
Problems

Plane Wall

3.1 Consider the plane wall of Figure 3.1, separating hot and cold fluids at temperatures $T_{a1}$ and $T_{a2}$, respectively. Using surface energy balances as boundary conditions at $x = 0$ and $x = L$ (see Equation 2.32), obtain the temperature distribution within the wall and the heat flux in terms of $T_{a1}$, $T_{a2}$, $h_i$, $h_o$, $k$, and $L$.

3.2 The rear window of an automobile is defogged by passing warm air over its inner surface.
(a) If the warm air is at $T_{a2} = 40^\circ$C and the corresponding convection coefficient is $h_o = 30$ W/m²·K, what are the inner and outer surface temperatures of 4-mm-thick window glass, if the outside ambient air temperature is $T_{a1} = -10^\circ$C and the associated convection coefficient is $h_i = 65$ W/m²·K?
(b) In practice $T_{a2}$ and $h_o$ vary according to weather conditions and car speed. For values of $h_o = 2$, 65, and 100 W/m²·K, compute and plot the inner and outer surface temperatures as a function of $T_{a2}$ for $-30 \leq T_{a2} \leq 0^\circ$C.

3.3 The rear window of an automobile is defogged by attaching a thin, transparent, film-type heating element to its inner surface. By electrically heating this element, a uniform heat flux may be established at the inner surface.
(a) For 4-mm-thick window glass, determine the electrical power required per unit window area to maintain an inner surface temperature of 15°C when the interior air temperature and convection coefficient are $T_{a1} = 25^\circ$C and $h_i = 10$ W/m²·K, while the exterior (ambient) air temperature and convection coefficient are $T_{a2} = -10^\circ$C and $h_o = 65$ W/m²·K.
(b) In practice $T_{a2}$ and $h_o$ vary according to weather conditions and car speed. For values of $h_o = 2$, 20, 65, and 100 W/m²·K, determine and plot the electrical power requirement as a function of $T_{a2}$ for $-30 \leq T_{a2} \leq 0^\circ$C. From your results, what can you conclude about the need for heater operation at low values of $h_o$? How is this conclusion affected by the value of $T_{a2}$? If $h \propto V^n$, where $V$ is the vehicle speed and $n$ is a positive exponent, how does the vehicle speed affect the need for heater operation?

3.4 In a manufacturing process, a transparent film is being bonded to a substrate as shown in the sketch. To cure the bond at a temperature $T_0$, a radiant source is used to provide a heat flux $q_0$ (W/m²), all of which is absorbed at the bonded surface. The back of the substrate is maintained at $T_1$ while the free surface of the film is exposed to air at $T_a$ and a convection heat transfer coefficient $h$.

(a) Show the thermal circuit representing the steady-state heat transfer situation. Be sure to label all elements, nodes, and heat rates. Leave in symbolic form.
(b) Assume the following conditions: $T_a = 20^\circ$C, $h = 50$ W/m²·K, and $T_1 = 30^\circ$C. Calculate the heat flux $q_0$ that is required to maintain the bonded surface at $T_0 = 60^\circ$C.
(c) Compute and plot the required heat flux as a function of the film thickness for $0 \leq L_f \leq 1$ mm.
(d) If the film is not transparent and all of the radiant heat flux is absorbed at its upper surface, determine...
by performing many time-consuming experiments at various operating temperatures. A new experimental design is proposed whereby the temperature dependence may be determined in one experiment. The proposed apparatus consists of multiple layers, with each layer consisting of a \( t_{\text{mm}} \)-thick stainless steel plate with \( k_{\text{mm}} = 15 \) W/m\( \cdot \)K. The resulting stainless steel–low thermal conductivity–stainless steel sandwiches then separate \( N = 5 \) layers of the liquid. The entire structure is heated from above to eliminate natural convection within the liquid, and cooled from below with a flowing liquid. The temperature of each stainless steel sheet is measured with a thermocouple, and the device is encased in insulation. The temperature range over which the thermal conductivity of a particular liquid is to be measured is \( 300 \text{ K} \leq T \leq 400 \text{ K} \). To resolve the temperature dependence of the liquid’s thermal conductivity, the temperature difference across each liquid layer is to be held at \( \Delta T = 2 \text{ °C} \). The nominal thermal conductivity of the liquid is \( k_{\text{L}} = 0.8 \) W/m\( \cdot \)K.

(a) Consider the low thermal conductivity material to be Bakelite. Determine the overall height, \( H \), of the experimental apparatus.

(b) Consider replacing the Bakelite with an aerogel characterized by \( k_{\text{g}} = 0.0065 \) W/m\( \cdot \)K. What is the overall height of the apparatus?

(c) To minimize heat losses through the sides of the device, the area of the heater \( (A_{\text{he}}) \) is made 10 times larger than the area of the sides \( (A_{\text{s}}) \) of the device. Compare the required heater area and required electrical power for devices constructed using Bakelite and aerogel low thermal conductivity materials.

3.11 The wall of a drying oven is constructed by sandwiching an insulation material of thermal conductivity \( k = 0.05 \) W/m\( \cdot \)K between thin metal sheets. The oven air is at \( T_{\text{air}} = 300\text{ °C} \), and the corresponding convection coefficient is \( h_{\text{c}} = 30 \) W/m\(^2\)·K. The inner wall surface absorbs a radiant flux of \( q_{\text{rad}} = 100 \) W/m\(^2\) from hotter objects within the oven. The room air is at \( T_{\text{room}} = 25\text{ °C} \), and the overall coefficient for convection and radiation from the outer surface is \( h_{\text{o}} = 10 \) W/m\(^2\)·K.

Absorbed radiation, \( q_{\text{rad}} \)

Insulation, \( k \)

Roan air

(a) Draw the thermal circuit for the wall and label all temperatures, heat rates, and thermal resistances.

(b) What insulation thickness \( L \) is required to maintain the outer wall surface at a safe-to-touch temperature of \( T_{\text{s}} = 40\text{ °C} \)?

