Automatic Control

Error Analysis

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Error Analysis

static error coefficient

Desired value for the response = $r(t)$

The response = $y(t)$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$e(t) = \mathcal{L}^{-1}E(s)$$

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$e_{ss} = \lim_{s \to 0} sE(s)$$

final value theorem
Steady state error is the difference between the desired value for the response and the actual response at the steady state (at $t = \infty$). Steady state error is termed static.

1. Input is step $R(s) = 1/s$:

\[
e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s)H(s)} = \frac{1}{1 + \lim_{s \to 0} G(s)H(s)}
\]

\[K_p = \text{static position error coefficient}\]

\[G(s)H(s) = \frac{K(1 + z_1s)(1 + z_2s) \ldots (1 + z_ms)}{s^N(1 + p_1s)(1 + p_2s) \ldots (1 + p_Qs)}, \quad m < Q + N\]

According to the value of $N$, one can define the type of the system, and consequently the steady state error.
For \( N = 0 \) (System type 0):

\[
Kp = \lim_{s \to 0} G(s)H(s) = k \implies e_{ss} = \frac{1}{1+k}
\]

For \( N = 1 \) (System type 1):

\[
Kp = \infty \implies e_{ss} = 0
\]

For \( N = 2 \) (System type 2):

\[
Kp = \infty \implies e_{ss} = 0
\]
2. Input is ramp \( R(s) = \frac{1}{s^2} \):

\[
e_{ss} = \lim_{s \to 0} \frac{s \left( \frac{1}{s^2} \right)}{1 + G(s)H(s)} = \lim_{s \to 0} \frac{1}{s + sG(s)H(s)} = \lim_{s \to 0} \frac{1}{G(s)H(s)}
\]

\( Kv = \) static velocity error coefficient

For \( N = 0 \) (System type 0):

\( Kv = 0 \implies e_{ss} = \infty \)

For \( N = 1 \) (System type 1):

\( Kv = k \implies e_{ss} = \frac{1}{k} \)

For \( N = 2 \) (System type 2):

\( Kv = \infty \implies e_{ss} = 0 \)
2. Input is parabolic $r(t) = t^2/2$:

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)H(s)} = \lim_{s \to 0} \frac{1}{s^2 G(s)H(s)}$$

$Ka = \text{static acceleration error coefficient}$

For $N = 0$ (System type 0):

$$Ka = 0 \implies e_{ss} = \infty$$

For $N = 1$ (System type 1):

$$Ka = 0 \implies e_{ss} = \infty$$

For $N = 2$ (System type 2):

$$Kv = k \implies e_{ss} = 1/k$$
Dynamic error coefficient

\[ E(s) = \frac{R(s)}{1 + G(s)H(s)} \]

\[ = R(s) \left[ \frac{1}{k_1} + \frac{1}{k_2} s + \frac{1}{k_3} s^2 + \ldots \right] \]

K1 = dynamic position error coefficient
K2 = dynamic velocity error coefficient
K3 = dynamic acceleration error coefficient

\[ e(t) = \frac{1}{k_1} r(t) + \frac{1}{k_2} \dot{r}(t) + \frac{1}{k_3} \ddot{r}(t) \]

which gives an indication to the error at any time t