10.2 Refrigerant 22 is the working fluid in a Carnot vapor refrigeration cycle for which the evaporator temperature is \(-30^\circ C\). Saturated vapor enters the condenser at 36°C, and saturated liquid exits at the same temperature. The mass flow rate of refrigerant is 10kg/min. Determine
(a) the rate of heat transfer to the refrigerant passing through the evaporator, in kW.
(b) the net power input to the cycle, in kW.
(c) the coefficient of performance.
(d) the refrigeration capacity, in tons.

10.3 A Carnot vapor refrigeration cycle operates between thermal reservoirs at 40°F and 90°F. For (a) Refrigerant 134a, (b) propane, (c) water, (d) Refrigerant 22, and (e) ammonia as the working fluid, determine the operating pressures in the condenser and evaporator, in lb/in.\(^2\), and the coefficient of performance.

10.4 A Carnot vapor refrigeration cycle is used to maintain a cold region at 40°F when the ambient temperature is 90°F. Refrigerant 134a enters the condenser as saturated vapor at 140 lb/in.\(^2\) and leaves as saturated liquid at the same pressure. The evaporator pressure is 40 lb/in.\(^2\). The mass flow rate of refrigerant is 6.2 lb/min. Calculate
(a) the compressor and turbine power, each in Btu/min.
(b) the coefficient of performance.

10.5 For the cycle in Problem 10.4, determine
(a) the rates of heat transfer, in Btu/min, for the refrigerant flowing through the evaporator and condenser, respectively.
(b) the rates and directions of exergy transfer accompanying each of these heat transfers, in Btu/min. Let \(T_0 = 90^\circ F\).

10.6 An ideal vapor-compression refrigeration cycle operates at steady state with Refrigerant 134a as the working fluid. Saturated vapor enters the compressor at 2 bar, and saturated liquid exits the condenser at 8 bar. The mass flow rate of refrigerant is 7 kg/min. Determine
(a) the compressor power, in kW.
(b) the refrigerating capacity, in tons.
(c) the coefficient of performance.

10.7 Plot each of the quantities in Problem 10.6 versus evaporator temperature for evaporator pressures ranging from 0.6 to 4 bar, while the condenser pressure remains fixed at 8 bar.

10.8 Refrigerant 134a is the working fluid in an ideal vapor-compression refrigeration cycle operating at steady state. Refrigerant enters the compressor at 1.4 bar, \(-12^\circ C\), and the condenser pressure is 9 bar. Liquid exits the condenser at 32°C. The mass flow rate of refrigerant is 7 kg/min. Determine
(a) the compressor power, in kW.
(b) the refrigeration capacity, in tons.
(c) the coefficient of performance.

10.9 An ideal vapor-compression refrigeration system operates at steady state with Refrigerant 134a as the working fluid. Superheated vapor enters the compressor at 10 lb/in.\(^2\), 0°F, and saturated liquid leaves the condenser at 180 lb/in.\(^2\). The refrigeration capacity is 8 tons. Determine
(a) the compressor power, in horsepower.
(b) the rate of heat transfer from the working fluid passing through the condenser, in Btu/min.
(c) the coefficient of performance.

10.10 Refrigerant 22 enters the compressor of an ideal vapor-compression refrigeration system as saturated vapor at \(-40^\circ C\) with a volumetric flow rate of 15 m\(^3\)/min. The refrigerant leaves the condenser at 19°C, 9 bar. Determine
(a) the compressor power, in kW.
(b) the refrigerating capacity, in tons.
(c) the coefficient of performance.

10.11 An ideal vapor-compression refrigeration cycle, with ammonia as the working fluid, has an evaporator temperature of \(-20^\circ C\) and a condenser pressure of 12 bar. Saturated vapor enters the compressor, and saturated liquid exits the condenser. The mass flow rate of the refrigerant is 3 kg/min. Determine
(a) the coefficient of performance.
(b) the refrigerating capacity, in tons.

10.12 Refrigerant 134a enters the compressor of an ideal vapor-compression refrigeration cycle as saturated vapor at \(-10^\circ C\). The condenser pressure is 160 lb/in.\(^2\). The mass flow rate of refrigerant is 6 lb/min. Plot the coefficient of performance and the refrigerating capacity, in tons, versus the condenser exit temperature ranging from \(-40\) to \(-10^\circ C\). All other conditions are the same as in Problem 10.11.

10.13 To determine the effect of changing the evaporator temperature on the performance of an ideal vapor-compression refrigeration cycle, plot the coefficient of performance and the refrigerating capacity, in tons, for the cycle in Problem 10.11 for saturated vapor entering the compressor at temperatures ranging from \(-40\) to \(-10^\circ C\). All other conditions are the same as in Problem 10.11.

10.14 To determine the effect of changing condenser pressure on the performance of an ideal vapor-compression refrigeration cycle, plot the coefficient of performance and the refrigerating capacity, in tons, for the cycle in Problem 10.11 for condenser pressures ranging from 8 to 16 bar. All other conditions are the same as in Problem 10.11.

10.15 A vapor-compression refrigeration cycle operates at steady state with Refrigerant 134a as the working fluid. Saturated vapor enters the compressor at 2 bar, and saturated liquid exits the condenser at 8 bar. The isentropic compressor efficiency is 80%. The mass flow rate of refrigerant is 7 kg/min. Determine
(a) the compressor power, in kW.
(b) the refrigeration capacity, in tons.
(c) the coefficient of performance.