3.12 The electrolytic membrane of the fuel cell in Example 1.4 is a thin composite structure consisting of sandwiched layers of delicate materials, as shown in the sketch. The thickness of the polymer core is \( t_{\text{pol}} = 0.20 \) mm, while the thickness of each of the catalyst layers is \( t_{\text{c}} = 0.01 \) mm. The gas diffusion layers are each \( t_{\text{diff}} = 0.1 \) mm thick. Since the membrane can soften and lose durability at temperatures exceeding 85°C, the materials engineer decides to strengthen the membrane by implanting long carbon nanotubes (diameter \( D_{\text{cm}} = 14 \) nm, \( k_{\text{cm}} = 3000 \) W/m\( \cdot \)K) lengthwise within both catalyst layers. Determine the value of the effective thermal conductivity, \( k_{\text{eff}} \), of the membrane assembly that is defined by the relation \( q_{\text{g}} = k_{\text{eff}} \cdot w \Delta T / L \), where \( L, w, t \) are the length, width, and total thickness of the membrane assembly respectively; \( q_{\text{g}} \) is the heat transfer rate along the assembly; and \( \Delta T \) is the temperature drop along a section of the assembly of length \( L \). Determine the value of \( k_{\text{eff}} \) for carbon nanotube loadings of \( f = 0, 10\%, 20\%, \) and \( 30\% \), where \( f \) is the volume fraction of carbon nanotubes in the catalyst layers. The thermal conductivity of the polymer core is \( k_{\text{pol}} = 0.25 \) W/m·K, and the thermal conductivities of the gas diffusion layers and catalyst layers are \( k_{\text{diff}} = 1.3 \) W/m·K and \( k_{\text{c}} = 1 \) W/m·K, respectively.
contact resistance of \( R_{nc} = 3 \times 10^{-4} \text{ m}^2 \cdot \text{K/W} \). The convection heat transfer coefficient at the outer surface of the sheath is 10 \( \text{W/m}^2 \cdot \text{K} \), and the temperature of the ambient air is 20°C. If the temperature of the insulation may not exceed 50°C, what is the maximum allowable electrical power that may be dissipated per unit length of the conductor? What is the critical radius of the insulation?

3.44 Electric current flows through a long rod generating thermal energy at a uniform volumetric rate of \( \dot{q} = 2 \times 10^6 \text{ W/m}^3 \). The rod is concentric with a hollow ceramic cylinder, creating an enclosure that is filled with air.

The thermal resistance per unit length due to radiation between the enclosure surfaces is \( R_{rad} = 0.30 \text{ m} \cdot \text{K/W} \), and the coefficient associated with free convection in the enclosure is \( h = 20 \text{ W/m}^2 \cdot \text{K} \).

(a) Construct a thermal circuit that can be used to calculate the surface temperature of the rod, \( T_r \). Label all temperatures, heat rates, and thermal resistances, and evaluate each thermal resistance.

(b) Calculate the surface temperature of the rod for the prescribed conditions.

3.45 The evaporator section of a refrigeration unit consists of thin-walled, 10-mm-diameter tubes through which refrigerant passes at a temperature of -18°C. Air is cooled as it flows over the tubes, maintaining a surface convection coefficient of 100 \( \text{W/m}^2 \cdot \text{K} \), and is subsequently routed to the refrigerator compartment.

(a) For the foregoing conditions and an air temperature of -3°C, what is the rate at which heat is extracted from the air per unit tube length?

(b) If the refrigerator's defrost unit malfunctions, frost will slowly accumulate on the outer tube surface. Assess the effect of frost formation on the cooling capacity of a tube for frost layer thicknesses in the range 0 ≤ δ ≤ 4 mm. Frost may be assumed to have a thermal conductivity of 0.4 \( \text{W/m} \cdot \text{K} \).

(c) The refrigerator is discontinue after the defrost unit malfunctions and a 2-mm-thick layer of frost has formed. If the tubes are in ambient air for which \( T_a = 20°C \) and natural convection maintains a convection coefficient of 2 \( \text{W/m}^2 \cdot \text{K} \), how long will it take for the frost to melt? The frost may be assumed to have a mass density of 700 \( \text{kg/m}^3 \) and a latent heat of fusion of 334 \( \text{kJ/kg} \).

3.46 A composite cylindrical wall is composed of two materials with thermal conductivities \( k_n \) and \( k_p \), which are separated by a very thin, electric resistance heater for which interfacial contact resistances are negligible. Liquid pumped through the tube is at a temperature \( T_{m,i} \) and provides a convection coefficient \( h_t \) at the inner surface of the composite. The outer surface is exposed to ambient air, which is at \( T_{m,a} \) and provides a convection coefficient of \( h_p \). Under steady-state conditions, a uniform heat flux of \( q_s^e \) dissipated by the heater.

(a) Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables.

(b) Obtain an expression that may be used to determine the heater temperature, \( T_h \).

(c) Obtain an expression for the ratio of heat flows to the outer and inner fluids, \( \dot{q}_o/\dot{q}_i \). How might the variables of the problem be adjusted to minimize this ratio?

3.47 An electrical current of 700 A flows through a stainless steel cable having a diameter of 5 mm and an electrical resistance of \( 6 \times 10^{-4} \text{ Ω/m} \) (i.e., per meter of cable length). The cable is in an environment having a temperature of 30°C, and the total coefficient associated with convection and radiation between the cable and the environment is approximately 25 \( \text{W/m}^2 \cdot \text{K} \).

(a) If the cable is bare, what is its surface temperature?

(b) If a very thin coating of electrical insulation is applied to the cable, with a contact resistance of 0.02 \( \text{m}^2 \cdot \text{K/W} \), what are the insulation and cable surface temperatures?
effects may be neglected, and the Pyrex may be assumed to have a thermal conductivity of 1.4 W/m·K.

3.55 Consider the liquid oxygen storage system and the laboratory environmental conditions of Problem 1.49. To reduce oxygen loss due to vaporization, an insulating layer should be applied to the outer surface of the container. Consider using a laminated aluminum foil/glass mat insulation, for which the thermal conductivity and surface emissivity are \( k = 0.0016 \) W/m·K and \( e = 0.20 \), respectively.

(a) If the container is covered with a 10-mm-thick layer of insulation, what is the percentage reduction in oxygen loss relative to the uncovered container?

(b) Compute and plot the oxygen evaporation rate (kg/s) as a function of the insulation thickness \( t \) for 0 \( \leq t \leq 50 \) mm.

3.56 In Example 3.5, an expression was derived for the critical insulation radius of an insulated, cylindrical tube. Derive the expression that would be appropriate for an insulated sphere.

3.57 A hollow aluminum sphere, with an electrical heater in the center, is used in tests to determine the thermal conductivity of insulating materials. The inner and outer radii of the sphere are 0.15 and 0.18 m, respectively, and testing is done under steady-state conditions with the inner surface of the aluminum maintained at 250°C.

