References


Problems

Conduction

1.1 The thermal conductivity of a sheet of rigid, extruded insulation is reported to be $k = 0.029 \text{ W/m} \cdot \text{K}$. The measured temperature difference across a 20-mm-thick sheet of the material is $T_1 - T_2 = 10^\circ\text{C}$.
(a) What is the heat flux through a 2 m $\times$ 2 m sheet of the insulation?
(b) What is the rate of heat transfer through the sheet of insulation?

1.2 A concrete wall, which has a surface area of 20 m$^2$ and is 0.30 m thick, separates conditioned room air from ambient air. The temperature of the inner surface of the wall is maintained at 25°C, and the thermal conductivity of the concrete is 1 W/m $\cdot$ K.
(a) Determine the heat loss through the wall for outer surface temperatures ranging from $-15^\circ\text{C}$ to $38^\circ\text{C}$, which correspond to winter and summer extremes, respectively. Display your results graphically.
(b) On your graph, also plot the heat loss as a function of the outer surface temperature for wall materials having thermal conductivities of 0.75 and 1.25 W/m $\cdot$ K. Explain the family of curves you have obtained.

1.3 The concrete slab of a basement is 11 m long, 8 m wide, and 0.20 m thick. During the winter, temperatures are nominally 17°C and 10°C at the top and bottom surfaces, respectively. If the concrete has a thermal conductivity of 1.4 W/m $\cdot$ K, what is the rate of heat loss through the slab? If the basement is heated by a gas furnace operating at an efficiency of $\eta = 0.90$ and natural gas is priced at $C_g = 30.01/\text{MJ}$, what is the daily cost of the heat loss?

1.4 The heat flux through a wood slab 50 mm thick, whose inner and outer surface temperatures are 40 and 20°C, respectively, has been determined to be 40 W/m$^2$. What is the thermal conductivity of the wood?

1.5 The inner and outer surface temperatures of a glass window 5 mm thick are 15 and 5°C. What is the heat loss through a window that is 1 m by 3 m on a side? The thermal conductivity of glass is 1.4 W/m $\cdot$ K.

1.6 A glass window of width $W = 1$ m and height $H = 2$ m is 5 mm thick and has a thermal conductivity of $k_g = 1.4 \text{ W/m} \cdot \text{K}$. If the inner and outer surface temperatures of the glass are 15°C and $-20^\circ\text{C}$, respectively, on a cold winter day, what is the rate of heat loss through the glass? To reduce heat loss through windows, it is customary to use a double pane construction in which adjoining panes are separated by an air space. If the spacing is 10 mm and the glass surfaces in contact with the air have temperatures of $10^\circ\text{C}$ and $-15^\circ\text{C}$, what is the rate of heat loss from a 1 m $\times$ 2 m window? The thermal conductivity of air is $k_a = 0.024 \text{ W/m} \cdot \text{K}$.

1.7 A freezer compartment consists of a cubical cavity that is 2 m on a side. Assume the bottom to be perfectly insulated. What is the minimum thickness of styrofoam insulation ($k = 0.030 \text{ W/m} \cdot \text{K}$) that must be applied to the top and side walls to ensure a heat load of less than 500 W, when the inner and outer surfaces are $-10$ and $35°C$?

1.8 An inexpensive food and beverage container is fabricated from 25-mm-thick polystyrene ($k = 0.023 \text{ W/m} \cdot \text{K}$) and has interior dimensions of 0.8 m $\times$ 0.6 m $\times$ 0.6 m. Under conditions for which an inner surface temperature of approximately 2°C is maintained by an ice-water mixture and an outer surface temperature of 20°C is maintained by the ambient, what is the heat flux through the container wall? Assuming negligible heat gain through the 0.8 m $\times$ 0.6 m base of the cooler, what is the total heat load for the prescribed conditions?

1.9 What is the thickness required of a masonry wall having thermal conductivity 0.75 W/m $\cdot$ K if the heat rate is to be 80% of the heat rate through a composite structural wall having a thermal conductivity of 0.25 W/m $\cdot$ K and a thickness of 100 mm? Both walls are subjected to the same surface temperature difference.

1.10 The 5-mm-thick bottom of a 200-mm-diameter pan may be made from aluminum ($k = 240 \text{ W/m} \cdot \text{K}$) or
copper \((k = 390 \text{ W/m} \cdot \text{K})\). When used to boil water, the surface of the bottom exposed to the water is nominally at 110°C. If heat is transferred from the stove to the pan at a rate of 600 W, what is the temperature of the surface in contact with the stove for each of the two materials?

1.11 A square silicon chip \((k = 150 \text{ W/m} \cdot \text{K})\) is of width \(w = 5 \text{ mm}\) on a side and of thickness \(t = 1 \text{ mm}\). The chip is mounted in a substrate such that its side and back surfaces are insulated, while the front surface is exposed to a coolant.

If 4 W are being dissipated in circuits mounted to the back surface of the chip, what is the steady-state temperature difference between back and front surfaces?