10.16 Modify the cycle in Problem 10.9 to have an isentropic compressor efficiency of 83% and let the temperature of the liquid leaving the condenser be 100°F. Determine, for the modified cycle,
(a) the compressor power, in horsepower.
(b) the rate of heat transfer from the working fluid passing through the condenser, in Btu/min.
(c) the coefficient of performance.
(d) the rates of exergy destruction in the compressor and expansion valve, each in Btu/min, for \(T_0 = 90^\circ F\).
10.17 A vapor-compression refrigeration system circulates Refrigerant 134a at a rate of 6 kg/min. The refrigerant enters the compressor at $-10^\circ$C, 1.4 bar, and exits at 7 bar. The isentropic compressor efficiency is 67%. There are no appreciable pressure drops as the refrigerant flows through the condenser and evaporator. The refrigerant leaves the condenser at 7 bar, 24°C. Ignoring heat transfer between the compressor and its surroundings, determine
(a) the coefficient of performance.
(b) the refrigerating capacity in tons.
(c) the rates of exergy destruction in the compressor and expansion valve, each in kW.
(d) the changes in specific flow exergy of the refrigerant passing through the evaporator and condenser, respectively, each in kJ/kg.

Let $T_0 = 21^\circ$C, $p_0 = 1$ bar.

10.18 A vapor-compression refrigeration system, using ammonia as the working fluid, has evaporator and condenser pressures of 30 and 200 lb/ft$^2$, respectively. The refrigerant passes through each heat exchanger with a negligible pressure drop. At the inlet and exit of the compressor, the temperatures are $10^\circ$F and $300^\circ$F, respectively. The heat transfer rate from the working fluid passing through the condenser is 50,000 Btu/h, and liquid exits at 200 lb/ft$^2$, 90°F. If the compressor operates adiabatically, determine
(a) the compressor power input, in hp.
(b) the coefficient of performance.

10.19 If the minimum and maximum allowed refrigerant pressures are 1 and 10 bar, respectively, which of the following can be used as the working fluid in a vapor-compression refrigeration system that maintains a cold region at 0°C, while discharging energy by heat transfer to the surrounding air at 30°C: Refrigerant 22, Refrigerant 134a, ammonia, propane?

10.20 Consider the following vapor-compression refrigeration cycle used to maintain a cold region at temperature $T_C$ when the ambient temperature is 80°F. Saturated vapor enters the compressor at 15°F below $T_C$, and the compressor operates adiabatically with an isentropic efficiency of 80%. Saturated liquid exits the condenser at 95°F. There are no pressure drops through the evaporator or condenser, and the refrigerating capacity is 1 ton. Plot refrigerant mass flow rate, in lb/min, coefficient of performance, and refrigerating efficiency, versus $T_C$ ranging from 40°F to $-25^\circ$F if the refrigerant is
(a) Refrigerant 134a.
(b) propane.
(c) Refrigerant 22.
(d) ammonia.

The refrigerating efficiency is defined as the ratio of the cycle coefficient of performance to the coefficient of performance of a Carnot refrigeration cycle operating between thermal reservoirs at the ambient temperature and the temperature of the cold region.

10.21 In a vapor-compression refrigeration cycle, ammonia exits the evaporator as saturated vapor at $-22^\circ$C. The refrigerant enters the condenser at 16 bar and 160°C, and saturated liquid exits at 16 bar. There is no significant heat transfer between the compressor and its surroundings, and the refrigerant passes through the evaporator with a negligible change in pressure. If the refrigerating capacity is 150 kW, determine
(a) the mass flow rate of refrigerant, in kg/s.
(b) the power input to the compressor, in kW.
(c) the coefficient of performance.
(d) the isentropic compressor efficiency.

10.22 A vapor-compression refrigeration system with a capacity of 10 tons has superheated Refrigerant 134a vapor entering the compressor at 15°C, 4 bar, and exiting at 12 bar. The compression process can be modeled by $p_1/v_1 = constant$. At the condenser exit, the pressure is 11.6 bar, and the temperature is 44°C. The condenser is water-cooled, with water entering at 20°C and leaving at 30°C with a negligible change in pressure. Heat transfer from the outside of the condenser can be neglected. Determine
(a) the mass flow rate of the refrigerant, in kg/s.
(b) the power input and the heat transfer rate for the compressor, each in kW.
(c) the coefficient of performance.
(d) the mass flow rate of the cooling water, in kg/s.
(e) the rates of exergy destruction in the condenser and expansion valve, each expressed as a percentage of the power input. Let $T_0 = 20^\circ$C.

10.23 The capacity of a propane vapor-compression refrigeration system is 5 tons. Saturated vapor at 0°F enters the compressor, and superheated vapor leaves at 120°F, 180 lb/ft$^2$. Heat transfer from the condenser to its surroundings occurs at a rate of 3.5 Btu per lb of refrigerant passing through the condenser. Liquid refrigerant enters the expansion valve at 85°F, 180 lb/ft$^2$. The condenser is water-cooled, with water entering at 65°F and leaving at 80°F with a negligible change in pressure. Determine
(a) the compressor power input, in Btu/min.
(b) the mass flow rate of cooling water through the condenser, in lb/min.
(c) the coefficient of performance.