In a particular test, a spherical shell of insulation is cast on the outer surface of the sphere to a thickness of 0.12 m. The system is in a room for which the air temperature is 20°C and the convection coefficient at the outer surface of the insulation is 30 W/m²·K. If 80 W are dissipated by the heater under steady-state conditions, what is the thermal conductivity of the insulation?

3.58 A spherical tank for storing liquid oxygen on the space shuttle is to be made from stainless steel of 0.80-m outer diameter and 5-mm wall thickness. The boiling point and latent heat of vaporization of liquid oxygen are 90 K and 213 kJ/kg, respectively. The tank is to be installed in a large compartment whose temperature is to be maintained at 240 K. Design a thermal insulation system that will maintain oxygen losses due to boiling below 1 kg/day.

3.59 A spherical, cryosurgical probe may be imbedded in diseased tissue for the purpose of freezing, and thereby destroying, the tissue. Consider a probe of 3-mm diameter whose surface is maintained at -30°C when imbedded in tissue that is at 37°C. A spherical layer of frozen tissue forms around the probe, with a temperature of 0°C existing at the phase front (interface) between the frozen and normal tissue. If the thermal conductivity of frozen tissue is approximately 1.5 W/m·K and heat transfer at the phase front may be characterized by an effective convection coefficient of 50 W/m²·K, what is the thickness of the layer of frozen tissue (assuming negligible perfusion)?

3.60 A spherical vessel used as a reactor for producing pharmaceuticals has a 10-mm-thick stainless steel wall (\( k = 17 \) W/m·K) and an inner diameter of 1 m. The exterior surface of the vessel is exposed to ambient air (\( T_a = 25°C \)) for which a convection coefficient of 6 W/m²·K may be assumed.

(a) During steady-state operation, an inner surface temperature of 50°C is maintained by energy generated within the reactor. What is the heat loss from the vessel?

(b) If a 20-mm-thick layer of fiberglass insulation (\( k = 0.040 \) W/m·K) is applied to the exterior of the vessel and the rate of thermal energy generation is unchanged, what is the inner surface temperature of the vessel?

3.61 The wall of a spherical tank of 1-m diameter contains an exothermic chemical reaction and is at 200°C when the ambient air temperature is 25°C. What thickness of urethane foam is required to reduce the exterior temperature to 40°C, assuming the convection coefficient is 20 W/m²·K for both situations? What is the percentage reduction in heat rate achieved by using the insulation?

3.62 A composite spherical shell of inner radius \( r_1 = 0.25 \) m is constructed from lead of outer radius \( r_2 = 0.30 \) m and AISI 302 stainless steel of outer radius \( r_3 = 0.31 \) m. The cavity is filled with radioactive wastes that generate heat at a rate of \( q = 5 \times 10^2 \) W/m³. It is proposed to submerge the container in oceanic waters that are at a temperature of \( T_w = 10°C \) and provide a uniform convection coefficient of \( h = 500 \) W/m²·K at the outer surface of the container. Are there any problems associated with this proposal?

3.63 As an alternative to storing radioactive materials in oceanic waters, it is proposed that the system of Problem 3.62 be placed in a large tank for which the flow of water, and hence the convection coefficient \( h \), can be controlled. Compute and plot the maximum temperature of the lead, \( T(r) \), as a function of \( h \) for 100 \( \leq h \leq 1000 \) W/m²·K. If the temperature of the lead is not to exceed 500 K, what is the minimum allowable value of \( h \)? To improve system reliability, it is desirable to increase the thickness of the stainless steel shell. For \( h = 300, 500, \) and 1000 W/m²·K, compute and plot the maximum lead temperature as a function of shell thickness for \( r_1 = 0.30 \) m. What are the corresponding values of the maximum allowable thickness?

3.64 The energy transferred from the anterior chamber of the eye through the cornea varies considerably depending on whether a contact lens is worn. Treat the eye as a
(a) Obtain equations for the temperature distributions $T_1(r)$ and $T_2(r)$ in the fuel and cladding, respectively. Express your results exclusively in terms of the foregoing variables.

(b) Consider a uranium oxide fuel pin for which $k_f = 2$ W/m·K and $r_1 = 6$ mm and cladding for which $k_c = 25$ W/m·K and $r_2 = 9$ mm. If $q = 2 \times 10^8$ W/m², $h = 2000$ W/m²·K, and $T_m = 300$ K, what is the maximum temperature in the fuel element?

(c) Compute and plot the temperature distribution, $T(r)$, for values of $h = 2000$, 5000, and 10,000 W/m²·K. If the operator wishes to maintain the centerline temperature of the fuel element below 1000 K, can she do so by adjusting the coolant flow and hence the value of $h$?

3.89 Consider the configuration of Example 3.8, where uniform volumetric heating within a stainless steel tube is induced by an electric current and heat is transferred by convection to air flowing through the tube. The tube wall has inner and outer radii of $r_1 = 25$ mm and $r_2 = 35$ mm, a thermal conductivity of $k = 15$ W/m·K, an electrical resistivity of $\rho_e = 0.7 \times 10^{-6}$ Ω·m, and a maximum allowable operating temperature of 1400 K.

(a) Assuming the outer tube surface to be perfectly insulated and the air flow to be characterized by a temperature and convection coefficient of $T_{w,1} = 400$ K and $h = 100$ W/m²·K, determine the maximum allowable electric current $I$.

(b) Compute and plot the radial temperature distribution in the tube wall for the electric current of part (a) and three values of $h$ (100, 500, and 1000 W/m²·K). For each value of $h$, determine the rate of heat transfer to the air per unit length of tube.

(c) In practice, even the best of insulating materials would be unable to maintain adiabatic conditions at the outer tube surface. Consider use of a refractory insulating material of thermal conductivity $k = 1.0$ W/m·K and neglect radiation exchange at its outer surface. For $h = 100$ W/m²·K and the maximum allowable current determined in part (a), compute and plot the temperature distribution in the composite wall for two values of the insulation thickness ($\delta = 25$ and 50 mm). The outer surface of the insulation is exposed to room air for which $T_m = 300$ K and $h_i = 25$ W/m²·K. For each insulation thickness, determine the rate of heat transfer per unit length to the inner air flow and the ambient air.