1.12 A gage for measuring heat flux to a surface or through a laminated material employs five thin-film, chromel/ alumel (type K) thermocouples deposited on the upper and lower surfaces of a wafer with a thermal conductivity of 1.4 \(\text{W/m} \cdot \text{K}\) and a thickness of 0.25 mm.

(a) Determine the heat flux \(q^*\) through the gage when the voltage output at the copper leads is 350 \(\mu\text{V}\). The Seebeck coefficient of the type-K thermocouple materials is approximately 40 \(\mu\text{V/}^\circ\text{C}\).

(b) What precaution should you take in using a gage of this nature to measure heat flow through the laminated structure shown?

Convection

1.13 You’ve experienced convection cooling if you’ve ever extended your hand out the window of a moving vehicle or into a flowing water stream. With the surface of your hand at a temperature of 30°C, determine the convection heat flux for (a) a vehicle speed of 35 km/h in air at −5°C with a convection coefficient of 40 \(\text{W/m}^2 \cdot \text{K}\) and (b) a velocity of 0.2 m/s in a water stream at 10°C with a convection coefficient of 900 \(\text{W/m}^2 \cdot \text{K}\). Which condition would feel colder? Contrast these results with a heat loss of approximately 30 \(\text{W/m}^2\) under normal room conditions.

1.14 Air at 40°C flows over a long, 25-mm-diameter cylinder with an embedded electrical heater. In a series of tests, measurements were made of the power per unit length, \(P^*\), required to maintain the cylinder surface temperature at 300°C for different freestream velocities \(V\) of the air. The results are as follows:

<table>
<thead>
<tr>
<th>Air velocity, (V) (m/s)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power, (P^*) (W/m)</td>
<td>450</td>
<td>658</td>
<td>983</td>
<td>1507</td>
<td>1963</td>
</tr>
</tbody>
</table>

(a) Determine the convection coefficient for each velocity, and display your results graphically.

(b) Assuming the dependence of the convection coefficient on the velocity to be of the form \(h = CV^n\), determine the parameters \(C\) and \(n\) from the results of part (a).

1.15 An electric resistance heater is embedded in a long cylinder of diameter 30 mm. When water with a temperature of 25°C and velocity of 1 m/s flows crosswise over the cylinder, the power per unit length required to maintain the surface at a uniform temperature of 90°C is 28 kW/m. When air, also at 25°C, but with a velocity of 10 m/s is flowing, the power per unit length required to maintain the same surface temperature is 400 W/m. Calculate and compare the convection coefficients for the flows of water and air.

1.16 A cartridge electrical heater is shaped as a cylinder of length \(L = 200 \text{ mm}\) and outer diameter \(D = 20 \text{ mm}\). Under normal operating conditions the heater dissipates 2 kW while submerged in a water flow that is at 20°C and provides a convection heat transfer coefficient of \(h = 5000 \text{ W/m}^2 \cdot \text{K}\). Neglecting heat transfer from the ends of the heater, determine its surface temperature \(T_s\). If the water flow is inadvertently terminated while the heater continues to operate, the heater surface is exposed to air that is also at 20°C but for which \(h = 50 \text{ W/m}^2 \cdot \text{K}\). What is the corresponding surface temperature? What are the consequences of such an event?
If the transmission efficiency is $\eta = 0.93$ and airflow over the case corresponds to $T_a = 30^\circ$C and $h = 200$ W/m$^2 \cdot$ K, what is the surface temperature of the transmission?

### Radiation

1.24 Under conditions for which the same room temperature is maintained by a heating or cooling system, it is not uncommon for a person to feel chilled in the winter but comfortable in the summer. Provide a plausible explanation for this situation (with supporting calculations) by considering a room whose air temperature is maintained at 20$^\circ$C throughout the year, while the walls of the room are nominally at 27$^\circ$C and 14$^\circ$C in the summer and winter, respectively. The exposed surface of a person in the room may be assumed to be at a temperature of 32$^\circ$C throughout the year and to have an emissivity of 0.90. The coefficient associated with heat transfer by natural convection between the person and the room air is approximately 2 W/m$^2 \cdot$ K.

1.25 A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150 W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature?

1.26 An instrumentation package has a spherical outer surface of diameter $D = 100$ mm and emissivity $\varepsilon = 0.25$. The package is placed in a large space simulation chamber whose walls are maintained at 77 K. If operation of the electronic components is restricted to the temperature range 40 $\leq T \leq$ 85$^\circ$C, what is the range of acceptable power dissipation for the package? Display your results graphically, showing also the effect of variations in the emissivity by considering values of 0.20 and 0.30.

1.27 Consider the conditions of Problem 1.22. However, now the plate is in a vacuum with a surrounding temperature of 25$^\circ$C. What is the emissivity of the plate? What is the rate at which radiation is emitted by the surface?