10.24 A window-mounted air conditioner supplies 19 m$^3$/min of air at 15°C, 1 bar to a room. Air returns from the room to the evaporator of the unit at 22°C. The air conditioner operates at steady state on a vapor-compression refrigeration cycle with Refrigerant-22 entering the compressor at 4 bar, 10°C. Saturated liquid refrigerant at 9 bar leaves the condenser. The compressor has an isentropic efficiency of 70%, and refrigerant exits the compressor at 9 bar. Determine the compressor power, in kW, the refrigeration capacity, in tons, and the coefficient of performance.

10.25 A vapor-compression refrigeration system for a household refrigerator has a refrigerating capacity of 1000 Btu/h.
(c) the refrigeration capacity, per unit mass of air flow, in kJ/kg.
(d) the coefficient of performance.

10.51 Reconsider Problem 10.50, but include in the analysis that the compressor and turbine have isentropic efficiencies of 85 and 88% respectively. Answer the same questions as in Problem 10.50.

10.52 Plot the quantities calculated in parts (a) through (d) of Problem 10.50 versus the compressor pressure ratio ranging from 3 to 7. Repeat for equal compressor and turbine isentropic efficiencies of 95%, 90%, and 80%.

10.53 Consider a Brayton refrigeration cycle with a regenerative heat exchanger. Air enters the compressor at 480°F, 15 lbf/in.² and is compressed isentropically to 40 lbf/in.². Compressed air enters the regenerative heat exchanger at 540°F and is cooled to 480°F before entering the turbine. The expansion through the turbine is isentropic. If the refrigeration capacity is 15 tons, calculate
(a) the volumetric flow rate at the compressor inlet, in ft³/min.
(b) the coefficient of performance.

10.54 Reconsider Problem 10.53, but include in the analysis that the compressor and turbine each have isentropic efficiencies of 88%. Answer the same questions for the modified cycle as in Problem 10.53.

10.55 Air at 2 bar, 380 K is extracted from a main jet engine compressor for cabin cooling. The extracted air enters a heat exchanger where it is cooled at constant pressure to 320 K through heat transfer with the ambient. It then expands adiabatically to 0.95 bar through a turbine and is discharged into the cabin. The turbine has an isentropic efficiency of 75%. If the mass flow rate of the air is 1.0 kg/s, determine
(a) the power developed by the turbine, in kW.
(b) the rate of heat transfer from the air to the ambient, in kW.

10.56 Air at 32 lbf/in.², 680°F is extracted from a main jet engine compressor for cabin cooling. The extracted air enters a heat exchanger where it is cooled at constant pressure to 600°F through heat transfer with the ambient. It then expands adiabatically to 14 lbf/in.² through a turbine and is discharged into the cabin at 500°F with a mass flow rate of 200 lb/min. Determine
(a) the power developed by the turbine, in horsepower.
(b) the isentropic turbine efficiency.
(c) the rate of heat transfer from the air to the ambient, in Btu/min.

10.57 Air undergoes a Stirling refrigeration cycle, which is the reverse of the Stirling power cycle introduced in Sec. 9.11. At the beginning of the isothermal compression, the pressure and temperature are 100 kPa and 300 K, respectively. The compression ratio is 6, and the temperature during the isothermal expansion is 100 K. Determine
(a) the heat transfer during the isothermal expansion, in kJ per kg of air.
(b) the net work for the cycle, in kJ per kg of air.
(c) the coefficient of performance.

10.58 Air undergoes an Ericsson refrigeration cycle, which is the reverse of the Ericsson power cycle introduced in Sec. 9.11. At the beginning of the isothermal compression, the pressure and temperature are 100 kPa and 310 K, respectively. The pressure ratio during the isothermal compression is 3. During the isothermal expansion the temperature is 270 K. Determine
(a) the heat transfer for the isothermal expansion, per unit mass of air flow, in kJ/kg.
(b) the net work, per unit mass of air flow, in kJ/kg.
(c) the coefficient of performance.

**Design & open ended problems: exploring engineering practice**

10.1D A vapor-compression refrigeration system using propane is being designed for a household food freezer. The refrigeration system must maintain a temperature of 0°F within the freezer compartment when the temperature of the room is 90°F. Under these conditions, the steady-state heat transfer rate from the room into the freezer compartment is 1500 Btu/h. Specify operating pressures and temperatures at key points within the refrigeration system and estimate the refrigerant mass flow rate and compressor power required. Investigate and report on safety issues related to the use of propane as a refrigerant.

10.2D Refrigerant 22 is widely used as the working fluid in air conditioners and industrial chillers. However, its use is likely to be phased out in the future due to concerns about ozone depletion. Investigate which environmentally-acceptable working fluids are under consideration to replace Refrigerant 22 for these uses. Determine the design issues for air conditioners and chillers that would result from changing refrigerants. Write a report of your findings.

10.3D An air-conditioning system is under consideration that will use a vapor-compression ice maker during the nighttime, when electric rates are lowest, to store ice for meeting the daytime air-conditioning load. The maximum loads are 100 tons during the day and 50 tons at night. Is it best to size the system to make enough ice at night to carry the entire daytime load or to use a smaller chiller that runs both day and night? Base your strategy on the day–night electric rate structure of your local electric utility company.