3.90 A homeowner, whose water pipes have frozen during a period of cold weather, decides to melt the ice by passing an electric current $I$ through the pipe wall. The inner and outer radii of the wall are designated as $r_1$ and $r_2$, and its electrical resistance per unit length is designated as $R'_e$ (Ω/m). The pipe is well insulated on the outside, and during melting the ice (and water) in the pipe remains at a constant temperature $T_m$ associated with the melting process.

(a) Assuming that steady-state conditions are reached shortly after application of the current, determine the form of the steady-state temperature distribution $T(r)$ in the pipe wall during the melting process.

(b) Develop an expression for the time $t_m$ required to completely melt the ice. Calculate this time for $I = 100$ A, $R'_e = 0.30$ Ω/m, and $r_1 = 50$ mm.

3.91 A high-temperature, gas-cooled nuclear reactor consists of a composite cylindrical wall for which a thorium fuel element ($k = 57$ W/m·K) is encased in graphite ($k \approx 3$ W/m·K) and gaseous helium flows through an annular coolant channel. Consider conditions for which the helium temperature is $T_w = 600$ K and the convection coefficient at the outer surface of the graphite is $h = 2000$ W/m²·K.

(a) If thermal energy is uniformly generated in the fuel element at a rate $q = 10^8$ W/m³, what are the temperatures $T_1$ and $T_2$ at the inner and outer surfaces, respectively, of the fuel element?

(b) Compute and plot the temperature distribution in the composite wall for selected values of $q$. What is the maximum allowable value of $q$?

3.92 A long cylindrical rod of diameter 200 mm with thermal conductivity of 0.5 W/m·K experiences uniform volumetric heat generation of 24,000 W/m³. The rod is encapsulated by a circular sleeve having an outer diameter of 400 mm and a thermal conductivity of 4 W/m·K. The outer surface of the sleeve is exposed to cross flow of air at 27°C with a convection coefficient of 25 W/m²·K.

(a) Find the temperature at the interface between the rod and sleeve and on the outer surface.

(b) What is the temperature at the center of the rod?
design in the same generation remains of a considerably higher wall thickness and preferably n.

(a) On the same graph, plot the steady-state dimensionless temperature, \( \left( T(x \text{ or } r) - T(0) \right) / (T(0) - T_0) \), versus the dimensionless characteristic length, \( x/a \) or \( r/a \), for each shape.

(b) Which shape has the smallest temperature difference between the center and the surface? Explain this behavior by comparing the ratio of the volume-to-surface area.

(c) Which shape would be preferred for use as a nuclear fuel element? Explain why.

Extended Surfaces

3.98 The radiation heat gage shown in the diagram is made from constantan metal foil, which is coated black and is in the form of a circular disk of radius \( R \) and thickness \( t \). The gage is located in an evacuated enclosure. The incident radiation flux absorbed by the foil, \( q_r^\text{rad} \), diffuses toward the outer circumference and into the larger copper ring, which acts as a heat sink at the constant temperature \( T(R) \). Two copper lead wires are attached to the center of the foil and to the ring to complete a thermocouple circuit that allows for measurement of the temperature difference between the foil center and the foil edge, \( \Delta T = T(0) - T(R) \).

Obtain the differential equation that determines \( T(r) \), the temperature distribution in the foil, under steady-state conditions. Solve this equation to obtain an expression relating \( \Delta T \) to \( q_r^\text{rad} \). You may neglect radiation exchange between the foil and its surroundings.

3.99 Copper tubing is joined to the absorber of a flat-plate solar collector as shown.

The aluminum alloy (2024-T6) absorber plate is 6 mm thick and well insulated on its bottom. The top surface of the plate is separated from a transparent cover plate by an evacuated space. The tubes are spaced a distance \( L \) of 0.20 m from each other, and water is circulated through the tubes to remove the collected energy. The water may be assumed to be at a uniform temperature of \( T_w = 60°C \). Under steady-state operating conditions for which the net radiation heat flux to the surface is \( q_{\text{rad}}^\text{net} = 800 \text{ W/m}^2 \), what is the maximum temperature on the plate and the heat transfer rate per unit length of tube? Note that \( q_{\text{rad}}^\text{net} \) represents the net effect of solar radiation absorption by the absorber plate and radiation exchange between the absorber and cover plate. You may assume the temperature of the absorber plate directly above a tube to be equal to that of the water.

3.100 Copper tubing is joined to a solar collector plate of thickness \( t \), and the working fluid maintains the temperature of the plate above the tubes at \( T_w \). There is a uniform net radiation heat flux \( q_{\text{rad}}^\text{net} \) to the top surface of the plate, while the bottom surface is well insulated. The top surface is also exposed to a fluid at \( T_f \) that provides for a uniform convection coefficient \( h \).

(a) Derive the differential equation that governs the temperature distribution \( T(x) \) in the plate.

(b) Obtain a solution to the differential equation for appropriate boundary conditions.

3.101 A thin flat plate of length \( L \), thickness \( t \), and width \( W \gg L \) is thermally joined to two large heat sinks that are maintained at a temperature \( T_0 \). The bottom of the
plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of \( q'_n \).

(a) Derive the differential equation that determines the steady-state temperature distribution \( T(x) \) in the plate.

(b) Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

3.102 Consider the flat plate of Problem 3.101, but with the heat sinks at different temperatures, \( T(0) = T_s \) and \( T(L) = T_o \), and with the bottom surface no longer insulated. Convection heat transfer is now allowed to occur between this surface and a fluid at \( T_w \), with a convection coefficient \( h \).

(a) Derive the differential equation that determines the steady-state temperature distribution \( T(x) \) in the plate.

(b) Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

(c) For \( q'_n = 20,000 \text{ W/m}^2 \), \( T_s = 100\degree \text{C} \), \( T_o = 35\degree \text{C} \), \( T_w = 25\degree \text{C} \), \( k = 25 \text{ W/m} \cdot \text{K} \), \( h = 50 \text{ W/m} \cdot \text{K} \), \( L = 100 \text{ mm} \), \( t = 5 \text{ mm} \), and a plate width of \( W = 30 \text{ mm} \), plot the temperature distribution and determine the sink heat rates, \( q_s(0) \) and \( q_s(L) \). On the same graph, plot three additional temperature distributions corresponding to changes in the following parameters, with the remaining parameters unchanged: (i) \( q'_n = 50,000 \text{ W/m}^2 \), (ii) \( h = 200 \text{ W/m} \cdot \text{K} \), and (iii) the value of \( q'_n \) for which \( q_s(0) = 0 \) when \( h = 200 \text{ W/m} \cdot \text{K} \).