1.28 An overhead 25-m-long, uninsulated industrial steam pipe of 100 mm diameter is routed through a building whose walls and air are at 25$^\circ$C. Pressurized steam maintains a pipe surface temperature of 150$^\circ$C, and the coefficient associated with natural convection is $h = 10$ W/m$^2 \cdot$ K. The surface emissivity is $\varepsilon = 0.8$.

(a) What is the rate of heat loss from the steam line?

(b) If the steam is generated in a gas-fired boiler operating at an efficiency of $\eta = 0.90$ and natural gas is priced at $C_g = $0.01 per MJ, what is the annual cost of heat loss from the line?

1.29 If $T_r = T_{sur}$ in Equation 1.9, the radiation heat transfer coefficient may be approximated as

$$h_{rad} = 4\varepsilon\sigma T^3$$

where $\bar{T} = (T_r + T_{sur})/2$. We wish to assess the validity of this approximation by comparing values of $h_r$ and $h_{rad}$ for the following conditions. In each case represent your results graphically and comment on the validity of the approximation.

(a) Consider a surface of either polished aluminum ($\varepsilon = 0.05$) or black paint ($\varepsilon = 0.9$), whose temperature may exceed that of the surroundings ($T_{sur} = 25^\circ$C) by 10 to 100$^\circ$C. Also compare your results with values of the coefficient associated with free convection in air ($T_r = T_{sur}$), where $h = (W/m^2 \cdot K) = 0.98 \Delta T^{0.4}$.

(b) Consider initial conditions associated with placing a workpiece at $T_r = 25^\circ$C in a large furnace whose wall temperature may be varied over the range 100 $\leq T_{sur} \leq$ 1000$^\circ$C. According to the surface finish or coating, its emissivity may assume values of 0.05, 0.2, and 0.9. For each emissivity, plot the relative error, $(h_r - h_{rad})/h_r$, as a function of the furnace temperature.

1.30 Consider the conditions of Problem 1.18. With heat transfer by convection to air, the maximum allowable chip power is found to be 0.35 W. If consideration is also given to net heat transfer by radiation from the chip surface to large surroundings at 15$^\circ$C, what is the percentage increase in the maximum allowable chip power afforded by this consideration? The chip surface has an emissivity of 0.9.

1.31 Chips of width $L = 15$ mm on a side are mounted to a substrate that is installed in an enclosure whose walls and air are maintained at a temperature of $T_{sur} = T_a = 25^\circ$C. The chips have an emissivity of $\varepsilon = 0.60$ and a maximum allowable temperature of $T_r = 85^\circ$C.

(a) If heat is rejected from the chips by radiation and natural convection, what is the maximum operating...
power of each chip? The convection coefficient depends on the chip-to-air temperature difference and may be approximated as $h = C(T_c - T_a)^{1/4}$, where $C = 4.2 \text{ W/m}^2 \cdot \text{K}^{1/4}$.

(b) If a fan is used to maintain air flow through the enclosure and heat transfer is by forced convection, with $h = 250 \text{ W/m}^2 \cdot \text{K}$, what is the maximum operating power?

1.32 A vacuum system, as used in sputtering electrically conducting thin films on microcircuits, is comprised of a baseplate maintained by an electrical heater at 300 K and a shroud within the enclosure maintained at 77 K by a liquid-nitrogen coolant loop. The circular baseplate, insulated on the lower side, is 0.3 m in diameter and has an emissivity of 0.25.

(a) How much electrical power must be provided to the baseplate heater?

(b) At what rate must liquid nitrogen be supplied to the shroud if its heat of vaporization is 125 kJ/kg?

(c) To reduce the liquid-nitrogen consumption, it is proposed to bond a thin sheet of aluminum foil ($\varepsilon = 0.09$) to the baseplate. Will this have the desired effect?

1.33 Consider the transmission case of Problem 1.23, but now allow for radiation exchange with the ground/chassis, which may be approximated as large surroundings at $T_{sur} = 30^\circ \text{C}$. If the emissivity of the case is $\varepsilon = 0.80$, what is the surface temperature?

Energy Balance and Multimode Effects

1.34 An electrical resistor is connected to a battery, as shown schematically. After a brief transient, the resistor assumes a nearly uniform, steady-state temperature of 95°C, while the battery and lead wires remain at the ambient temperature of 25°C. Neglect the electrical resistance of the lead wires.

(a) Consider the resistor as a system about which a control surface is placed and Equation 1.11c is applied. Determine the corresponding values of $\dot{E}_{\text{in}}(\text{W})$, $\dot{E}_{\text{out}}(\text{W})$, $\dot{E}_{\text{rad}}(\text{W})$, and $\dot{E}_{\text{con}}(\text{W})$. If a control surface is placed about the entire system, what are the values of $\dot{E}_{\text{in}}$, $\dot{E}_{\text{out}}$, $\dot{E}_{\text{rad}}$, and $\dot{E}_{\text{con}}$?

(b) If electrical energy is dissipated uniformly within the resistor, which is a cylinder of diameter $D = 60 \text{ mm}$ and length $L = 250 \text{ mm}$, what is the volumetric heat generation rate, $\dot{q} (\text{W/m}^3)$?

