1. Text (Sipser, second edition) Chapter 1 (p.88) 1.29b [14%] \[ A_2 = \{ \omega \omega \omega \mid \omega \text{ is in } \{a,b\}^* \} \]

Assume to the contrary that \( A_2 \) is regular, and let \( p \) be the pumping length given by pumping lemma. Choose \( s = a^p ba^p ba^p b \), which can be divided into three pieces \( s = xyz \), where \( |xy| \leq p \). This means \( xy \) contains only \( a \)'s. Since \( |y| > 0 \), let \( y = a^k \), \( k > 0 \). However, \( xy^2z = a^{p+k} ba^p ba^p b \), where \( p+k > p \), is not in \( A_2 \). That is \( s \) cannot be pumped. This is a contradiction. Thus, \( A_2 \) is not regular.

2. Text (Sipser, second edition) Chapter 1 (p.89) 1.31 [14%]

Let \( M = (Q, \Sigma, \delta, q_0, F) \) be the DFA that recognizes \( A \). We can build an NFA \( M^R = (Q^R, \Sigma^R, \delta^R, q_0^R, F^R) \) that recognizes the reverse language \( A^R \) as follows: We pick \( q_0^R \notin Q \) as the start state, \( \Sigma^R = \Sigma \), \( Q^R = Q \cup \{ q_0^R \} \), \( F^R = \{ q_0 \} \), and

\[
\delta^R(q, a) = \bigcup_{\delta(p, a) = q} \{ p \}
\]
3. Text (Sipser, second edition) Chapter 1 (p.89) 1.34 [14%]
The following DFA recognizes D. Thus, D is regular.

4. Text (Sipser, second edition) Chapter 1 (p.89) 1.37 [14%]
We only need to construct a DFA to keep track of the remainder of the input seen so far (from left to right) divided by n. If it ends up with remainder zero, accept; otherwise reject. Notice the following relations:

If \((\omega \mod n) = k\), where \(0 \leq k \leq n - 1\), we have \(\omega = nq + k\).

For case \(\omega 0\): the remainder is 
\[
[2(nq + k) \mod n] = [2k \mod n]
\]

For case \(\omega 1\): the remainder is 
\[
[2(nq + k) + 1 \mod n] = [2k + 1 \mod n]
\]

Construct DFA \(M=({q_0,q_1, \ldots, q_n}, \{0,1\}, \delta, q_0, \{q_0\})\),

\[
\delta(q_k,a) = q_j, \text{ where } j = \begin{cases} 
2k \mod n \quad & \text{if } a = 0 \\
2k + 1 \mod n \quad & \text{if } a = 1
\end{cases}
\]

\(M\) recognizes \(C_n\), thus \(C_n\) is regular.

5. Text (Sipser, second edition) Chapter 1 (p.90) 1.46a, 1.46c, 1.46d [10% each]

1.46a \(\{0^n1^m0^n \mid m, n \geq 0\}\)
Use pumping lemma, and choose \(s = 0^n1^n = xyz\). Here \(xy\) contains only 0’s. Let, \(y = 0^k\), \(k>0\). Thus \(xy^0z = 0^n-^k1^n\) is not in the language. Thus, it’s not regular.
14.6c \{w \mid w \text{ in } \{0,1\}^* \text{ is not a palindrome}\}
Let its compliment language \( L = \{w \mid w \text{ in } \{0,1\}^* \text{ is a palindrome}\}, \) we can prove the original language is not regular by showing that \( L \) is not regular.
Choose \( s = 0^p10^p = xyz \), a palindrome in \( L \). Here \( xy \) contains only 0’s. Let, \( y = 0^k \), \( k > 0 \). Thus \( xy^0z = 0^{p-k}10^p \) is not a palindrome, thus it’s not in the language \( L \). Thus, \( L \) is not regular.

1.46d \{wtw \mid w, t \text{ are in } \{0,1\}^+\}
Use pumping lemma, and choose \( s = 0^p10^p = xyz \). Here \( xy \) contains only 0’s. Let, \( y = 0^k \), \( k > 0 \). Thus \( xy^0z = 0^{p-k}10^p \) is not in the language. Thus, it’s not regular.

6. Text (Sipser, second edition) Chapter 1 (p.90) 1.48 [14%]
The following DFA recognize \( D \), thus \( D \) is regular.