10.4D A vertical, closed-loop geothermal heat pump is under consideration for a new 50,000-ft² school building. The design capacity is 100 tons for both heating and cooling. The local water table is 150 ft, and the ground water temperature is 55°F. Specify a ground source heat pump as well as
\[ \mu_i = \bar{G}_i + RT \frac{\ln(y_i \rho_w)}{p_{ref}} \]  

(11.144)  

Chemical potential of component \( i \) in an ideal gas mixture

**Exercises: things engineers think about**

1. What is an advantage of using the Redlich–Kwong equation of state in the generalized form given by Eq. 11.9 instead of Eq. 11.7? A disadvantage?
2. To determine the specific volume of superheated water vapor at a known pressure and temperature, when would you use each of the following: the steam tables, the generalized compressibility chart, an equation of state, the ideal gas model?
3. If the function \( p = p(T, v) \) is an equation of state, is \( (\partial p/\partial T)_v \) a property? What are the independent variables of \( (\partial p/\partial T)_v \)?
4. In the expression \( (\partial u/\partial T)_p \), what does the subscript \( v \) signify?
5. Explain how a Mollier diagram provides a graphical representation of the fundamental function \( h(s, p) \).
6. How is the Clapeyron equation used?
7. For a gas whose equation of state is \( p \bar{v} = RT \), are the specific heats \( C_p \) and \( C_v \) necessarily functions of temperature alone?
8. Referring to the \( p-T \) diagram for water, explain why ice melts under the blade of an ice skate.
9. Can you devise a way to determine the specific heat \( C_p \) of a gas by direct measurement? Indirectly, using other measured data?
10. For an ideal gas, what is the value of the Joule–Thomson coefficient?
11. At what states is the entropy departure negligible? The fugacity coefficient, \( f/p \), closely equal to unity?
12. In Eq. 11.107, what do the subscripts \( T, p \), and \( n_i \) signify? What does \( n \) denote?
13. How does Eq. 11.108 reduce for a system consisting of a pure substance? Repeat for an ideal gas mixture.
14. If two different liquids of known volumes are mixed, is the final volume necessarily equal to the sum of the original volumes?
15. For a binary solution at temperature \( T \) and pressure \( p \), how would you determine the specific heat \( C_p \)? Repeat for an ideal solution and for an ideal gas mixture.

**Problems: developing engineering skills**

**Using Equations of State**

11.1 Owing to safety requirements, the pressure within a 19.3 ft³ cylinder should not exceed 52 atm. Check the pressure within the cylinder if filled with 100 lb of CO₂ maintained at 212°F using the
(a) van der Waals equation.
(b) compressibility chart.
(c) ideal gas equation of state.

11.2 Ten pounds mass of propane have a volume of 2 ft³ and a pressure of 600 lb/in.² Determine the temperature, in °R, using the
(a) van der Waals equation.
(b) compressibility chart.
(c) ideal gas equation of state.
(d) propane tables.
11.3 The pressure within a 23.3-m³ tank should not exceed 105 bar. Check the pressure within the tank if filled with 1000 kg of water vapor maintained at 360°C using the
(a) ideal gas equation of state.
(b) van der Waals equation.
(c) Redlich–Kwong equation.
(d) compressibility chart.
(e) steam tables.

11.4 Estimate the pressure of water vapor at a temperature of 500°C and a density of 24 kg/m³ using the
(a) steam tables.
(b) compressibility chart.
(c) Redlich–Kwong equation.
(d) van der Waals equation.
(e) ideal gas equation of state.

11.5 Methane gas flows through a pipeline with a volumetric flow rate of 11 ft³/s at a pressure of 183 atm and a temperature of 56°F. Determine the mass flow rate, in lb/s, using the
(a) ideal gas equation of state.
(b) van der Waals equation.
(c) compressibility chart.

11.6 Determine the specific volume of water vapor at 20 MPa and 400°C, in m³/kg, using the
(a) steam tables.
(b) compressibility chart.
(c) Redlich–Kwong equation.
(d) van der Waals equation.
(e) ideal gas equation of state.

11.7 A vessel whose volume is 1 m³ contains 4 kmol of methane at 100°C. Owing to safety requirements, the pressure of the methane should not exceed 12 MPa. Check the pressure using the
(a) ideal gas equation of state.
(b) Redlich–Kwong equation.
(c) Benedict–Webb–Rubin equation.

11.8 Methane gas at 100 atm and −18°C is stored in a 10-m³ tank. Determine the mass of methane contained in the tank, in kg, using the
(a) ideal gas equation of state.
(b) van der Waals equation.
(c) Benedict–Webb–Rubin equation.

11.9 Using the Benedict–Webb–Rubin equation of state, determine the volume, in m³, occupied by 165 kg of methane at a pressure of 200 atm and temperature of 400 K. Compare with the results obtained using the ideal gas equation of state and the generalized compressibility chart.

11.10 A rigid tank contains 1 kg of oxygen (O₂) at p₁ = 40 bar, T₁ = 180 K. The gas is cooled until the temperature drops to 150 K. Determine the volume of the tank, in m³, and the final pressure, in bar, using the
(a) ideal gas equation of state.
(b) Redlich–Kwong equation.
(c) compressibility chart.

11.11 One pound mass of air initially occupying a volume of 0.4 ft³ at a pressure of 1000 lbf/in² expands isothermally and without irreversibilities until the volume is 2 ft³. Using the Redlich–Kwong equation of state, determine the
(a) temperature, in °R.
(b) final pressure, in lbf/in².
(c) work developed in the process, in Btu.