(c) Neglecting radiation from the resistor, what is the convection coefficient?

1.35 An aluminum plate 4 mm thick is mounted in a horizontal position, and its bottom surface is well insulated. A special, thin coating is applied to the top surface such that it absorbs 80% of any incident solar radiation, while having an emissivity of 0.25. The density $\rho$ and specific heat $c$ of aluminum are known to be 2700 kg/m$^3$ and 900 J/kg · K, respectively.

(a) Consider conditions for which the plate is at a temperature of 25°C and its top surface is suddenly exposed to ambient air at $T_a = 20^\circ \text{C}$ and to solar radiation that provides an incident flux of 900 W/m$^2$. The convection heat transfer coefficient between the surface and the air is $h = 20 \text{ W/m}^2 \cdot \text{K}$. What is the initial rate of change of the plate temperature?

(b) What will be the equilibrium temperature of the plate when steady-state conditions are reached?

(c) The surface radiative properties depend on the specific nature of the applied coating. Compute and plot the steady-state temperature as a function of the emissivity for $0.05 \leq \varepsilon \leq 1$, with all other conditions remaining as prescribed. Repeat your calculations for values of $\alpha_s = 0.5$ and 1.0, and plot the results with those obtained for $\alpha_s = 0.8$. If the intent is to maximize the plate temperature, what is the most desirable combination of the plate emissivity and its absorptivity to solar radiation?

1.36 A blood warmer is to be used during the transfusion of blood to a patient. This device is to heat blood taken from the blood bank at 10°C to 37°C at a flow rate of 200 ml/min. The blood passes through tubing of length...
(a) For an initial condition corresponding to a wafer temperature of $T_{wa} = 300$ K and the position of the wafer shown schematically, determine the corresponding time rate of change of the wafer temperature, $(dT_w/dt)$.

(b) Determine the steady-state temperature reached by the wafer if it remains in this position. How significant is convection heat transfer for this situation? Sketch how you would expect the wafer temperature to vary as a function of vertical distance.

1.44 Radioactive wastes are packed in a long, thin-walled cylindrical container. The wastes generate thermal energy nonuniformly according to the relation $\dot{q} = \dot{q}_0 [1 - (n r_o)^2]$, where $\dot{q}$ is the local rate of energy generation per unit volume, $\dot{q}_0$ is a constant, and $r_o$ is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid that is at $T_o$ and provides a uniform convection coefficient $h$.

Obtain an expression for the total rate at which energy is generated in a unit length of the container. Use this result to obtain an expression for the temperature $T_i$ of the container wall.

1.45 Consider the conducting rod of Example 1.3 under steady-state conditions. As suggested in Comment 3, the temperature of the rod may be controlled by varying the speed of air flow over the rod, which, in turn, alters the convection heat transfer coefficient. To consider the effect of the convection coefficient, generate plots of $T$ versus $I$ for values of $h = 50, 100,$ and $250$ W/m$^2$ · K. Would variations in the surface emissivity have a significant effect on the rod temperature?

1.46 A long bus bar (cylindrical rod used for making electrical connections) of diameter $D$ is installed in a large conduit having a surface temperature of $30^\circ$C and in which the ambient air temperature is $T_a = 30^\circ$C. The electrical resistivity, $\rho_e (\mu \Omega$ · m), of the bar material is a function of temperature, $\rho_e = \rho_{e0} [1 + \alpha(T - T_a)]$, where $\rho_{e0} = 0.0171$ $\mu \Omega$ · m, $T_e = 25^\circ$C, and $\alpha = 0.00396$ $K^{-1}$. The bar experiences free convection in the ambient air, and the convection coefficient depends on the bar diameter, as well as on the difference between the surface and ambient temperatures. The governing relation is of the form, $h = CD^{0.25}(T - T_a)^{0.25}$, where $C = 1.21$ W · m$^{-1.75}$ · K$^{-1.25}$. The emissivity of the bar surface is $\varepsilon = 0.85$.

(a) Recognizing that the electrical resistance per unit length of the bar is $R_e = \rho_e/A_e$, where $A_e$ is its cross-sectional area, calculate the current-carrying capacity of a 20-mm-diameter bus bar if its temperature is not to exceed $65^\circ$C. Compare the relative importance of heat transfer by free convection and radiation exchange.

(b) To assess the trade-off between current-carrying capacity, operating temperature, and bar diameter, for diameters of 10, 20, and 40 mm, plot the bar temperature $T$ as a function of current for the range $100 \leq I \leq 5000$ A. Also plot the ratio of the heat transfer by convection to the total heat transfer.

1.47 A small sphere of reference-grade iron with a specific heat of 447 J/kg · K and a mass of 0.515 kg is suddenly immersed in a water-ice mixture. Fine thermocouple wires suspend the sphere, and the temperature is observed to change from 15 to $14^\circ$C in 6.35 s. The experiment is repeated with a metallic sphere of the same diameter, but of unknown composition with a mass of 1.263 kg. If the same observed temperature change occurs in 4.59 s, what is the specific heat of the unknown material?

