4-51 \[ \Sigma M_0 = 0 = 6 F_{AC} - 11(100) \quad \Rightarrow \quad F_{AC} = 183.3 \text{ lbf} \]

The deflection at point \( A \) in the negative \( y \) direction is equal to the elongation of the rod \( AC \). From Table A-5, \( E_s = 30 \text{ Mpsi} \).

\[ y_A = \left( \frac{FL}{AE} \right)_{AC} = \frac{183.3(12)}{\pi \left( \frac{0.5^2}{4} \right) 30(10^6)} = -3.735 \left( 10^{-4} \right) \text{ in} \]

By similar triangles the deflection at \( B \) due to the elongation of the rod \( AC \) is

\[ \frac{y_A}{6} = \frac{y_B}{18} \quad \Rightarrow \quad y_B = 3y_A = 3(-3.735)10^{-4} = -0.00112 \text{ in} \]

From Table A-5, \( E_a = 10.4 \text{ Mpsi} \)

The bar can then be treated as a simply supported beam with an overhang \( AB \). From Table A-9, beam 10

\[ y_B = y_{B1} + y_{B2} = y_{B1} \]

\[ y_{B1} = \left( \frac{BD}{6} \right) \left( \frac{\frac{\partial y_{BC}}{\partial x}}{\frac{\partial x}{\partial x}} \right)_{x_{wl+a}} \frac{Fa^2}{3EI} (l + a) = \frac{d}{dx} \left( \frac{F(x-l)}{6EI} (x-l)^2 - a(x-l) \right)_{x_{wl+a}} \frac{Fa^2}{3EI} (l + a) = \frac{7Fa^2}{6EI} (2l+3a) - \frac{Fa^2}{3EI} (l + a) \]

\[ = \frac{7(100)5}{6(10.4)10^6 \left( \frac{0.25(2^2)}{12} \right)} [2(6)+3(5)] - \frac{100 \left( \frac{5^2}{3(10.4)10^6 \left( \frac{0.25(2^2)}{12} \right)} \right) (6+5)} = -0.01438 \text{ in} \]

\[ y_B = y_{B1} + y_{B2} = -0.00112 - 0.01438 = -0.0155 \text{ in} \quad \text{Ans.} \]
\[ I = \pi (0.5^4)/64 = 3.068 \, (10^{-3}) \, \text{in}^4, \quad J = 2 \, I = 6.136 \, (10^{-3}) \, \text{in}^4, \quad A = \pi (0.5^2)/4 = 0.1963 \, \text{in}^2. \]

Consider \( x \) to be in the direction of \( OA \), \( y \) vertically upward, and \( z \) in the direction of \( AB \). Resolve the force \( F \) into components in the \( x \) and \( y \) directions obtaining 0.6 \( F \) in the horizontal direction and 0.8 \( F \) in the negative vertical direction. The 0.6 \( F \) force creates strain energy in the form of bending in \( AB \) and \( OA \), and tension in \( OA \). The 0.8 \( F \) force creates strain energy in the form of bending in \( AB \) and \( OA \), and torsion in \( OA \). Use the dummy variable \( \bar{x} \) to originate at the end where the loads are applied on each segment,

\[
0.6 \, F: \quad \begin{align*}
AB & \quad M = 0.6 \, F \, \bar{x} & \frac{\partial M}{\partial F} &= 0.6 \, \bar{x} \\
OA & \quad M = 4.2 \, F & \frac{\partial M}{\partial F} &= 4.2 \\
F_0 &= 0.6 \, F & \frac{\partial F_0}{\partial F} &= 0.6
\end{align*}
\]

\[
0.8 \, F: \quad \begin{align*}
AB & \quad M = 0.8 \, F \, \bar{x} & \frac{\partial M}{\partial F} &= 0.8 \, \bar{x} \\
OA & \quad M = 0.8 \, F \, \bar{x} & \frac{\partial M}{\partial F} &= 0.8 \, \bar{x} \\
T &= 5.6 \, F & \frac{\partial T}{\partial F} &= 5.6
\end{align*}
\]

Once the derivatives are taken the value of \( F = 15 \, \text{lbf} \) can be substituted in. The deflection of \( B \) in the direction of \( F \) is

\[
(\delta_a)_F = \frac{\partial U}{\partial F} = \left( \frac{F_0 L}{AE} \right)_{OA} \frac{\partial F_0}{\partial F} + \left( \frac{TL}{JG} \right)_{OA} \frac{\partial T}{\partial F} + \frac{1}{EI} \sum \int M \frac{\partial M}{\partial F} \, d\bar{x}
\]

\[
= \frac{0.6(15)15}{0.1963(30)10^6(0.6)} + \frac{5.6(15)15}{6.136(10^{-3})11.5(10^6)(5.6)}
\]

\[
+ \frac{15}{30(10^6)3.068(10^{-3})} \int_0^7 (0.6\bar{x})^2 \, d\bar{x} + \frac{15(4.2^2)}{30(10^6)3.068(10^{-3})} \int_0^7 d\bar{x}
\]

\[
+ \frac{15}{30(10^6)3.068(10^{-3})} \int_0^7 (0.8\bar{x})^2 \, d\bar{x} + \frac{15}{30(10^6)3.068(10^{-3})} \int_0^7 (0.8\bar{x})^2 \, d\bar{x}
\]

\[
= 1.38(10^{-5}) + 0.1000 + 6.71(10^{-3}) + 0.0431 + 0.0119 + 0.1173
\]

\[
= 0.279 \, \text{in.} \quad \text{Ans.}
\]
*Note. This is not the actual deflection of point $B$. For this, dummy forces must be placed on $B$ in the $x$, $y$, and $z$ directions. Determine the energy due to each, take derivatives, and then substitute the values of $F_x = 9$ lbf, $F_y = -12$ lbf, and $F_z = 0$. This can be done separately and then use superposition. The actual deflections of $B$ are

$$\delta_B = 0.0831 \, i - 0.2862 \, j - 0.00770 \, k \, \text{ in}$$

From this, the deflection of $B$ in the direction of $F$ is

$$ (\delta_B)_F = 0.6(0.0831) + 0.8(0.2862) = 0.279 \, \text{ in}$$

which agrees with our result.
1. Choose $R_B$ as redundant reaction.

2. Statics. $R_C = wl - R_B \quad (1)$

\[
M_C = \frac{1}{2}wl^2 - R_B (l - a) \quad (2)
\]

3. Deflection equation for point $B$. Superposition of beams 2 and 3 of Table A-9,

\[
y_B = \frac{R_B (l - a)^3}{3EI} + \frac{w(l - a)^2}{24EI} \left[ 4l(l - a) - (l - a)^2 - 6l^2 \right] = 0
\]

4. Solving for $R_B$.

\[
R_B = \frac{w}{8(l - a)} \left[ 6l^2 - 4l(l - a) + (l - a)^2 \right]
\]

\[
= \frac{w}{8(l - a)} \left( 3l^2 + 2al + a^2 \right) \quad \text{Ans.}
\]

Substituting this into Eqs. (1) and (2) gives

\[
R_C = wl - R_B = \frac{w}{8(l - a)} \left( 5l^2 - 10al - a^2 \right) \quad \text{Ans.}
\]

\[
M_C = \frac{1}{2}wl^2 - R_B (l - a) = \frac{w}{8} \left( l^2 - 2al - a^2 \right) \quad \text{Ans.}
\]