11.12 Water vapor initially at 240°C, 1 MPa expands in a piston–cylinder assembly isothermally and without internal irreversibilities to a final pressure of 0.1 MPa. Evaluate the work done, in kJ/kg. Use a truncated virial equation of state with the form

\[ Z = 1 + \frac{B}{v} + \frac{C}{v^2} \]

where \( B \) and \( C \) are evaluated from steam table data at 240°C and pressures ranging from 0 to 1 MPa.

11.13 Referring to the virial series, Eqs. 3.30 and 3.31, show that \( B = B/R T, C = (C - B^2/3R) T^2 \).

11.14 Express Eq. 11.5, the van der Waals equation, in terms of the compressibility factor \( Z \)
(a) as a virial series in \( (v/k) \). [Hint: Expand the \( (v/k - 1/8)^{-1} \) term of Eq. 11.5 in a series]
(b) as a virial series in \( p_R \).
(c) Dropping terms involving \( (p_R)^2 \) and higher in the virial series of part (b), obtain the following approximate form:

\[ Z = 1 + \left( \frac{1}{8} - \frac{27/64}{p_R} \right) \frac{p_R}{T_R} \]

(d) Compare the compressibility factors determined from the equation of part (c) with tabulated compressibility factors from the literature for 0 < \( p_R < 0.5 \) and each of \( T_R = 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 \). Comment on the realm of validity of the approximate form.

11.15 The Berthelot equation of state has the form

\[ p = \frac{RT}{b} - \frac{a}{b^2} \]

(a) Using Eqs. 11.3, show that

\[ a = \frac{27 \bar{T}^3}{64 \bar{p}} \quad \text{and} \quad b = \frac{1 \bar{T}^3}{8 \bar{p}} \]

(b) Express the equation in terms of the compressibility factor \( Z \), the reduced temperature \( T_R \), and the pseudoreduced specific volume, \( v_R \).

11.16 The Beattie–Bridgeman equation of state can be expressed as

\[ p = \frac{RT(1 - \varepsilon)(\nu + B)}{\nu^3} - \frac{A}{\nu^2} \]

where

\[ A = A_0 \left( 1 - \frac{a}{v} \right) \quad \text{and} \quad B = B_0 \left( 1 - \frac{b}{v} \right) \]

\[ \varepsilon = \frac{c}{\nu T^3} \]

and \( A_0, B_0, a, b, \) and \( c \) are constants. Express this equation of state in terms of the reduced pressure, \( p_R \), reduced
temperature, \( T_R \), pseudoreduced specific volume, \( v'_R \), and appropriate dimensionless constants.

11.17 The Dieterici equation of state is

\[
p = \left(\frac{RT}{v - b}\right) \exp \left( -\frac{a}{RTv} \right)
\]

(a) Using Eqs. 11.3, show that

\[
a = \frac{4RT^2}{\rho e^2}, \quad b = \frac{RT}{\rho e^2}
\]

(b) Show that the equation of state can be expressed in terms of compressibility chart variables as

\[
Z = \left(\frac{v'_R}{v_R - 1/\rho^2}\right) \exp \left( -\frac{a}{T_Rv'\rho e^2} \right)
\]

(c) Convert the result of part (b) to a virial series in \( v'_R \).

(Hint: Expand the \((v'_R - 1/\rho^2)^{-1}\) term in a series. Also expand the exponential term in a series.)

11.18 The Peng–Robinson equation of state has the form

\[
p = \frac{RT}{v - b} - \frac{a}{v^2 - \rho^2}
\]

Using Eqs. 11.3, evaluate the constants \( a, b, c \) in terms of the critical pressure \( p_c \), critical temperature \( T_c \), and critical compressibility factor \( Z_c \).

11.19 The \( p-v-T \) relation for chlorofluorinated hydrocarbons can be described by the Carnahan–Starling–DeSantis equation of state

\[
\frac{p}{\rho} = \frac{1 + \beta + \beta^2 - \beta^3}{(1 + \beta)^3} - \frac{a}{RT(\rho + b)}
\]

where \( \beta = b/\rho_a \), \( a = a_0 \exp (a_1 T + a_2 T^2) \), and \( b = b_0 + b_1 T + b_2 T^2 \). For Refrigerants 12 and 13, the required coefficients for \( T \) in K, \( \rho \) in \( g/L/(mol)^3 \), and \( b \) in \( L/mol \) are given in Table P11.19. Specify which of the two refrigerants would allow the smaller amount of mass to be stored in a 10-m³ vessel at 0.2 MPa, 80°C.

Using Relations from Exact Differentials

11.20 The differential of pressure obtained from a certain equation of state is given by one of the following expressions. Determine the equation of state.

\[
dp = \frac{2(v - b)}{RT} dv + \frac{(v - b)^2}{RT^2} dT
\]

\[
dp = \frac{RT}{(v - b)^2} dv + \frac{R}{v - b} dT
\]

11.21 Introducing \( \delta Q_{\text{rev}} = T \, dS \) into Eq. 6.8 gives

\[
\delta Q_{\text{rev}} = dU + p \, dv
\]

Using this expression together with the test for exactness, demonstrate that \( Q_{\text{rev}} \) is not a property.

11.22 Show that Eq. 11.16 is satisfied by an equation of state with the form \( p = [RT(v - b)] + a \).

11.23 For the functions \( x = x(y, w), y = y(z, w), z = z(x, w) \), demonstrate that

\[
\frac{\partial x}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial x} = 1
\]

11.24 Using Eq. 11.35, check the consistency of

(a) the steam tables at 2 MPa, 400°C.