1.48 A spherical, stainless steel (AISI 302) canister is used to store reacting chemicals that provide for a uniform heat flux $q_i$ to its inner surface. The canister is suddenly submerged in a liquid bath of temperature $T_w < T_o$, where $T_i$ is the initial temperature of the canister wall.

(a) Assuming negligible temperature gradients in the canister wall and a constant heat flux $q_i$, develop an equation that governs the variation of the wall temperature with time during the transient process. What is the initial rate of change of the wall temperature if $q_i = 10^5$ W/m$^2$?

(b) What is the steady-state temperature of the wall?

(c) The convection coefficient depends on the velocity associated with fluid flow over the canister and whether or not the wall temperature is large enough to induce boiling in the liquid. Compute and plot the steady-state temperature as a function of $h$ for
bodily functions and is ultimately lost as heat from our bodies. Consider a person who consumes 2100 kcal per day (note that what are commonly referred to as food calories are actually kilocalories), of which 2000 kcal is converted to thermal energy. (The remaining 100 kcal is used to do work on the environment.) The person has a surface area of 1.8 m² and is dressed in a bathing suit.

(a) The person is in a room at 20°C, with a convection heat transfer coefficient of 3 W/m²·K. At this air temperature, the person is not perspiring much. Estimate the person’s average skin temperature.

(b) If the temperature of the environment were 33°C, what rate of perspiration would be needed to maintain a comfortable skin temperature of 33°C?

1.58 Single fuel cells such as the one of Example 1.4 can be sealed up by arranging them into a fuel cell stack. A stack consists of multiple electrolytic membranes that are sandwiched between electrically conducting bipolar plates. Air and hydrogen are fed to each membrane through flow channels within each bipolar plate, as shown in the sketch. With this stack arrangement, the individual fuel cells are connected in series, electrically, producing a stack voltage of $E_{\text{stack}} = N \times E_s$, where $E_s$ is the voltage produced across each membrane and $N$ is the number of membranes in the stack. The electrical current is the same for each membrane. The cell voltage, $E_s$, as well as the cell efficiency, increases with temperature (the air and hydrogen fed to the stack are humidified to allow operation at temperatures greater than in Example 1.4), but the membranes will fail at temperatures exceeding $T = 85°C$. Consider $L \times W$ membranes, where $L = W = 100$ mm, of thickness $t_m = 0.43$ mm that each produce $E_s = 0.6$ volts at $I = 60$ A, and $E_{\text{inlet}} = 45$ W of thermal energy when operating at $T = 80°C$. The external surfaces of the stack are exposed to air at $T_s = 25°C$ and surroundings at $T_{\text{sur}} = 30°C$, with $e = 0.88$ and $h = 150$ W/m²·K.

1.55 The roof of a car in a parking lot absorbs a solar radiant flux of 800 W/m², while the underside is perfectly insulated. The convection coefficient between the roof and the ambient air is 12 W/m²·K.

(a) Neglecting radiation exchange with the surroundings, calculate the temperature of the roof under steady-state conditions if the ambient air temperature is 20°C.

(b) For the same ambient air temperature, calculate the temperature of the roof if its surface emissivity is 0.8.

c) The convection coefficient depends on airflow conditions over the roof, increasing with increasing air speed. Compute and plot the roof temperature as a function of $h$ for $2 \leq h \leq 200$ W/m²·K.

1.56 Consider the conditions of Problem 1.22, but the surroundings temperature is 25°C and radiation exchange with the surroundings is not negligible. If the convection coefficient is 6.4 W/m²·K and the emissivity of the plate is $\varepsilon = 0.42$, determine the time rate of change of the plate temperature, $dT/dt$, when the plate temperature is 225°C. Evaluate the heat loss by convection and the heat loss by radiation.

1.57 Most of the energy we consume as food is converted to thermal energy in the process of performing all our
Problems

2.3 A spherical shell with inner radius $r_1$ and outer radius $r_2$ has surface temperatures $T_1$ and $T_2$, respectively, where $T_1 > T_2$. Sketch the temperature distribution on $T-r$ coordinates assuming steady-state, one-dimensional conduction with constant properties. Briefly justify the shape of your curve.

2.4 Assume steady-state, one-dimensional heat conduction through the symmetric shape shown.

Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1 - x)$, $T(x) = 300(1 - 2x - x^2)$, and $q = 6000 \text{ W/m}$, where $A$ is in square meters, $T$ in kelvins, and $x$ in meters.

2.5 A solid, truncated cone serves as a support for a system that maintains the top (truncated) face of the cone at a temperature $T_2$, while the base of the cone is at a temperature $T_1 < T_2$.

The thermal conductivity of the solid depends on temperature according to the relation $k = k_o + aT$, where $a$ is a positive constant, and the sides of the cone are well insulated. Do the following quantities increase, decrease, or remain the same with increasing $x$: the heat transfer rate $q_x$, the heat flux $q_x^*$, the thermal conductivity $k$, and the temperature gradient $dT/dx$?

