PROBLEM 2.58

KNOWN: A gas within a piston-cylinder assembly undergoes two different processes between the same end states.

FIND: Sketch the processes on p-V coordinates. For each process evaluate \( W \) and \( Q \).

SCHEMATIC & GIVEN DATA:

Data: \( p_1 = 10 \text{ bar}, V_1 = 0.1 \text{ m}^3, U_1 = 400 \text{ kJ} \) and \( p_2 = 1 \text{ bar}, V_2 = 1.0 \text{ m}^3, U_2 = 200 \text{ kJ} \).

Process A: Process from 1 to 2 during which the pressure-volume relation is \( pV = \text{constant} \).

Process B: Constant-volume process from state 1 to a pressure of 2 bar, followed by a linear pressure-volume process to state 2.

ENERGY MODEL:
1. The gas is the closed system.
2. Kinetic and potential energy effects can be ignored.
3. The p-V relation is specified for each process.
4. The moving boundary is the only work mode.

ANALYSIS: For Process A, \( W_A = \int p \, dV = \frac{d}{dV} \left[ \frac{pV^n}{n-1} \right] \bigg|_{V_1}^{V_2} = \frac{200 \text{ bar} \cdot \left(0.1 \text{ m}^3\right)}{2} \right] \left[ \frac{10^6 \text{ N/m}^2}{1 \text{ bar}} \right] \left[ \frac{1 \text{ m}^3}{10^6 \text{ N/m}^2} \right] = 200 \text{ bar} \cdot \left(0.1 \text{ m}^3\right) = 20 \text{ kJ} \)

Thus, \( W_A = (20 \text{ bar})(0.1 \text{ m}^3) = 2 \text{ kJ} \) and \( U_A = U_1 + W_A = (200 - 400) \text{ kJ} + 20 \text{ kJ} = 180 \text{ kJ} \)

For Process B, the piston does not move for the first step (constant volume) and thus no work is done. The work can be evaluated geometrically for the second step, during which the p-V relation is linear.

\( W_B = \text{Rate} \left[ V_1 - V_i \right] \left[ \frac{2 \text{ bar} + 1 \text{ bar}}{2} \right] \left[ 1.0 \text{ m}^3 - 0.1 \text{ m}^3 \right] \left[ \frac{10^6 \text{ N/m}^2}{1 \text{ bar}} \right] \left[ \frac{1000 \text{ N/m}^2}{10^6 \text{ N/m}^2} \right] = 13.5 \text{ kJ} \)

Thus, \( W_B = 13.5 \text{ kJ} \).

\( \Delta U + \Delta K + \Delta P = Q_B - W_B \Rightarrow Q_B = \Delta U + \Delta P \)

\( Q_B = (200 - 400) \text{ kJ} + 13.5 \text{ kJ} = -65 \text{ kJ} \)