^{0.4} \]

Estimate the surface temperature of the chip if it is dissipating 30 mW.

6.27 Consider the electronic elements that are cooled by forced convection in Problem 6.26. The cooling system is designed and tested at sea level (\( P = 1 \text{ atm} \)), but the circuit board is sold to a customer in Mexico City, with an elevation of 2250 m and atmospheric pressure of 76.5 kPa.

(a) Estimate the surface temperature of the chip located 120 mm from the leading edge of the board when the board is operated in Mexico City. The dependence of various thermophysical properties upon pressure is noted in Problem 6.17.

(b) It is highly desirable for the chip operating temperature to be independent of the location of the customer. What air velocity is required for operation in Mexico City if the chip temperature is to be the same as at sea level?

6.28 Consider the chip on the circuit board of Problem 6.26. To ensure reliable operation over extended periods, the chip temperature should not exceed 85°C. Assuming the availability of forced air at \( T_a = 25°C \) and applicability of the prescribed heat transfer correlation, compute and plot the maximum allowable chip power dissipation \( P_c \) as a function of air velocity for \( 1 \leq V \leq 25 \text{ m/s} \). If the chip surface has an emissivity of 0.80 and the board is mounted in a large enclosure whose walls are at 25°C, what is the effect of radiation on the \( P_c - V \) plot?

6.29 A major contributor to product defects in electronic modules relates to stresses induced during thermal
6.32 The defroster of an automobile functions by discharging warm air on the inner surface of the windshield. To prevent condensation of water vapor on the surface, the temperature of the air and the surface convection coefficient \( (T_{aw}, h) \) must be large enough to maintain a surface temperature \( T_{sd} \) that is at least as high as the dewpoint \( T_{dp} \).

Consider a windshield of length \( L = 800 \text{ mm} \) and thickness \( t = 6 \text{ mm} \) and driving conditions for which the vehicle moves at a velocity of \( V = 70 \text{ mph} \) in ambient air at \( T_{aw} = -15^\circ \text{C} \). From laboratory experiments performed on a model of the vehicle, the average convection coefficient on the outer surface of the windshield is known to be correlated by an expression of the form \( N_{u,s} = 0.030 Re_s^{0.8} Pr^{0.3} \), where \( Re_s = VL/\nu \). Properties of the ambient air may be approximated as \( k = 0.023 \text{ W/m} \cdot \text{K}, \nu = 12.5 \times 10^{-6} \text{ m}^2/\text{s}, \) and \( Pr = 0.71 \). If \( T_{dp} = 10^\circ \text{C} \) and \( T_{aw} = 50^\circ \text{C} \), what is the smallest value of \( h \) required to prevent condensation on the inner surface?

6.33 A microscale detector monitors a steady flow \( (T_{aw} = 27^\circ \text{C}, V = 10 \text{ m/s}) \) of air for the possible presence of small, hazardous particulate matter that may be suspended in the room. The sensor is heated to a slightly higher temperature in order to induce a chemical reaction associated with certain substances of interest that might impinge on the sensor's active surface. The active surface produces an electric current if such surface reactions occur; the electric current is then sent to an alarm. To maximize the sensor head's surface area and, in turn, the probability of capturing and detecting a particle, the sensor head is designed with a very complex shape. The value of the average heat transfer coefficient associated with the heated sensor must be known so that the required electrical power to the sensor can be determined.

Consider a sensor with a characteristic dimension of \( L_s = 80 \mu \text{m} \). A scale model of the sensor is placed in a
5 \times 10^5$, for what values of $Re_c$, would the total heat transfer be independent of orientation?

7.14 In fuel cell stacks, it is desirable to operate under conditions that promote uniform surface temperatures for the electrolytic membranes. This is especially true in high-temperature fuel cells where the membrane is constructed of a brittle ceramic material. Electrochemical reactions in the electrolytic membranes generate thermal energy, while gases flowing above and below the membranes cool it. The stack designer may specify top and bottom flows that are in the same, opposite, or orthogonal directions. A preliminary study of the effect of the relative flow directions is conducted whereby a 150 mm $\times$ 150 mm thin sheet of material, producing a uniform heat flux of 100 W/m$^2$, is cooled (top and bottom) by air with a free stream temperature and velocity of 25°C and 2 m/s, respectively.

(a) Determine the minimum and maximum local membrane temperatures for top and bottom flows that are in the same, opposite, and orthogonal directions. Which flow configuration minimizes the membrane temperature? Hints: For the opposite and orthogonal flow cases, the boundary layers are subject to boundary conditions that are neither uniform temperature nor uniform heat flux. It is, however, reasonable to expect that the resulting temperatures would be bracketed by your answers based on the constant heat flux and constant temperature boundary conditions.

(b) Plot the surface temperature distribution $T(x)$ for the cases involving flow in the opposite and same directions. Thermal stresses are undesirable and are related to the spatial temperature gradient along the membrane. Which configuration minimizes spatial temperature gradients?