2.6 To determine the effect of the temperature dependence of the thermal conductivity on the temperature distribution in a solid, consider a material for which this dependence may be represented as

$$k = k_o + aT$$

where $k_o$ is a positive constant and $a$ is a coefficient that may be positive or negative. Sketch the steady-state temperature distribution associated with heat transfer in a plane wall for three cases corresponding to $a > 0$, $a = 0$, and $a < 0$.

2.7 A young engineer is asked to design a thermal protection barrier for a sensitive electronic device that might be exposed to irradiation from a high-powered infrared laser. Having learned as a student that a low thermal conductivity material provides good insulating characteristics, the engineer specifies use of a nanostructured aerogel, characterized by a thermal conductivity of $k_o = 0.005 \text{ W/m} \cdot \text{K}$, for the protective barrier. The engineer's boss questions the wisdom of selecting the aerogel because it has a low thermal conductivity. Consider the sudden laser irradiation of (a) pure aluminum, (b) glass, and (c) aerogel. The laser provides irradiation of $G = 10 \times 10^6 \text{ W/m}^2$. The absorptivities of the materials are $\alpha = 0.2$, 0.9, and 0.8 for the aluminum, glass, and aerogel, respectively, and the initial temperature of the barrier is $T_1 = 300 \text{ K}$. Explain why the boss is concerned. Hint: All materials experience thermal expansion (or contraction), and local stresses that develop within a material are, to a first approximation, proportional to the local temperature gradient.

2.8 Consider steady-state conditions for one-dimensional conduction in a plane wall having a thermal conductivity $k = 50 \text{ W/m} \cdot \text{K}$ and a thickness $L = 0.25 \text{ m}$, with no internal heat generation.

Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux.

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_1(\degree \text{C})$</th>
<th>$T_2(\degree \text{C})$</th>
<th>$dT/dx \text{ (K/m)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>-20</td>
<td></td>
</tr>
<tr>
<td>2</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td></td>
<td>200</td>
</tr>
</tbody>
</table>

2.9 Consider a plane wall 100 mm thick and of thermal conductivity 100 W/m · K. Steady-state conditions are known to exist with $T_1 = 400 \text{ K}$ and $T_2 = 600 \text{ K}$. Determine the heat flux $q_x^*$ and the temperature gradient $dT/dx$ for the coordinate systems shown.
2.10 A cylinder of radius \( r_o \) length \( L \), and thermal conductivity \( k \) is immersed in a fluid of convection coefficient \( h \) and unknown temperature \( T_o \). At a certain instant the temperature distribution in the cylinder is \( T(r) = a + br^2 \), where \( a \) and \( b \) are constants. Obtain expressions for the heat transfer rate at \( r_o \) and the fluid temperature.

2.11 In the two-dimensional body illustrated, the gradient at surface \( A \) is found to be \( \partial T/\partial y = 30 \text{ K/m} \). What are \( \partial T/\partial y \) and \( \partial T/\partial x \) at surface \( B \)?

2.12 Sections of the trans-Alaska pipeline run above the ground and are supported by vertical steel shafts (\( k = 25 \text{ W/m} \cdot \text{K} \)) that are 1 m long and have a cross-sectional area of 0.005 m². Under normal operating conditions, the temperature variation along the length of a shaft is known to be governed by an expression of the form

\[ T = 100 - 150x + 10x^2 \]

where \( T \) and \( x \) have units of °C and meters, respectively. Temperature variations are small over the shaft cross section. Evaluate the temperature and conduction heat rate at the shaft-pipeline joint (\( x = 0 \)) and at the shaft-ground interface (\( x = 1 \text{ m} \)). Explain the difference in the heat rates.

2.13 Steady-state, one-dimensional conduction occurs in a rod of constant thermal conductivity \( k \) and variable cross-sectional area \( A_s(x) = A_s e^{ax} \), where \( A_s \) and \( a \) are constants. The lateral surface of the rod is well insulated.

(a) Write an expression for the conduction heat rate, \( q_s(x) \). Use this expression to determine the temperature distribution \( T(x) \) and qualitatively sketch the distribution for \( T(0) > T(L) \).

(b) Now consider conditions for which thermal energy is generated in the rod at a volumetric rate \( q = \dot{q}_s \exp(-ax) \), where \( \dot{q}_s \) is a constant. Obtain an expression for \( q_s(x) \) when the left face (\( x = 0 \)) is well insulated.

