5.77 A quantity of water within a piston-cylinder assembly executes a Carnot power cycle. During isothermal expansion, the water is heated from saturated liquid at 50 bar until it is a saturated vapor. The vapor then expands adiabatically to a pressure of 5 bar while doing 364.31 kJ/kg of work.

(a) Sketch the cycle on $p$-$v$ coordinates.
(b) Evaluate the heat transfer per unit mass and work per unit mass for each process, in kJ/kg.
(c) Evaluate the thermal efficiency.

**KNOWN:** Water executes a closed system Carnot power cycle between two known pressures and with specified work produced during one process.

**FIND:** Sketch the cycle on $p$-$v$ coordinates, determine heat and work during each process, and evaluate thermal efficiency.

**SCHEMATIC AND GIVEN DATA:**

![Diagram of Carnot cycle with hot and cold reservoirs and processes labeled.

<table>
<thead>
<tr>
<th>Process</th>
<th>$Q/m$ [kJ/kg]</th>
<th>$W/m$ [kJ/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-3</td>
<td>0</td>
<td>364.31</td>
</tr>
<tr>
<td>3-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State</th>
<th>$p$ [bar]</th>
<th>$T$ [°C]</th>
<th>$x$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>264.0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>264.0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>151.9</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>151.9</td>
<td></td>
</tr>
</tbody>
</table>

**ENGINEERING MODEL:**
1. The closed system defined by the dashed line on the accompanying diagram undergoes a Carnot power cycle.
2. Kinetic and potential energy effects can be neglected.

**ANALYSIS:**
(a) The $p$-$v$ diagram is shown below. Process 1-2 is isothermal expansion from State 1, a saturated liquid, to State 2, a saturated vapor, both at 50 bar. In the saturated mixture region temperature and pressure are dependent and thus constant during process 1-2. Temperature is
the saturation temperature at 50 bar. Specific volume increases during process 1-2 expansion. Process 2-3 is adiabatic expansion in which specific volume continues to increase while pressure drops to 5 bar. Process 3-4 is isothermal compression during which specific volume decreases. Since the water is still in the saturated mixture region, pressure and temperature are both constant. The temperature is the saturation temperature at 5 bar. Process 4-1 is adiabatic compression, during which specific volume continues to decrease while pressure increases back to 50 bar.

\[ p-v \] Diagram

To determine the energy transfers during each process, the energy balance for a closed system and the expansion/compression work equation apply.

**Process 1-2 (Isothermal Expansion)**
State 1 is saturated liquid at \( p_1 = 50 \) bar. From Table A-3:
- \( v_1 = v_n = 0.0012859 \text{ m}^3/\text{kg} \),
- \( u_1 = u_n = 1147.8 \text{ kJ/kg} \)

State 2 is saturated vapor at \( p_2 = 50 \) bar. From Table A-3:
- \( v_2 = v_g = 0.03944 \text{ m}^3/\text{kg} \),
- \( u_2 = u_g = 2597.1 \text{ kJ/kg} \)

Applying Eq. 2.17 for the constant-pressure expansion

\[
\frac{W_{12}}{m} = p_1(v_2 - v_1)
\]

Substituting values and applying appropriate conversion factors give
\[ \frac{W_{12}}{m} = (50 \text{ bar}) \left( 0.03944 \frac{m^3}{kg} - 0.0012859 \frac{m^3}{kg} \right) \left| 10^3 \frac{N}{m^2} \right| \left| 10^3 \frac{N \cdot m}{kg} \right| = 190.77 \text{ kJ/kg} \]

The positive sign associated with work indicates work is from the system.

Applying the closed system energy balance (neglecting kinetic and potential energy effects):
\[ u_2 - u_1 = (Q_{12}/m) - (W_{12}/m) \]

Solving for heat transfer per unit mass gives
\[ (Q_{12}/m) = (W_{12}/m) + u_2 - u_1 \]

Substituting values gives
\[ (Q_{12}/m) = 190.77 \text{ kJ/kg} + 2597.1 \text{ kJ/kg} - 1147.8 \text{ kJ/kg} = 1640.07 \text{ kJ/kg} \]

The positive sign associated with energy transfer by heat indicates heat is into the system.