3.103 A bonding operation utilizes a laser to provide a constant heat flux, \( q'_n \), across the top surface of a thin adhesive-backed, plastic film to be affixed to a metal strip as shown in the sketch. The metal strip has a thickness \( d = 1.25 \text{ mm} \) and its width is large relative to that of the film. The thermophysical properties of the strip are \( \rho = 7850 \text{ kg/m}^3 \), \( c_p = 435 \text{ J/kg} \cdot \text{K} \), and \( k = 60 \text{ W/m} \cdot \text{K} \). The thermal resistance of the plastic film of width \( w_1 = 40 \text{ mm} \) is negligible. The upper and lower surfaces of the strip (including the plastic film) experience convection with air at 25\degree \text{C} and a convection coefficient of 10 \text{ W/m}^2 \cdot \text{K}. The strip and film are very long in the direction normal to the page. Assume the edges of the metal strip are at the air temperature \( T_w \).

(a) Derive an expression for the temperature distribution in the portion of the steel strip with the plastic film \(( -w_1/2 \leq x \leq +w_1/2 \)).

(b) If the heat flux provided by the laser is 10,000 \text{ W/m}^2, determine the temperature of the plastic film at the center \(( x = 0 \)) and its edges \(( x = \pm w_1/2 \)).

(c) Plot the temperature distribution for the entire strip and point out its special features.

3.104 A thin metallic wire of thermal conductivity \( k \), diameter \( D \), and length \( 2L \) is annealed by passing an electrical current through the wire to induce a uniform volumetric heat generation \( q \). The ambient air around the wire is at a temperature \( T_w \) while the ends of the wire at \( x = \pm L \) are also maintained at \( T_w \). Heat transfer from the wire to the air is characterized by the convection coefficient \( h \). Obtain an expression for the steady-state temperature distribution \( T(x) \) along the wire.

3.105 A motor draws electric power \( P_{\text{elec}} \) from a supply line and delivers mechanical power \( P_{\text{mech}} \) to a pump through a rotating copper shaft of thermal conductivity \( k_s \), length \( L \), and diameter \( D \). The motor is mounted on a square pad of width \( W \), thickness \( t \), and thermal conductivity \( k_p \). The surface of the housing exposed to ambient air at \( T_w \) is of area \( A_h \), and the corresponding convection coefficient is \( h_s \). Opposite ends of the shaft are at temperatures \( T_1 \) and \( T_w \), and heat transfer from the shaft to the ambient air is characterized by the convection coefficient \( h_p \). The base of the pad is at \( T_w \).
### Problems

#### Simple Fins

3.119 A long, circular aluminum rod is attached at one end to a heated wall and transfers heat by convection to a cold fluid.

(a) If the diameter of the rod is tripled, by how much would the rate of heat removal change?

(b) If a copper rod of the same diameter is used in place of the aluminum, by how much would the rate of heat removal change?

3.120 A brass rod 100 mm long and 5 mm in diameter extends horizontally from a casting at 200°C. The rod is in an air environment with $T_a = 20^\circ$C and $h = 30$ W/m$^2$·K. What is the temperature of the rod 25, 50, and 100 mm from the casting?

3.121 The extent to which the tip condition affects the thermal performance of a fin depends on the fin geometry and thermal conductivity, as well as the convection coefficient. Consider an alloyed aluminum ($k = 180$ W/m·K) rectangular fin of length $L = 10$ mm, thickness $t = 1$ mm, and width $w \gg t$. The base temperature of the fin is $T_b = 100^\circ$C, and the fin is exposed to a fluid of temperature $T_a = 25^\circ$C.

(a) Assuming a uniform convection coefficient of $h = 100$ W/m$^2$·K over the entire fin surface, determine the fin heat transfer rate per unit width $q_i$, efficiency $\eta_i$, effectiveness $\varepsilon_i$, thermal resistance per unit width $R_{i,f}$, and the tip temperature $T_L$ for Cases A and B of Table 3.4. Contrast your results with those based on an infinite fin approximation.

(b) Explore the effect of variations in $L$ on the heat rate for $3 < L < 50$ mm. Also consider the effect of such variations for a stainless steel fin ($k = 15$ W/m·K).

3.123 A straight fin fabricated from 2024 aluminum alloy ($k = 185$ W/m·K) has a base thickness of $t = 3$ mm and a length of $L = 15$ mm. Its base temperature is $T_b = 100^\circ$C, and it is exposed to a fluid for which $T_a = 20^\circ$C and $h = 50$ W/m$^2$·K. For the foregoing conditions and a fin of unit width, compare the fin heat rate, efficiency, and volume for rectangular, triangular, and parabolic profiles.

3.124 Two long copper rods of diameter $D = 10$ mm are soldered together end to end, with solder having a melting point of 650°C. The rods are in air at 25°C with a convection coefficient of 10 W/m$^2$·K. What is the minimum power input needed to effect the soldering?

3.125 Circular copper rods of diameter $D = 1$ mm and length $L = 25$ mm are used to enhance heat transfer from a surface that is maintained at $T_{a1} = 100^\circ$C. One end of the rod is attached to this surface (at $x = 0$), while the other end ($x = 25$ mm) is joined to a second surface, which is maintained at $T_{a2} = 0^\circ$C. Air flowing between the surfaces (and over the rods) is also at a temperature of $T_w = 0^\circ$C, and a convection coefficient of $h = 100$ W/m$^2$·K is maintained.

(a) What is the rate of heat transfer by convection from a single copper rod to the air?

(b) What is the total rate of heat transfer from a 1-m by 1-m section of the surface at 100°C, if a bundle of the rods is installed on 4-mm centers?

3.126 During the initial stages of the growth of the nanowire of Problem 3.109, a slight perturbation of the liquid catalyst droplet can cause it to be suspended on the top of the nanowire in an off-center position. The resulting uniform deposition of solid at the solid-liquid interface can be manipulated to form engineered shapes such as a nanospring, that is characterized by a spring radius, $r$, spring pitch, $s$, overall chord length, $L_c$ (length running along the spring), and end-to-end length, $L$, as shown in the sketch. Consider a silicon carbide nanospring of diameter $D = 15$ nm, $r = 30$ nm, $s = 25$ nm, and $L_c = 425$ nm. From experiments, it is known that the average spring pitch $\bar{s}$ varies with average temperature $\bar{T}$ by the relation $\bar{s} = 0.1$ mm/K. Using this information, a student suggests that a nanospring...
3.134 As more and more components are placed on a single integrated circuit (chip), the amount of heat that is dissipated continues to increase. However, this increase is limited by the maximum allowable chip operating temperature, which is approximately 75°C. To maximize heat dissipation, it is proposed that a $4 \times 4$ array of copper pin fins be metallurgically joined to the outer surface of a square chip that is 12.7 mm on a side.