### Thermophysical Properties

2.14 Consider a 300 mm × 300 mm window in an aircraft. For a temperature difference of 80°C from the inner to the outer surface of the window, calculate the heat loss through \( L = 10 \text{-mm-thick polycarbonate, soda lime glass, and aerogel windows, respectively. The thermal conductivities of the aerogel and polycarbonate are } k_{aerogel} = 0.014 \text{ W/m} \cdot \text{K} \text{ and } k_{polycarbonate} = 0.21 \text{ W/m} \cdot \text{K} \text{, respectively. Evaluate the thermal conductivity of the soda lime glass at 500 K. If the aircraft has 130 windows and the cost to heat the cabin air is $1/kW \cdot \text{h}, compare the } \text{costs associated with the heat loss through the windows for an 8-hour intercontinental flight.}

2.15 Gold is commonly used in semiconductor packaging to form interconnections (also known as interconnects) that carry electrical signals between different devices in the package. In addition to being a good electrical conductor, gold interconnects are also effective at protecting the heat-generating devices to which they are attached by conducting thermal energy away from the devices to surrounding, cooler regions. Consider a thin film of gold that has a cross section of 60 mm × 250 nm.

(a) For an applied temperature difference of 20°C, determine the energy conducted along a 1-μm-long, thin-film interconnect. Evaluate properties at 300 K.

(b) Plot the lengthwise (in the 1-μm direction) and spanwise (in the thinnest direction) thermal conductivities of the gold film as a function of the film thickness, \( L \), for \( 30 \leq L \leq 140 \text{ nm} \).

2.16 A TV advertisement by a well-known insulation manufacturer states: it isn’t the thickness of the insulating material that counts, it’s the R-value. The ad shows that to obtain an R-value of 19, you need 18 ft of rock, 15 in. of wood, or just 6 in. of the manufacturer’s insulation. Is this advertisement technically reasonable? If you are like most TV viewers, you don’t know the R-value is defined as \( L/k \), where \( L \) (in.) is the thickness of the insulation and \( k \) (Btu · in./hr · ft² · °F) is the thermal conductivity of the material.

2.17 An apparatus for measuring thermal conductivity employs an electrical heater sandwiched between two
Problems

(a) With two samples of SS316 in the apparatus, the heater draws 0.353 A at 100 V and the differential thermocouples indicate $\Delta T_1 = 25.0^\circ C$. What is the thermal conductivity of the stainless steel sample material? What is the average temperature of the samples? Compare your result with the thermal conductivity value reported for this material in Appendix A.

(b) By mistake, an Armco iron sample is placed in the lower position of the apparatus with one of the SS316 samples from part (a) in the upper portion. For this situation, the heater draws 0.601 A at 100 V and the differential thermocouples indicate $\Delta T_2 = 15.0^\circ C$. What are the thermal conductivity and average temperature of the Armco iron sample?

(c) What is the advantage in constructing the apparatus with two identical samples sandwiching the heater rather than with a single heater-sample combination? When would heat leakage out of the lateral surfaces of the samples become significant? Under what conditions would you expect $\Delta T_1 \neq \Delta T_2$?

2.18 An engineer desires to measure the thermal conductivity of an aerogel material. It is expected that the aerogel will have an extremely small thermal conductivity.

(a) Explain why the apparatus of Problem 2.17 cannot be used to obtain an accurate measurement of the aerogel’s thermal conductivity.

(b) The engineer designs a new apparatus for which an electric heater of diameter $D = 150$ mm is sandwiched between two thin plates of aluminum. The steady-state temperatures of the 5-mm-thick aluminum plates, $T_1$ and $T_2$, are measured with thermocouples. Aerogel sheets of thickness $t = 5$ mm are placed outside the aluminum plates, while a coolant with an inlet temperature of $T_{ci} = 25^\circ C$ maintains the exterior surfaces of the aerogel at a lower temperature. The circular aerogel sheets are formed so that they encase the heater and aluminum sheets, providing insulation to minimize radial heat losses. At steady state, $T_1 = T_2 = 55^\circ C$ and the heater draws 125 mA at 10 V. Determine the value of the aerogel thermal conductivity $k_a$.

(c) Calculate the temperature difference across the thickness of the 5-mm-thick aluminum plates. Comment on whether it is important to know the axial locations at which the temperatures of the aluminum plates are measured.

(d) If liquid water is used as the coolant with a total flow rate of $\dot{m} = 1$ kg/min (0.5 kg/min for each of the two streams), calculate the outlet temperature of the water, $T_{cw}$.

2.19 A method for determining the thermal conductivity $k$ and the specific heat $c_p$ of a material is illustrated in the sketch. Initially the two identical samples of diameter $D = 60$ mm and thickness $L = 10$ mm and the thin heater are at a uniform temperature of $T_i = 23.00^\circ C$, while surrounded by an insulating powder. Suddenly the heater is energized to provide a uniform heat flux $q_i^*$, on each of the sample interfaces, and the heat flux is maintained constant for a period of time, $\Delta t$. A short time after sudden heating is initiated, the temperature at this interface $T_a$ is related to the heat flux as

$$T_a(t) - T_i = 2q_i^* \left( \frac{t}{\pi pc_k} \right)^{1/2}$$
For a particular test run, the electrical heater dissipates 15.0 W for a period of $\Delta t = 120 \text{ s}$ and the temperature at the interface is $T_i = 24.57^\circ \text{C}$ after 30 s of heating. A long time after the heater is deenergized, $t > \Delta t$, the samples reach the uniform temperature of $T_i(\infty) = 33.50^\circ \text{C}$. The density of the sample materials, determined by measurement of volume and mass, is $\rho = 3965 \text{ kg/m}^3$.