(b) the Refrigerant 134a tables at 2 bar, 50°C.

11.25 Using Eq. 11.35, check the consistency of

(a) the steam tables at 100 lb/in², 600°F.

(b) the Refrigerant 134a tables at 40 lb/in², 100°F.

11.26 At a pressure of 1 atm, liquid water has a state of maximum density at about 4°C. What can be concluded about \( \partial s/\partial p \) at

(a) 3°C?

(b) 4°C?

(c) 5°C?

11.27 A gas enters a compressor operating at steady state and is compressed isentropically. Does the specific enthalpy increase or decrease as the gas passes from the inlet to the exit?

11.28 Show that \( T, p, h, \psi, \) and \( g \) can each be determined from a fundamental thermodynamic function of the form \( u = u(s, v) \).

11.29 Evaluate \( p, s, u, h, c_v, \) and \( c_p \) for a substance for which the Helmholtz function has the form

\[
\psi = -RT \ln \frac{v}{v'} - cT \left[ 1 - \frac{T}{T'} + \frac{c}{T'} \ln \frac{T}{T'} \right]
\]

where \( v' \) and \( T' \) denote specific volume and temperature, respectively, at a reference state, and \( c \) is a constant.

11.30 The Mollier diagram provides a graphical representation of the fundamental thermodynamic function \( h = h(s, p) \). Show that at any state fixed by \( s \) and \( p \) the properties \( T, v, u, \psi, \) and \( g \) can be evaluated using data obtained from the diagram.

11.31 Derive the relation \( c_p = -T(\partial^2 g/\partial T^2)_p \).

Evaluating \( \Delta s, \Delta u, \) and \( \Delta h \)

11.32 Using \( p-v-T \) data for saturated ammonia from Table A-13E, calculate at 20°F

(a) \( h_g - h_f \)

(b) \( u_g - u_f \)

(c) \( s_g - s_f \)

Compare with the values obtained using table data.

<table>
<thead>
<tr>
<th>Substance</th>
<th>( a_0 \times 10^3 )</th>
<th>( a_1 \times 10^5 )</th>
<th>( a_2 \times 10^6 )</th>
<th>( b_0 )</th>
<th>( b_1 \times 10^6 )</th>
<th>( b_2 \times 10^8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-12</td>
<td>3.52412</td>
<td>-2.77230</td>
<td>-0.67318</td>
<td>0.15376</td>
<td>-1.84195</td>
<td>-5.03644</td>
</tr>
<tr>
<td>R-13</td>
<td>2.29813</td>
<td>-3.41828</td>
<td>-1.52430</td>
<td>0.12814</td>
<td>-1.68474</td>
<td>-10.7951</td>
</tr>
</tbody>
</table>

Table P11.19
(c) Obtain an expression for the change in specific enthalpy, 
\[ h(p_2, T_2) - h(p_1, T_1) \]

11.51 For a gas whose \( p-v-T \) behavior is described by \( Z = 1 + Bp/RT \), where \( B \) is a function of temperature, derive expressions for the specific enthalpy, internal energy, and entropy changes, \( [h(p_2, T_2) - h(p_1, T_1)], [u(p_2, T_2) - u(p_1, T_1)] \), and \( [s(p_2, T_2) - s(p_1, T_1)] \).

11.52 For a gas whose \( p-v-T \) behavior is described by \( Z = 1 + Bv + Cv^2 \), where \( B \) and \( C \) are functions of temperature, derive an expression for the specific entropy change, 
\[ s(T_2, v_2) - s(T_1, v_1) \].

Using Other Thermodynamic Relations

11.53 The volume of a 1-kg copper sphere is not allowed to vary by more than 0.1%. If the pressure exerted on the sphere is increased from 10 bar while the temperature remains constant at 300 K, determine the maximum allowed pressure, in bar. Average values of \( \rho, \beta, \) and \( \kappa \) are 8888 kg/m\(^3\), 49.2 \times 10\(^{-7}\) (K\(^{-1}\)), and 0.776 \times 10\(^{-11}\) \text{m}^2/\text{N} respectively.

11.54 The volume of a 1-lb copper sphere is not allowed to vary by more than 0.1%. If the pressure exerted on the sphere is increased from 1 atm while the temperature remains constant at 80°F, determine the maximum allowed pressure, in atm. Average values of \( \rho, \beta, \) and \( \kappa \) are 555 lb/ft\(^3\), 2.75 \times 10\(^{-5}\) (°F\(^{-1}\)), and 3.72 \times 10\(^{-16}\) ft\(^2\)/lbf, respectively.

11.55 Develop expressions for the volume expansivity \( \beta \) and the isothermal compressibility \( \kappa \) for
(a) an ideal gas.
(b) a gas whose equation of state is \( p(v - b) = RT \).
(c) a gas obeying the van der Waals equation.

11.56 Derive expressions for the volume expansivity \( \beta \) and the isothermal compressibility \( \kappa \) in terms of \( T, \rho, Z, \) and the first partial derivatives of \( Z \). For gas states with \( p_R < 3, T_R < 2 \), determine the sign of \( \kappa \). Discuss.

11.57 Show that the isothermal compressibility \( \kappa \) is always greater than or equal to the isentropic compressibility \( \alpha \).