7.15 Air at a pressure of 1 atm and a temperature of 50°C is in parallel flow over the top surface of a flat plate that is heated to a uniform temperature of 100°C. The plate has a length of 0.20 m (in the flow direction) and a width of 0.10 m. The Reynolds number based on the plate length is 40,000. What is the rate of heat transfer from the plate to the air? If the free stream velocity of the air is doubled and the pressure is increased to 10 atm, what is the rate of heat transfer?

7.16 Consider a rectangular fin that is used to cool a motorcycle engine. The fin is 0.15 m long and at a temperature of 250°C, while the motorcycle is moving at 80 km/h in air at 27°C. The air is in parallel flow over both surfaces of the fin, and turbulent flow conditions may be assumed to exist throughout.

(a) What is the rate of heat removal per unit width of the fin?

7.17 The Weather channel reports that it is a hot, muggy day with an air temperature of 90°F, a 10 mph breeze out of the southwest, and bright sunshine with a solar irradiation of 400 W/m$^2$. Consider the wall of a metal building over which the prevailing wind blows. The length of the wall in the wind direction is 10 m, and the emissivity is 0.93. Assume that all the solar irradiation is absorbed, that irradiation from the sky is negligible, and that flow is fully turbulent over the wall. Estimate the average wall temperature.

7.18 A photovoltaic solar panel consists of a sandwich of (top to bottom) a 3-mm-thick ceria-doped glass ($k_g = 1.4$ W/m·K), a 0.1-mm-thick optical grade adhesive ($k_a = 145$ W/m·K), a very thin silicon semiconducting material, a 0.1-mm-thick solder layer ($k_s = 50$ W/m·K) and a 2-mm-thick aluminum nitride substrate ($k_n = 120$ W/m·K). The solar-to-electrical conversion efficiency within the semiconductor depends on the silicon temperature $T_s$ and is described by the expression $\eta = 0.28 - 0.001T_s$, where $T_s$ is in °C, for 25°C $\leq T_s \leq 250$°C. Ten percent of the solar irradiation is absorbed at the top surface of the glass, while 83% of the solar irradiation is transmitted to and absorbed by the silicon (the remaining 7% is reflected away from the cell). The glass has an emissivity of 0.90.

7.19 Consider an $L = 1$ m long, $w = 0.1$ m wide solar cell that is placed on an insulated surface. Determine the silicon temperature and the electric power produced by the solar cell for an air velocity of 4 m/s parallel to the long direction, with air and surroundings temperatures of 25°C. The solar irradiation is 700 W/m$^2$. The boundary layer is tripped to a turbulent condition at the leading edge of the panel.
(b) Repeat part (a), except now the panel is oriented with its short side parallel to the air flow, that is, \( L = 0.1 \text{ m} \) and \( w = 1 \text{ m} \).

(c) Plot the electric power output and the silicon temperature versus air velocity over the range \( 0 \leq u_w \leq 10 \text{ m/s} \) for the \( L = 0.1 \text{ m} \) and \( w = 1 \text{ m} \) case.

7.19 Concentration of sunlight onto photovoltaic cells is desired since the concentrating mirrors and lenses are less expensive than the photovoltaic material. Consider the solar photovoltaic cell of Problem 7.18. A 100 mm \( \times \) 100 mm photovoltaic cell is irradiated with concentrated solar energy. Since the concentrating lens is glass, it absorbs 10% of the irradiation instead of the top surface of the solar cell, as in Problem 7.18. The remaining irradiation is reflected from the system (7%) or is absorbed in the silicon semiconductor material of the photovoltaic cell (83%). The photovoltaic cell is cooled by air directed parallel to its top and bottom surfaces. The air temperature and velocity are 25°C and 5 m/s, respectively, and the bottom surface is coated with a high-emissivity paint, \( \varepsilon_b = 0.95 \).

(a) Determine the electric power produced by the photovoltaic cell and the silicon temperature for a square concentrating lens with \( L_{	ext{max}} = 400 \text{ mm} \), which focuses the irradiation falling on the lens to the smaller area of the photovoltaic cell. Assume the concentrating lens temperature is 25°C and does not interfere with boundary layer development over the photovoltaic cell's top surface. The top and bottom boundary layers are both tripped to turbulent conditions at the leading edge of the photovoltaic material.

(b) Determine the electric power output of the photovoltaic cell and the silicon temperature over the range 100 mm \( \leq L_{	ext{max}} \leq 600 \text{ mm} \).

7.20 The roof of a refrigerated truck compartment is of composite construction, consisting of a layer of foamed urethane insulation (\( t_i = 50 \text{ mm} \), \( k_i = 0.026 \text{ W/m} \cdot \text{K} \)) sandwiched between aluminum alloy panels (\( t_p = 5 \text{ mm} \), \( k_p = 180 \text{ W/m} \cdot \text{K} \)). The length and width of the roof are \( L = 10 \text{ m} \) and \( W = 3.5 \text{ m} \), respectively, and the temperature of the inner surface is \( T_{i,0} = -10°C \). Consider conditions for which the truck is moving at a speed of \( v = 105 \text{ km/h} \), the air temperature is \( T_{w} = 32°C \), and the solar irradiation is \( G = 750 \text{ W/m}^2 \). Turbulent flow may be assumed over the entire length of the roof.