Determine the specific heat and thermal conductivity of the test material. By looking at values of the thermophysical properties in Table A.1 or A.2, identify the test sample material.

The Heat Equation

2.20 At a given instant of time the temperature distribution within an infinite homogeneous body is given by the function

$$T(x, y, z) = x^2 - 2y^2 + z^2 - xy + 2yz$$

Assuming constant properties and no internal heat generation, determine the regions where the temperature changes with time.

2.21 A pan is used to boil water by placing it on a stove, from which heat is transferred at a fixed rate $q_a$. There are two stages to the process. In Stage 1, the water is taken from its initial (room) temperature $T_i$ to the boiling point, as heat is transferred from the pan by natural convection. During this stage, a constant value of the convection coefficient $h$ may be assumed, while the bulk temperature of the water increases with time, $T = T_i(t)$. In Stage 2, the water has come to a boil, and its temperature remains at a fixed value, $T = T_m$, as heating continues. Consider a pan bottom of thickness $L$ and diameter $D$, with a coordinate system corresponding to $x = 0$ and $x = L$ for the surfaces in contact with the stove and water, respectively.

(a) Write the form of the heat equation and the boundary/initial conditions that determine the variation of temperature with position and time, $T(x, t)$, in the pan bottom during Stage 1. Express your result in terms of the parameters $q_a, D, L, h$, and $T_m$, as well as appropriate properties of the pan material.

(b) During Stage 2, the surface of the pan in contact with the water is at a fixed temperature, $T(L, t) = T_m > T_b$. Write the form of the heat equation and boundary conditions that determine the temperature distribution, $T(x)$, in the pan bottom. Express your result in terms of the parameters $q_a, D, L, T_b$, and $T_m$, as well as appropriate properties of the pan material.

2.22 Uniform internal heat generation at $\dot{q} = 5 \times 10^5 \text{ W/m}^3$ is occurring in a cylindrical nuclear reactor fuel rod of 50-mm diameter, and under steady-state conditions the temperature distribution is of the form $T(r) = a + br^2$, where $T$ is in degrees Celsius and $r$ is in meters, while $a = 800^\circ \text{C}$ and $b = -4.167 \times 10^6 \text{ C/m}^3$. The fuel rod properties are $k = 30 \text{ W/m} \cdot \text{K}$, $\rho = 1100 \text{ kg/m}^3$, and $c_p = 600 \text{ J/kg} \cdot \text{K}$.

(a) What is the rate of heat transfer per unit length of the rod at $r = 0$ (the centerline) and at $r = 25 \text{ mm}$ (the surface)?

(b) If the reactor power level is suddenly increased to $\dot{q} = 10^6 \text{ W/m}^3$, what is the initial time rate of temperature change at $r = 0$ and $r = 25 \text{ mm}$?

2.23 The steady-state temperature distribution in a onedimensional wall of thermal conductivity $30 \text{ W/m} \cdot \text{K}$ and thickness 50 mm is observed to be $T(\text{Celsius}) = a + bx^2$, where $a = 200^\circ \text{C}$, $b = -200^\circ \text{C/m}^2$, and $x$ is in meters.

(a) What is the heat generation rate $\dot{q}$ in the wall?

(b) Determine the heat fluxes at the two wall faces. In what manner are these heat fluxes related to the heat generation rate?

2.24 The temperature distribution across a wall 0.3 m thick at a certain instant of time is $T(x) = a + bx + cx^2$, where $T$ is in degrees Celsius and $x$ is in meters, $a = 200^\circ \text{C}$, $b = -200^\circ \text{C/m}$, and $c = 30^\circ \text{C/m}^2$. The wall has a thermal conductivity of $1 \text{ W/m} \cdot \text{K}$.

(a) On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

(b) If the cold surface is exposed to a fluid at $100^\circ \text{C}$, what is the convection coefficient?

2.25 A plane wall of thickness $2L = 40 \text{ mm}$ and thermal conductivity $k = 5 \text{ W/m} \cdot \text{K}$ experiences uniform volumetric heat generation at a rate $\dot{q}$, while convection heat transfer occurs at both of its surfaces ($x = -L, +L$), each of which is exposed to a fluid of temperature $T_w = 20^\circ \text{C}$. Under steady-state conditions, the temperature distribution in the wall is of the form $T(x) = a + bx + cx^2$, where $a = 82.0^\circ \text{C}$, $b = -210^\circ \text{C/m}$, $c = -2 \times 10^4 \text{ C/m}^2$, and $x$ is in meters. The origin of the $x$-coordinate is at the midplane of the wall.

(a) Sketch the temperature distribution and identify significant physical features.