11.58 Prove that \( \beta(T)p_r = -(\partial \mu/\partial T)_{p} \).

11.59 For aluminum at 0°C, \( \rho_r = 2700 \text{ kg/m}^3 \), \( \beta = 71.4 \times 10^{-9} \text{ (K)}^{-1} \), \( \kappa = 1.34 \times 10^{-12} \text{ m}^2/\text{N} \), and \( c_p = 0.9211 \text{ kJ/kg \cdot K} \). Determine the percent error in \( c_p \) that would result if it were assumed that \( c_p = c_w \).

11.60 Estimate the temperature rise, in °C, of mercury, initially at 0°C and 1 bar if its pressure were raised to 100 bar isentropically. For mercury at 0°C, \( c_p = 28.0 \text{ kJ/kmol \cdot K} \), \( u = 0.0147 \text{ m}^3/\text{kmol} \), and \( \beta = 17.8 \times 10^{-5} \text{ (K)}^{-1} \).

11.61 At certain states, the \( p-v-T \) data for a particular gas can be represented as \( Z = 1 - Ap/T^4 \), where \( Z \) is the compressibility factor and \( A \) is a constant. Obtain an expression for the specific heat \( c_p \) in terms of the gas constant \( R \), specific heat ratio \( k \), and \( Z \). Verify that your expression reduces to Eq. 3.47a when \( Z = 1 \).

11.62 For a gas obeying the van der Waals equation of state,
(a) show that \( (\partial c_v/\partial v)_T = 0 \).
(b) develop an expression for \( c_p - c_w \).
(c) develop expressions for \( [u(T_2, v_2) - u(T_1, v_1)] \) and \( [s(T_2, v_2) - s(T_1, v_1)] \).

11.63 If the value of the specific heat \( c_p \) of air is 0.1965 Btu/ lb \cdot °R at \( T_1 = 1000°F \), \( v_1 = 36.8 \text{ ft}^3/\text{lb} \), determine the value of \( c_p \) at \( T_2 = 1000°F \), \( v_2 = 0.0555 \text{ ft}^3/\text{lb} \). Assume that air obeys the Berthelot equation of state
\[ P = \frac{RT}{v - b} - \frac{a}{T^2 v} \]
where
\[ a = \frac{27 R^2 T_c^3}{64 p_c}, \quad b = \frac{1}{8} \frac{RT_c}{p_c} \]

11.64 Show that the specific heat ratio \( k \) can be expressed as \( k = c_p/\rho (c_p \kappa - T_0 \beta^2) \). Using this expression together with data from the steam tables, evaluate \( k \) for water vapor at 200 lb/ft\(^2\), 500°F.

11.65 For liquid water at 40°C, 1 atm estimate
(a) \( c_w \) in kJ/kg \cdot K
(b) the velocity of sound, in m/s.
Use Data from Table 11.2, as required.

11.66 Using steam table data, estimate the velocity of sound in liquid water at (a) 20°C, 50 bar, (b) 50°F, 1500 lb/ft\(^2\).

11.67 At a certain location in a wind tunnel, a stream of air is at 500°F, 1 atm and has a velocity of 2115 ft/s. Determine the Mach number at this location.

11.68 For a gas obeying the equation of state \( p(v - b) = RT \), where \( b \) is a positive constant, can the temperature be reduced in a Joule–Thomson expansion?

11.69 A gas is described by \( u = RT\mu - A/T + B \), where \( A, B, C \) are constants. For the gas (a) obtain an expression for the temperatures at the Joule–Thomson inversion states.
(b) obtain an expression for \( c_p - c_w \).

11.70 Determine the maximum Joule–Thomson inversion temperature in terms of the critical temperature \( T_c \) predicted by the

(a) van der Waals equation.
(b) Redlich–Kwong equation.
(c) Dieterici equation given in Problem 11.17.

11.71 Derive an equation for the Joule–Thomson coefficient as a function of \( T \) and \( v \) for a gas that obeys the van der Waals equation of state and whose specific heat \( c_v \) is given by \( c_v = A + BT + CT^2 \), where \( A, B, C \) are constants. Evaluate the temperatures at the inversion states in terms of \( R, v \), and the van der Waals constants \( a \) and \( b \).

11.72 Show that Eq. 11.77 can be written as
\[ \mu_1 = \frac{T^2}{c_p} \left( \frac{\partial(u/T)}{\partial T} \right)_p \]
(a) Using this result, obtain an expression for the Joule–Thomson coefficient for a gas obeying the equation of state
\[ v = \frac{RT}{P} - \frac{A}{T^2} \]
where \( A \) is a constant.
where \( f_{\text{CO}_2} \) is an empirical interaction parameter, determine the pressure, in kPa, at \( v = 0.005 \text{ m}^3/\text{kg} \), \( T = 180^\circ \text{C} \) for a mixture of Refrigerants 12 and 13, in which Refrigerant 12 is 40% by mass. For a mixture of Refrigerants 12 and 13, \( f_{\text{CO}_2} = 0.035 \).

11.103 A rigid vessel initially contains carbon dioxide gas at 32°C and pressure \( p \). Ethylene gas is allowed to flow into the tank until a mixture consisting of 20% carbon dioxide and 80% ethylene (molar basis) exists within the tank at a temperature of 43°C and a pressure of 110 bar. Determine the pressure \( p \), in bar, using Kay's rule together with the generalized compressibility chart.