(a) For equivalent values of the solar absorptivity and the emissivity of the outer surface (\( \alpha_s = \varepsilon = 0.5 \)), estimate the average temperature \( T_{a,0} \) of the outer surface. What is the corresponding heat load imposed on the refrigeration system?

(b) A special finish (\( \alpha_s = 0.15 \), \( \varepsilon = 0.8 \)) may be applied to the outer surface. What effect would such an application have on the surface temperature and the heat load?

(c) If, with \( \alpha_s = \varepsilon = 0.5 \), the roof is not insulated (\( t_i = 0 \)), what are the corresponding values of the surface temperature and the heat load?

7.21 The top surface of a heated compartment consists of very smooth (A) and highly roughened (B) portions, and the surface is placed in an atmospheric airstream. In the interest of minimizing total convection heat transfer from the surface, which orientation, (1) or (2), is preferred? If \( T_f = 100°C \), \( T_w = 20°C \), and \( u_w = 20 \text{ m/s} \), what is the convection heat transfer from the entire surface for this orientation?
distance $L$ between two rollers at a velocity $V$. In this problem, we consider cooling of an aluminum alloy (2024-T6) by an air stream moving at a velocity $u_\infty$ in counter flow over the top surface of the sheet. A turbulence promoter is used to provide turbulent boundary layer development over the entire surface.

(a) By applying conservation of energy to a differential control surface of length $dx$, which either moves with the sheet or is stationary and through which the sheet passes, derive a differential equation that governs the temperature distribution along the sheet. Because of the low emissivity of the aluminum, radiation effects may be neglected. Express your result in terms of the velocity, thickness, and properties of the sheet ($V, \delta, \rho, c_p, \delta$), the local convection coefficient $h_x$ associated with the counter flow, and the air temperature. For a known temperature of the sheet ($T_s$) at the onset of cooling and a negligible effect of the sheet velocity on boundary layer development, solve the equation to obtain an expression for the outlet temperature $T_o$.

(b) For $\delta = 2 \text{ mm}, V = 0.10 \text{ m/s}, L = 5 \text{ m}, W = 1 \text{ m}, u_\infty = 20 \text{ m/s}, T_s = 20^\circ \text{C}, \text{ and } T_i = 300^\circ \text{C}$, what is the outlet temperature $T_o$?

7.26 In the production of sheet metals or plastics, it is customary to cool the material before it leaves the production process for storage or shipment to the customer. Typically, the process is continuous, with a sheet of thickness $\delta$ and width $W$ cooled as it transits the distance $L$ between two rollers at a velocity $V$. In this problem, we consider cooling of plain carbon steel by an air stream moving at a velocity $u_\infty$ in cross flow over the top and bottom surfaces of the sheet. A turbulence promoter is used to provide turbulent boundary layer development over the entire surface.

(a) By applying conservation of energy to a differential control surface of length $dx$, which either moves with the sheet or is stationary and through which the sheet passes, and assuming a uniform sheet temperature in the direction of air flow, derive a differential equation that governs the temperature distribution, $T(x)$, along the sheet. Consider the effects of radiation, as well as convection, and express your result in terms of the velocity, thickness, and properties of the sheet ($V, \delta, \rho, c_p, \delta$), the average convection coefficient $h_x$, and the environmental temperatures ($T_\text{in}, T_\text{out}$).

(b) Neglecting radiation, obtain a closed form solution to the foregoing equation. For $\delta = 3 \text{ mm}, V = 0.10 \text{ m/s}, L = 10 \text{ m}, W = 1 \text{ m}, u_\infty = 20 \text{ m/s}, T_s = 20^\circ \text{C}$, and a sheet temperature of $T_s = 500^\circ \text{C}$ at the onset of cooling, what is the outlet temperature $T_o$? Assume a negligible effect of the sheet velocity on boundary layer development in the direction of air flow. The density and specific heat of the steel are $\rho = 7850 \text{ kg/m}^3$ and $c_p = 620 \text{ J/kg} \cdot \text{K}$, while properties of the air may be taken to be $k = 0.044 \text{ W/m} \cdot \text{K}$, $\nu = 4.5 \times 10^{-5} \text{ m}^2/\text{s}$, $Pr = 0.68$.

(c) Accounting for the effects of radiation, with $\varepsilon = 0.70$ and $T_\text{out} = 20^\circ \text{C}$, numerically integrate the differential equation derived in part (a) to determine the temperature of the sheet at $L = 10 \text{ m}$. Explore the effect of $V$ on the temperature distribution along the sheet.

7.27 A steel strip emerges from the hot roll section of a steel mill at a speed of 20 m/s and a temperature of 1200 K. Its length and thickness are $L = 100 \text{ m}$ and $\delta = 0.003 \text{ m}$, respectively, and its density and specific heat are 7900 kg/m$^3$ and 640 J/kg · K, respectively.