Automatic Control

System Excitation and Transient Response

Dr. Aly Mousaad Aly
Department of Mechanical Engineering
Faculty of Engineering, Alexandria University
Types of input signals

**Deterministic:** Step, Ramp, Pulse, Impulse, Sine, Cosine, ... etc.

**Stochastic** (Random): Earthquake Loads, Wind Loads, ... etc.

These signals are governed by mathematical equations

Equations do not exist

**Step input:**

\[ u(t) = f(t) = \begin{cases} 
1 & t \geq 0 \\
0 & t < 0 
\end{cases} \]

\[ U(s) = \frac{1}{s} \]
Shifted step input:

\[ u(t) = \begin{cases} 
1 & \text{if } t \geq t_0 \\
0 & \text{if } t < t_0 
\end{cases} \]

\[ U(s) = \frac{1}{s} e^{-s t_0} \]

Ramp input:

\[ u(t) = \begin{cases} 
 t & \text{if } t \geq 0 \\
0 & \text{if } t < 0 
\end{cases} \]

\[ U(s) = \frac{1}{s^2} \]

Rectangular pulse:

\text{area} = \text{pulse strength}
Unit impulse:

\[ u(t) = \begin{cases} 
  1/t_0 & 0 \leq t \leq t_0 \\
  0 & t < 0 \\
  0 & t > 0 
\end{cases} \]

\[ U(s) = \frac{1}{t_0 s} - \frac{1}{t_0 s} e^{-st_0} = \frac{1}{t_0 s} \left(1 - e^{-st_0}\right) \]

Impulse function:

\[ U(s) = \lim_{t_0 \to 0} \frac{1}{t_0 s} \left(1 - e^{-st_0}\right) = \lim_{t_0 \to 0} \frac{s e^{-st_0}}{s} = 1 \]
Step response of first order systems

\[ Y(s) = \frac{K}{T s + 1} \Rightarrow Y(s) = \frac{K}{s (T s + 1)} = \frac{K}{s} \cdot \frac{1}{s + 1/T} \]

\[ y(t) = \frac{K}{T} \int_0^t e^{-t/T} \, dt = \frac{K}{T} (-T) e^{-t/T} \bigg|_0^t = K \left( 1 - e^{-t/T} \right) \]
Step response of first order systems

**K**: Gain
Final (steady-state) value

\[ K = \lim_{t \to \infty} y(t) \]

**T**: Time constant
Time when response rises 63% of final value
Indication of speed of response (convergence)
Impulse response of 1st order systems

\[
\frac{Y(s)}{U(s)} = \frac{K}{T \, s + 1}
\]

\[
Y(s) = 1 \frac{K}{(T \, s + 1)} = \frac{K}{T} \frac{1}{(s + 1/T)}
\]

\[
y(t) = \frac{K}{T} e^{-t/T}
\]
Step response for 2nd order system

\[
Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \quad \Rightarrow \quad Y(s) = \frac{1}{s} \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

\[
y(t) = 1 - \frac{\omega_n}{\omega_d} e^{-\zeta \omega_n t} \sin(\omega_d + \phi), \quad 0 < \zeta < 1
\]

\[
\omega_d = \omega_n \sqrt{1 - \zeta^2}, \quad \phi = \cos^{-1}(\zeta)
\]

\(\omega_n = \text{natural frequency}, \quad \zeta = \text{damping ratio}\)

\(\omega_d = \text{damped natural frequency}\)
Step response for 2nd order system

Note:
When $\zeta=0$ the system will never reach a target position (imagine a robot arm, it is impossible to catch an object)

$\zeta=1$ the system is slow

Best damping when $\zeta=0.707$
Step response for 2nd order system

\[ y_{\text{peak}} = 1 + e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}} \]

\[ M_p = 100 \times e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}} \]

\( M_p = \text{percent overshoot} \)

\[ t_r = \frac{\pi - \phi}{\omega_d} = \text{rise time} \]

\[ t_s\ 5\% = \frac{3}{\zeta \omega_n} \]

\[ t_s\ 2\% = \frac{4}{\zeta \omega_n} \]

\[ y(t) = 1 - \frac{\omega_n}{\omega_d} e^{-\zeta \omega_n t} \sin (\omega_d + \phi) \]

\( \zeta = 0.15 \)

Settling time = \( t_s \)

5% or 2%

Slides 6: System response
Example

\[ T.F. = \frac{25}{s^2 + 2s + 25} \]

\[ \omega_n = 5 \text{ rad/s}, \quad 2\zeta\omega_n = 2 \quad \Rightarrow \quad \zeta = 0.2 \]

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} = 4.9 \text{ rad/s} \]

\[ \phi = \cos^{-1}(\zeta) = 1.3694 \text{ rad} \]
\[ t_r = \frac{\pi - \phi}{\omega_d} = 0.36 \text{ sec} \]

\[ t_p = \frac{\pi}{\omega_d} = 0.64 \text{ sec} \]

\[ M_p = 100 \times e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}} = 52.66 \% \]

\[ t_{s\ 5\%} = \frac{3}{\zeta \omega_n} = 3 \text{ sec} \]

\[ t_{s\ 2\%} = \frac{4}{\zeta \omega_n} = 4 \text{ sec} \]
Error Analysis

\[ E(s) = \frac{1}{R(s) + G(s)H(s)} \]

\[ E(s) = \frac{R(s)}{1 + G(s)H(s)} \]