**Process 2-3 (Adiabatic Expansion)**

\[ (Q_{23}/m) = 0 \text{ kJ/kg (adiabatic process)} \]

\[ (W_{23}/m) = 364.31 \text{ kJ/kg (given)} \]

To fix State 3, a property in addition to pressure is required. Applying the closed system energy balance (neglecting kinetic and potential energy effects):
\[ u_3 - u_2 = (Q_{23}/m) - (W_{23}/m) \]

Solving for internal energy at State 3 gives:
\[ u_3 = (Q_{23}/m) - (W_{23}/m) + u_2 \]
\[ u_3 = 0 \text{ kJ/kg} - 364.31 \text{ kJ/kg} + 2597.1 \text{ kJ/kg} = 2232.8 \text{ kJ/kg} \]

State 3 is a two-phase mixture at \( p_3 = 5 \text{ bar} \). From Table A-3:
\[ u_3 = 639.68 \text{ kJ/kg} \]
\[ u_3 = 2561.2 \text{ kJ/kg} \]
\[ u_3 = 2232.8 \text{ kJ/kg} \text{ (computed from energy balance for process 2-3)} \]
\[ y_3 = 0.0010926 \text{ m}^3/\text{kg} \]
\[ y_3 = 0.3749 \text{ m}^3/\text{kg} \]
\[ h_3 = 640.23 \text{ kJ/kg} \]
\[ h_{g3} = 2108.5 \text{ kJ/kg} \]

Quality is determined from the relationship
PROBLEM 5.77 - Continued (4)

\[ x = \frac{u - u_f}{u_e - u_i} \]

Solving for quality at State 3 gives

\[ x_3 = \frac{2232.8 \text{ kJ/kg} - 639.68 \text{ kJ/kg}}{2561.2 \text{ kJ/kg} - 639.68 \text{ kJ/kg}} = 0.8291 \]

Specific volume at State 3 can be determined from the quality relation

\[ v_3 = v_{\text{fg}} + x_3 (v_{\text{fg}} - v_{\text{fg}3}) \]

\[ v_3 = 0.0010926 \text{ m}^3/\text{kg} + (0.8291)(0.3749 \text{ m}^3/\text{kg} - 0.0010926 \text{ m}^3/\text{kg}) = 0.3110 \text{ m}^3/\text{kg} \]

Specific enthalpy at State 3 is convenient later in the analysis. It can be determined from the quality relation

\[ h_3 = h_{\text{fg}} + x_3 h_{\text{fg}3} \]

\[ h_3 = 640.23 \text{ kJ/kg} + (0.8291)(2108.5 \text{ kJ/kg}) = 2388.39 \text{ kJ/kg} \]

**Process 3-4 (Isothermal Compression)**

The heat transfer during Process 3-4 is the heat transfer of the cycle associated with the cold reservoir while heat transfer during Process 1-2 is the heat transfer associated with the hot reservoir. Since the cycle is a Carnot cycle, and thus reversible, the following relationship introduced in Sec. 5.8.1 is applicable:

\[ \left( \frac{Q_C}{Q_H} \right)_{\text{cycle}} = \frac{T_C}{T_H} \]

Expressing heat transfer on a per unit mass basis and solving for \((Q_C/m)\) yield

\[ \frac{Q_C}{m} = \left( \frac{Q_H}{m} \right) \left( \frac{T_C}{T_H} \right) = \left( 1640.07 \text{ kJ/kg} \right) \left( \frac{424.9 \text{ K}}{537.0 \text{ K}} \right) = 1297.70 \text{ kJ/kg} \]

Here \(Q_C\) is a magnitude. Heat transfer during process 3-4 is from the system to the cold reservoir and thus is out of the system. Consequently, \((Q_H/m) = -1297.70 \text{ kJ/kg}\).

To determine the work per unit mass during process 3-4, apply the closed system energy balance (neglecting kinetic and potential energy effects):
\[ u_4 - u_3 = (Q_{34}/m) - (W_{34}/m) \]

Applying Eq. 2.17 for the constant-pressure compression

\[ \frac{W_{34}}{m} = p_3 (v_4 - v_3) \]

Substituting for work per unit mass in the energy equation gives

\[ u_4 - u_3 = (Q_{34}/m) - p_3 (v_4 - v_3) \]

Rearranging terms gives

\[ 0 = (Q_{34}/m) + (u_3 - v_3) - (u_4 + p_4 v_4) \]

1. Substituting enthalpy, \( h = u + pv \), yields

\[ 0 = (Q_{34}/m) + h_3 - h_4 \]