(a) Sketch the equivalent thermal circuit for the pin–chip–board assembly, assuming one-dimensional, steady-state conditions and negligible contact resistance between the pins and the chip. In variable form, label appropriate resistances, temperatures, and heat rates.

(b) For the conditions prescribed in Problem 3.27, what is the maximum rate at which heat can be dissipated in the chip when the pins are in place? That is, what is the value of $q_c$ for $T_c = 75°C$? The pin diameter and length are $D_p = 1.5$ mm and $L_p = 15$ mm.

3.135 In Problem 3.134, the prescribed value of $h_w = 1000$ W/m²·K is large and characteristic of liquid cooling. In practice it would be far more preferable to use air cooling, for which a reasonable upper limit to the convection coefficient would be $h_w = 250$ W/m²·K. Assess the effect of changes in the pin fin geometry on the chip heat rate if the remaining conditions of Problem 3.134, including a maximum allowable chip temperature of 75°C, remain in effect. Parametric variations that may be considered include the total number of pins, $N$, in the square array, the pin diameter $D_p$, and the pin length $L_p$. However, the product $N D_p^2$ should not exceed 9 mm to ensure adequate air flow passage through the array. Recommend a design that enhances chip cooling.

3.136 As a means of enhancing heat transfer from high-performance logic chips, it is common to attach a heat sink to the chip surface in order to increase the surface area available for convection heat transfer. Because of the ease with which it may be manufactured (by taking orthogonal sawcuts in a block of material), an attractive option is to use a heat sink consisting of an array of square fins of width $w$ on a side. The spacing between adjoining fins would be determined by the width of the sawblade, with the sum of this spacing and the fin width designated as the fin pitch $S$. The method by which the heat sink is joined to the chip would determine the interfacial contact resistance, $R_{sc}$.

Consider a square chip of width $W_c = 16$ mm and conditions for which cooling is provided by a dielectric liquid with $T_c = 25°C$ and $h = 1500$ W/m²·K. The heat sink is fabricated from copper ($k = 400$ W/m·K), and its characteristic dimensions are $w = 0.25$ mm, $S = 0.50$ mm, $L_f = 6$ mm, and $L_x = 3$ mm. The prescribed values of $w$ and $S$ represent minima imposed by manufacturing constraints and the need to maintain adequate flow in the passages between fins.

(a) If a metallurgical joint provides a contact resistance of $R_{sc} = 5 \times 10^{-6}$ m²·K/W and the maximum allowable chip temperature is 85°C, what is the maximum allowable chip power dissipation $q_c$? Assume all of the heat to be transferred through the heat sink.

(b) It may be possible to increase the heat dissipation by increasing $w$, subject to the constraint that $(S - w) \geq 0.25$ mm, and/or increasing $L_f$ (subject to manufacturing constraints that $L_f \leq 10$ mm). Assess the effect of such changes.

3.137 Because of the large number of devices in today’s PC chips, finned heat sinks are often used to maintain the chip at an acceptable operating temperature. Two fin
4.12 An electrical heater 100 mm long and 5 mm in diameter is inserted into a hole drilled normal to the surface of a large block of material having a thermal conductivity of 5 W/m·K. Estimate the temperature reached by the heater when dissipating 50 W with the surface of the block at a temperature of 25°C.

4.13 Two parallel pipelines spaced 0.5 m apart are buried in soil having a thermal conductivity of 0.5 W/m·K. The pipes have outer diameters of 100 and 75 mm with surface temperatures of 175°C and 5°C, respectively. Estimate the heat transfer rate per unit length between the two pipelines.

4.14 A tube of diameter 50 mm having a surface temperature of 85°C is embedded in the center plane of a concrete slab 0.1 m thick with upper and lower surfaces at 20°C. Using the appropriate tabulated relation for this configuration, find the shape factor. Determine the heat transfer rate per unit length of the tube.

4.15 Pressurized steam at 450 K flows through a long, thin-walled pipe of 0.5-m diameter. The pipe is enclosed in a concrete casing that is of square cross section and 1.5 m on a side. The axis of the pipe is centered in the casing, and the outer surfaces of the casing are maintained at 300 K. What is the heat loss per unit length of pipe?

4.16 Hot water at 85°C flows through a thin-walled copper tube of 30 mm diameter. The tube is enclosed by an eccentric cylindrical shell that is maintained at 35°C and has a diameter of 120 mm. The eccentricity, defined as the separation between the centers of the tube and shell, is 20 mm. The space between the tube and shell is filled with an insulating material having a thermal conductivity of 0.05 W/m·K. Calculate the heat loss per unit length of the tube and compare the result with the heat loss for a concentric arrangement.

4.17 A furnace of cubical shape, with external dimensions of 0.35 m, is constructed from a refractory brick (fireclay). If the wall thickness is 50 mm, the inner surface temperature is 600°C, and the outer surface temperature is 75°C, calculate the heat loss from the furnace.

4.18 The temperature distribution in laser-irradiated materials is determined by the power, size, and shape of the laser beam, along with the properties of the material being irradiated. The beam shape is typically Gaussian, and the local beam irradiation flux (often referred to as the laser fluence) is

\[ q(x, y) = q^0(x = y = 0) \exp(-x^2a^2) \exp(-y^2b^2) \]

where \( a \) and \( b \) are the standard deviations of the Gaussian distribution. The laser power \( P \) is incident on a circular area of radius \( r_0 \) and the fluence is defined as the energy per unit area.

