**Problem 2.73**

**Known:** Data are provided for a power cycle executed by a gas in a piston-cylinder assembly.

**Find:** For each process, evaluate $W$, find $Q$ for processes 1-2, 2-3. Evaluate the thermal efficiency.

**Schematic and Given Data:**

**Engine Model**

1. The gas is the closed system.
2. Volume change is the only work mode.
3. For each process, $\Delta KE = \Delta PE = 0$.

$V_1 = 15 \text{kJ}$

$Q_{31} = 10 \text{kJ}$

**Fig. P2.73**

**Analysis:** (a) The work can be evaluated from Eq. 2.17. For Process 1-2, the piston does not move (volume is constant). Thus, $W_{12} = 0$.

For Processes 1-2 and 2-3, the work can be evaluated geometrically. That is,

$$W_{12} = \text{R} \left( \frac{V_2 - V_1}{V_2 - V_1} \right) = \left( \frac{P_1 + P_2}{2} \right) \left( V_2 - V_1 \right) = \left[ \frac{5}{2} \right] k \frac{R}{k} \left( 5 - 1 \right) m^2 \left( \frac{10^{12} \text{N} \cdot \text{m}}{1 \text{kJ}} \right) = 5 \text{kJ}$$

$$W_{23} = \text{R} \left( \frac{V_3 - V_2}{V_2 - V_1} \right) = \left( \frac{P_2 + P_3}{2} \right) \left( V_3 - V_2 \right) = \left[ \frac{3}{2} \right] k \frac{R}{k} \left( 1 - 5 \right) m^2 \left( \frac{10^{12} \text{N} \cdot \text{m}}{1 \text{kJ}} \right) = 10 \text{kJ}$$

(b) $Q_{31}$ is given. For Process 1-2, $\Delta U + Q_{12} + \Delta PE = Q_{31} - W_{12}$

$$\Rightarrow Q_{12} = \Delta U + W_{12} = 15 \text{kJ} + 10 \text{kJ} = 25 \text{kJ}$$

For Process 2-3, $\Delta U + Q_{23} + \Delta PE = Q_{31} - W_{23}$

$$\Rightarrow Q_{23} = \left( V_3 - V_2 \right) + W_{23}$$

To find $\left( V_3 - V_2 \right)$, note that since internal energy is a property

$$\left( C_p - T_{2} \right) + \left( C_p - T_{3} \right) + \left( C_p - T_{1} \right) = 0$$

$$\Rightarrow \left( T_{3} - T_{2} \right) = - \left( \frac{T_{3} - T_{2}}{C_p} \right) = \frac{T_{3} - T_{2}}{10 \text{kJ}}$$

$$\Rightarrow \left( C_p - T_{2} \right) = \left( \frac{T_{3} - T_{2}}{10 \text{kJ}} \right) \left( T_{2} - T_{1} \right) = 10 \text{kJ}$$

(c) For any power cycle, the thermal efficiency is $\eta = \frac{\text{Work in}}{\text{Heat in}}$.

Here, $\text{Work in} = W_{12} + W_{23} + W_{31} = 15 - 25 + 0 = 8 \text{kJ}$

$\text{Heat in} = Q_{31} + Q_{31} = 31 + 10 = 41 \text{kJ}$

$$\Rightarrow \eta = \frac{8 \text{kJ}}{41 \text{kJ}} = 0.195 \text{ (19.5\%)}$$

1. Also, note that for any cycle, $W_{\text{cycle}} = W_{\text{cycle}}$ (Eq. 2.40). Thus

$W_{12} + W_{23} + W_{31} = Q_{31} + Q_{31} + Q_{31} \Rightarrow Q_{31} = W_{12} + W_{23} + W_{31} - Q_{31} - Q_{31}$, or

$Q_{31} = 16 - (-9) + 0 - 31 - 10 = -33 \text{kJ}$.