Solving for the exit enthalpy, \( h_4 \), gives

\[ h_4 = (Q_{34}/m) + h_3 \]

Substituting values yields

\[ h_4 = (-1297.70 \text{ kJ/kg}) + 2388.39 \text{ kJ/kg} = 1090.69 \text{ kJ/kg} \]

State 4 is a two-phase mixture at \( p_4 = 5 \) bar. From Table A-3:

- \( h_{fg} = 640.23 \text{ kJ/kg} \)
- \( h_{fg} = 2108.5 \text{ kJ/kg} \)
- \( h_4 = 1090.69 \text{ kJ/kg} \) (computed from energy balance for process 3-4)
- \( v_{fg} = 0.0010926 \text{ m}^3/\text{kg} \)
- \( v_{fg} = 0.3749 \text{ m}^3/\text{kg} \)
- \( u_{fg} = 639.68 \text{ kJ/kg} \)
- \( u_{fg} = 2561.2 \text{ kJ/kg} \)

State 4 now can be fixed by \( h_4 \) and \( p_4 \). Quality at State 4 is determined from the relationship

\[ x = \frac{h - h_4}{h_{fg}} \]

Solving for quality at State 4 gives
\[
\begin{align*}
x_4 &= \frac{1090.69 \text{ kJ/kg} - 640.23 \text{ kJ/kg}}{2108.5 \text{ kJ/kg}} = 0.2136
\end{align*}
\]

Specific volume at State 4 can be determined from the quality relation

\[\nu_4 = \nu_{v_4} + x_4(\nu_{\rho_4} - \nu_{v_4})\]

\[\nu_4 = 0.0010926 \text{ m}^3/\text{kg} + (0.2136)(0.3749 \text{ m}^3/\text{kg} - 0.0010926 \text{ m}^3/\text{kg}) = 0.08094 \text{ m}^3/\text{kg}\]

Substituting values into Eq. (1) and applying appropriate conversion factors to solve for work per unit mass give

\[
\frac{W_{41}}{m} = (5 \text{ bar}) \left(0.08094 \frac{\text{m}^3}{\text{kg}} - 0.3110 \frac{\text{m}^3}{\text{kg}}\right) \left(10^5 \frac{\text{N}}{\text{m}^2}\right) \left(10^7 \frac{\text{N} \cdot \text{m}}{\text{kJ}}\right) = -115.03 \text{ kJ/kg}
\]

The negative sign associated with work indicates work is to the system.

**Process 4-1 (Adiabatic Compression)**

\((Q_{41}/m) = 0 \text{ kJ/kg (adiabatic process)}\)

To determine the work per unit mass during process 4-1, apply the closed system energy balance (neglecting kinetic and potential energy effects):

\[u_1 - u_4 = (Q_{41}/m) - (W_{41}/m)\]

Substituting \((Q_{41}/m) = 0\) and solving for work give

\[(W_{41}/m) = u_4 - u_1\]

Applying the quality relation to determine \(u_4\) gives

\[u_4 = u_{\rho_4} + x_4(u_{\rho_4} - u_{\rho_5})\]

\[u_4 = 639.68 \text{ kJ/kg} + (0.2136)(2561.2 \text{ kJ/kg} - 639.68 \text{ kJ/kg}) = 1050.12 \text{ kJ/kg}\]

Substituting values and solving for work per unit mass give

\[(W_{41}/m) = 1050.12 \text{ kJ/kg} - 1147.8 \text{ kJ/kg} = -97.68 \text{ kJ/kg}\]

The negative sign associated with work indicates work is to the system.

Summarizing energy transfers
As expected for a cycle, the net heat transfer is equal to the net work.

(c) By definition, power cycle thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{H}} - Q_{\text{L}}} = \frac{\left(\frac{W_{\text{cycle}}}{m}\right)}{\left(\frac{Q_{\text{H}}}{m}\right)} = \frac{342.37 \, \frac{\text{kJ}}{\text{kg}}}{1640.07 \, \frac{\text{kJ}}{\text{kg}}} = 0.2088$$

Alternatively, since the cycle is a Carnot cycle, the thermal efficiency can be determined from the maximum thermal efficiency equation

$$\eta_{\text{max}} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} = 1 - \frac{424.9 \, \text{K}}{537.0 \, \text{K}} = 0.2088$$

[1] The introduction of enthalpy here is only for computational convenience using table data.