4.19 Laser beams are used to thermally process materials in a wide range of applications. Often, the beam is scanned along the surface of the material in a desired pattern. Consider the laser heating process of Problem 4.18, except now the laser beam scans the material at a scanning velocity of \( U \). A dimensionless maximum surface temperature can be well correlated by an expression of the form [Nisson, Y. I., A. Lietoila, R. G. Gold, and J. F. Gibbons, J. Appl. Phys., 51, 274, 1980]

\[
\frac{T_{1, \text{max}, U=0} - T_2}{T_{1, \text{max}, U=0} - T_2} = 1 + 0.301Pe - 0.0108Pe^2
\]

for the range \( 0 < Pe < 10 \) where \( Pe \) is a dimensionless velocity known as the Peclet number. For this problem, \( Pe = UR_0\sqrt{2\alpha} \) where \( \alpha \) is the thermal diffusivity of the material. The maximum material temperature does not occur directly below the laser beam, but a lag distance, \( \delta \), behind the center of the moving beam. The dimensionless lag distance can be correlated to \( Pe \) by [Sheng, I. C., and Y. Chen, J. Thermal Stresses, 14, 129, 1991]

\[
\frac{\delta U}{a} = 0.944Pe^{1.55}
\]

(a) For the laser beam size and shape, and material of Problem 4.18, determine the laser power required to achieve \( T_{1, \text{max}} = 200°C \) for \( U = 2 \text{ m/s} \). The density and specific heat of the material are \( \rho = 2000 \text{ kg/m}^3 \) and \( c = 800 \text{ J/kg} \cdot \text{K} \), respectively.

(b) Determine the lag distance, \( \delta \), associated with \( U = 2 \text{ m/s} \).

(c) Plot the required laser power to achieve \( T_{\text{max},1} = 200°C \) for \( 0 \leq U \leq 2 \text{ m/s} \).
Consider conditions for which a long aluminum pin fin of diameter $D = 5$ mm is attached to a base material whose temperature far from the junction is maintained at $T_b = 100$°C. Fin convection conditions correspond to $h = 50 \text{ W/m}^2 \cdot \text{K}$ and $T_w = 25$°C.

(a) What are the fin heat rate and junction temperature when the base material is (i) aluminum ($k = 240 \text{ W/m} \cdot \text{K}$) and (ii) stainless steel ($k = 15 \text{ W/m} \cdot \text{K}$)?

(b) Repeat the foregoing calculations if a thermal contact resistance of $R^c_{th} = 3 \times 10^{-5} \text{ m}^2 \cdot \text{K/W}$ is associated with the method of joining the pin fin to the base material.

(c) Considering the thermal contact resistance, plot the heat rate as a function of the convection coefficient over the range $10 \leq h \leq 100 \text{ W/m}^2 \cdot \text{K}$ for each of the two materials.

4.25 An igloo is built in the shape of a hemisphere, with an inner radius of $1.8$ m and walls of compacted snow that are $0.5$ m thick. On the inside of the igloo the surface heat transfer coefficient is $6 \text{ W/m}^2 \cdot \text{K}$; on the outside, under normal wind conditions, it is $15 \text{ W/m}^2 \cdot \text{K}$. The thermal conductivity of compacted snow is $0.15 \text{ W/m} \cdot \text{K}$. The temperature of the ice cap on which the igloo sits is $-20$°C and has the same thermal conductivity as the compacted snow.

(a) Assuming that the occupants' body heat provides a continuous source of $320$ W within the igloo, calculate the inside air temperature when the outside air temperature is $T_a = -40$°C. Be sure to consider heat losses through the floor of the igloo.

(b) Using the thermal circuit of part (a), perform a parameter sensitivity analysis to determine which variables have a significant effect on the inside air temperature. For instance, for very high wind conditions, the outside convection coefficient could double, or even triple. Does it make sense to construct the igloo with walls half or twice as thick?

4.26 Consider the thin integrated circuit (chip) of Problem 3.136. Instead of attaching the heat sink to the chip surface, an engineer suggests that sufficient cooling might be achieved by mounting the top of the chip onto a large copper ($k = 400 \text{ W/m} \cdot \text{K}$) surface that is located nearby. The metallurgical joint between the chip and the substrate provides a contact resistance of $R^c_{th} = 5 \times 10^{-5} \text{ m}^2 \cdot \text{K/W}$, and the maximum allowable chip temperature is $85$°C. If the large substrate temperature is $T_2 = 25$°C at locations far from the chip, what is the maximum allowable chip power dissipation $q_e$?

4.27 An electronic device, in the form of a disk $20$ mm in diameter, dissipates $100$ W when mounted flush on a large aluminum alloy (2024) block whose temperature is maintained at $27$°C. The mounting arrangement is such that a contact resistance of $R^c_{th} = 5 \times 10^{-5} \text{ m}^2 \cdot \text{K/W}$ exists at the interface between the device and the block.

(a) Calculate the temperature the device will reach, assuming that all the power generated by the device must be transferred by conduction to the block.

(b) In order to operate the device at a higher power level, a circuit designer proposes to attach a finned heat sink to the top of the device. The pin fins and base material are fabricated from copper ($k = 400 \text{ W/m} \cdot \text{K}$) and are exposed to an airstream at $27$°C for which the convection coefficient is $1000 \text{ W/m}^2 \cdot \text{K}$. For the device temperature computed in part (a), what is the permissible operating power?

4.28 An aluminum heat sink ($k = 240 \text{ W/m} \cdot \text{K}$) used to cool an array of electronic chips consists of a square channel of inner width $w = 25$ mm, through which liquid flow may be assumed to maintain a uniform surface temperature of
Problem

A grid has been constructed and the nodes labeled. The temperatures for nodes 1, 3, 6, 8, and 9 are identified.

\[ T_1 = 300 \text{ K}, \quad T_3 = 300 \text{ K} \]
\[ T_6 = 394 \text{ K}, \quad T_8 = 600 \text{ K} \]

(a) Beginning with properly defined control volumes, derive the finite-difference equations for nodes 2, 4, and 7 and determine the temperatures \( T_2, T_4, \) and \( T_7 \) (K).

(b) Calculate the heat loss per unit length from the channel.

4.45 Steady-state temperatures (K) at three nodal points of a long rectangular rod are as shown. The rod experiences a uniform volumetric generation rate of \( 5 \times 10^6 \text{ W/m}^3 \) and has a thermal conductivity of \( 20 \text{ W/m} \cdot \text{K} \). Two of its sides are maintained at a constant temperature of 300 K, while the others are insulated.

(a) Determine the temperatures at nodes 1, 4, 7, and 9.

(b) Calculate the heat rate per unit length (W/m) from the outer surface A to the adjacent fluid.

(c) Calculate the heat rate per unit length from the inner fluid to surface B.