11.104 Two tanks having equal volumes are connected by a valve. One tank contains carbon dioxide gas at 100°F and pressure \( p \). The other tank contains ethylene gas at 100°F and 1480 lbf/in.². The valve is opened and the gases mix, eventually attaining equilibrium at 100°F and pressure \( p' \) with a composition of 20% carbon dioxide and 80% ethylene (molar basis). Using Kay's rule and the generalized compressibility chart, determine in lbf/in.²:
(a) the initial pressure of the carbon dioxide, \( p \).
(b) the final pressure of the mixture, \( p' \).

Analyzing Multicomponent Systems

11.105 A binary solution at 25°C consists of 59 kg of ethyl alcohol (\( \text{C}_2\text{H}_5\text{OH} \)) and 41 kg of water. The respective partial molal volumes are 0.0573 and 0.0172 \( \text{m}^3/\text{kmol} \). Calculate the total volume, in \( \text{m}^3 \). Compare with the volume calculated using the molar specific volumes of the pure components, each a liquid at 25°C, in the place of the partial molal volumes.

11.106 The following data are for a binary solution of ethane (\( \text{C}_2\text{H}_6 \)) and pentane (\( \text{C}_5\text{H}_{12} \)) at a certain temperature and pressure:

<table>
<thead>
<tr>
<th>mole fraction of ethane</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.109</th>
<th>0.107</th>
<th>0.107</th>
<th>0.11</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume (in m³) per kmol of solution</td>
<td>0.119</td>
<td>0.116</td>
<td>0.112</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Estimate

(a) the specific volumes of pure ethane and pure pentane, each in \( \text{m}^3/\text{kmol} \).
(b) the partial molal volumes of ethane and pentane for an equimolar solution, each in \( \text{m}^3/\text{kmol} \).

11.107 The following data are for a binary mixture of carbon dioxide and methane at a certain temperature and pressure:

<table>
<thead>
<tr>
<th>mole fraction of methane</th>
<th>0.000</th>
<th>0.204</th>
<th>0.406</th>
<th>0.606</th>
<th>0.847</th>
<th>1.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume (in ft³) per lbmol of mixture</td>
<td>1.506</td>
<td>3.011</td>
<td>3.540</td>
<td>3.892</td>
<td>4.149</td>
<td>4.277</td>
</tr>
</tbody>
</table>

Estimate

(a) the specific volumes of pure carbon dioxide and pure methane, each in \( \text{ft}^3/\text{lbmol} \).
(b) the partial molal volumes of carbon dioxide and methane for an equimolar mixture, each in \( \text{ft}^3/\text{lbmol} \).

11.108 Using \( p-v-T \) data from the steam tables, determine the fugacity of water as a saturated vapor at (a) 280°C, (b) 500°F. Compare with the values obtained from the generalized fugacity chart.

11.109 Determine the fugacity, in atm, for
(a) butane at 555 K, 150 bar.
(b) methane at 120°F, 800 lbf/in.².
(c) benzene at 890°F, 135 atm.

11.110 Using the equation of state of Problem 11.14 (c), evaluate the fugacity of ammonia at 750 K, 100 atm and compare with the value obtained from Fig. A-6.

11.111 Using tabulated compressibility data from the literature, evaluate \( f/p \) at \( T_R = 1.40 \) and \( p_R = 2.0 \). Compare with the value obtained from Fig. A-6.

11.112 Consider the truncated virial expansion

\[ Z = 1 + \tilde{B}(T_R)p_R + \tilde{C}(T_R)p_R^2 + \tilde{D}(T_R)p_R^3 \]

(a) Using tabulated compressibility data from the literature, evaluate the coefficients \( \tilde{B}, \tilde{C}, \) and \( \tilde{D} \) for \( 0 < p_R < 1.0 \) and each of \( T_R = 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 \).
(b) Obtain an expression for \( \ln (f/p) \) in terms of \( T_R \) and \( p_R \).

(c) Using the coefficients of part (a), evaluate \( f/p \) at selected states and compare with tabulated values from the literature.

11.113 Derive the following approximation for the fugacity of a liquid at temperature \( T \) and pressure \( p \):

\[ f(T, p) = f_{\text{sat}}(T) \exp \left( \frac{v_f(T)}{RT} \left[ p - p_{\text{sat}}(T) \right] \right) \]

where \( f_{\text{sat}}(T) \) is the fugacity of the saturated liquid at temperature \( T \). For what range of pressures might the approximation \( f(T, p) = f_{\text{sat}}(T) \) apply?

11.114 Beginning with Eq. 11.122,

(a) evaluate \( \ln f \) for a gas obeying the Redlich–Kwong equation of state.

(b) Using the result of part (a), evaluate the fugacity, in bar, for Refrigerant 134a at 90°C, 10 bar. Compare with the fugacity value obtained from the generalized fugacity chart.

11.115 Consider a one-inlet, one-exit control volume at steady state through which the flow is internally reversible and isothermal. Show that the work per unit of mass flowing can be expressed in terms of the fugacity \( f \) as

\[ \left( \frac{W_{\text{ce}}}{m} \right)_{\text{res}} = -RT \ln \left( \frac{f_\text{in}}{f_i} \right) + \frac{V_i^2 - V_\text{out}^2}{2} + g(z_1 - z_2) \]

11.116 Methane expands isothermally and without irreversibilities through a turbine operating at steady state, entering at 60 atm, 77°F, and exiting at 9 atm. Using data from the generalized fugacity chart, determine the work developed, in Btu per lb of methane flowing. Ignore kinetic and potential energy effects.