(d) Verify that your results are consistent with an overall energy balance on the channel section.

4.47 Consider an aluminum heat sink \( (k = 240 \text{ W/m} \cdot \text{K}) \), such as that shown schematically in Problem 4.28. The inner and outer widths of the square channel are \( w = 20 \text{ mm} \) and \( W = 40 \text{ mm} \), respectively, and an outer surface temperature of \( T_s = 50^\circ \text{C} \) is maintained by the array of electronic chips. In this case, it is not the inner surface temperature that is known, but conditions \((T_w, h)\) associated with coolant flow through the channel, and the question is to determine the rate of heat transfer to the coolant per unit length of channel. For this purpose, consider a symmetrical section of the channel and a two-dimensional grid with \( \Delta x = \Delta y = 5 \text{ mm} \).

(a) For \( T_w = 20^\circ \text{C} \) and \( h = 5000 \text{ W/m}^2 \cdot \text{K} \), determine the unknown temperatures, \( T_1, \ldots, T_7 \), and the rate of heat transfer per unit length of channel, \( q' \).

(b) Assess the effect of variations in \( h \) on the unknown temperatures and the heat rate.
4.48 The steady-state temperatures (°C) associated with selected nodal points of a two-dimensional system having a thermal conductivity of 1.5 W/m·K are shown on the accompanying grid.

(a) Determine the temperatures at nodes 1, 2, and 3.
(b) Calculate the heat transfer rate per unit thickness normal to the page from the system to the fluid.

4.49 A steady-state, finite-difference analysis has been performed on a cylindrical fin with a diameter of 12 mm and a thermal conductivity of 15 W/m·K. The convection process is characterized by a fluid temperature of 25°C and a heat transfer coefficient of 25 W/m²·K.

(a) The temperatures for the first three nodes, separated by a spatial increment of \( \Delta x = 10 \text{ mm} \), are given in the sketch. Determine the fin heat rate.
(b) Determine the temperature at node 3, \( T_3 \).

Solving the Finite-Difference Equations

4.50 A long bar of rectangular cross section is 60 mm by 90 mm on a side and has a thermal conductivity of 1 W/m·K. One surface is exposed to a convection process with air at 100°C and a convection coefficient of 100 W/m²·K, while the remaining surfaces are maintained at 50°C.

(a) Determine the temperatures at nodes 1, 2, 3, and 4. Estimate the midpoint temperature.
(b) Reducing the mesh size by a factor of 2, determine the corresponding nodal temperatures. Compare your results with those from the coarser grid.
(c) From the results for the finer grid, plot the 75, 150, and 250°C isotherms.

4.52 Consider a long bar of square cross section (0.8 m to the side) and of thermal conductivity 2 W/m·K. Three of these sides are maintained at a uniform temperature of 300°C. The fourth side is exposed to a fluid at 100°C for which the convection heat transfer coefficient is 10 W/m²·K.
4.53 A long conducting rod of rectangular cross section (20 mm × 30 mm) and thermal conductivity $k = 20 \text{ W/m} \cdot \text{K}$ experiences uniform heat generation at a rate $q = 5 \times 10^7 \text{ W/m}^2$, while its surfaces are maintained at 300 K.

(a) Using a finite-difference method with a grid spacing of 5 mm, determine the temperature distribution in the rod.

(b) With the boundary conditions unchanged, what heat generation rate will cause the midpoint temperature to reach 600 K?

4.54 A flue passing hot exhaust gases has a square cross section, 300 mm to a side. The walls are constructed of refractory brick 150 mm thick with a thermal conductivity of 0.85 W/m·K. Calculate the heat loss from the flue per unit length when the interior and exterior surfaces are maintained at 350 and 25°C, respectively. Use a grid spacing of 75 mm.

4.55 Consider the system of Problem 4.54. The interior surface is exposed to hot gases at 350°C with a convection coefficient of 100 W/m²·K, while the exterior surface experiences convection with air at 25°C and a convection coefficient of 5 W/m²·K.

(a) Using a grid spacing of 75 mm, calculate the temperature field within the system and determine the heat loss per unit length by convection from the outer surface of the flue to the air. Compare this result with the heat gained by convection from the hot gases to the air.

(b) Determine the effect of grid spacing on the temperature field and heat loss per unit length to the air. Specifically, consider a grid spacing of 25 mm and plot appropriately spaced isotherms on a schematic of the system. Explore the effect of changes in the convection coefficients on the temperature field, and heat loss.

4.56 A common arrangement for heating a large surface area is to move warm air through rectangular ducts below the surface. The ducts are square and located midway between the top and bottom surfaces that are exposed to room air and insulated, respectively.

![Diagram showing air ducts with temperatures $T_1 = 30°C$ and $T_2 = 80°C$.]

For the condition when the floor and duct temperatures are 30 and 80°C, respectively, and the thermal conductivity of concrete is 1.4 W/m·K, calculate the heat rate from each duct, per unit length of duct. Use a grid spacing with $\Delta x = 2 \Delta y$, where $\Delta y = 0.125 L$ and $L = 150$ mm.

4.57 Consider the gas turbine cooling scheme of Example 4.4. In Problem 3.23, advantages associated with applying a thermal barrier coating (TBC) to the exterior surface of a turbine blade are described. If a 0.5-mm-thick zirconia coating ($k=1.3 \text{ W/m} \cdot \text{K}$, $R_{th}^e = 10^{-4} \text{ m}^2 \cdot \text{K/W}$) is applied to the outer surface of the air-cooled blade, determine the temperature field in the blade for the operating conditions of Example 4.4.

4.58 A long bar of rectangular cross section, 0.4 m × 0.6 m on a side and having a thermal conductivity of 1.5 W/m·K, is subjected to the boundary conditions shown below. Two of the sides are maintained at a uniform temperature of 200°C. One of the sides is adiabatic, and the remaining side is subjected to a convection process with $T_a = 30°C$ and $h = 50 \text{ W/m}^2 \cdot \text{K}$. Using an appropriate numerical technique with a grid spacing of 0.1 m, determine the temperature distribution in the bar and the heat transfer rate between the bar and the fluid per unit length of the bar.

![Diagram showing uniform temperature, $T = 200°C$.]

4.59 The top surface of a plate, including its grooves, is maintained at a uniform temperature of $T_1 = 200°C$. The lower surface is at $T_2 = 20°C$, the thermal conductivity is 15 W/m·K, and the groove spacing is 0.16 m.