Abstract—Physical-layer network coding (PLNC) allows two remote terminals to exchange information in two time slots via a half-duplex relay. In this paper, we consider PLNC in underwater acoustic (UWA) two-way relay networks, with particular attention on the recovery of the network-coded codeword at the relay. Considering multiple paths in UWA channels with large delay spreads and temporal variations, orthogonal-frequency division-multiplexing (OFDM) is used as an underlying modulation. Distinct from existing works which mainly focus on the flat-fading channel and the non-iterative PLNC decoding, we investigate three iterative receivers at the relay in doubly-selective UWA channels. To estimate the network-coded codeword, the three receivers respectively adopt i) a conventional decoding scheme to recover the codewords from two terminals independently; ii) a decoding scheme to recover the network-coded codeword directly, and iii) a joint decoding scheme to recover a composite codeword formed by the codewords from the two terminals. In the time-varying channel, the intercarrier interference (ICI) in the received signal has been explicitly addressed in three receivers. Extensive simulations and experimental results reveal that the decoding scheme for direct recovery of the network-coded codeword cannot work well in the multipath channel, and that with a lower complexity the performance of the iterative separate-decoding scheme can catch up with that of the joint decoding scheme.

Index Terms—Physical-layer network coding (PLNC), underwater acoustic two-way relay networks, OFDM, turbo equalization, doubly selective

I. INTRODUCTION

Network coding is envisioned to offer a broadband wireless interface with ubiquitous coverage in large areas with high capacity [2]. Extensive investigation has been carried out in the last decade; see, e.g., [2]–[11] and references therein.

Consider a two-way relay network where two terminals A and B desire to exchange information with the help of a relay R. The conventional approach without network coding requires a total of four time slots for messages exchange, with each terminal having two time slots individually. By applying the network coding, the number of required time slots can be reduced, as illustrated in Fig. 1. Typically, existing network coding schemes applied to two-way relay channels fall into two categories.

- **Network-layer network coding (NLNC):** The scheme applying network coding at the network layer reduces the number of required time slots from four to three by exploiting the broadcasting nature of the message from relay (see, e.g., [3], [4]). In this scheme, A and B take turns to send their messages to R in the first two time slots, and R transmits the XOR-ed version of the two messages in the third time slot.

- **Physical-layer network coding (PLNC):** This scheme further reduces the number of required time slots to two by exploiting the superposition nature of wireless signals (see, e.g., [6]–[8] and reference therein). In the first time slot which is known as the multiple-access (MAC) phase, both A and B transmit their own messages to R simultaneously. In the second time slot which is known as the broadcast (BC) phase, R broadcasts the XOR-ed version of the two messages. Recently, it has been pointed out that the coding operation at the relay node can be performed not only in the Galois field [6], [7] but also in the complex filed [12], [13], and a more general computation coding can be found in [14]. One variant of PLNC under extensive investigation recently in radio channels is the analog network coding (ANC) [5], where the relay node simply amplifies and forwards the received superimposed signals; see recent progress in, e.g., [9]–[11]. This is usually adopted when the relay node has limited capability.
A. Decoding at the Relay Node

In this paper, we consider the decode-and-forward (DF) based PLNC, in which the critical process is to recover the XOR-ed codeword at the relay node based on the received superimposed signal from two terminals.

To achieve a good decoding performance, the combination of PLNC with channel coding in a block manner is considered [15], [16]. For the additive white Gaussian noise channel (AWGN), it has been pointed out in [8], [15] that it is not necessary to recover individual messages from two terminals. Leveraging the fact that with the same linear channel code at both terminals the XOR-ed version of both terminal codewords is also a valid codeword, the relay node can demodulate and decode the XOR-ed version directly, which however, has been shown to suffer information loss during the demodulation [8], [15].

Ref. [16] proposed a generalized sum-product algorithm (G-SPA) based channel decoding approach which leads to improved performance. The basic idea is to construct a virtual nonbinary code for decoding which is followed by PLNC mapping. For example, with binary low-density parity-check (LDPC) coding and BPSK modulation, a GF(4) nonbinary LDPC code can be constructed for information recovery. However, since the decoding complexity per bit for a nonbinary LDPC code over GF(\(q\)) scales with \(q \log(q)\) (see [17] and references therein), this method requires a high complexity when the modulation constellation is large.

B. PLNC with OFDM Modulation

Another critical issue of PLNC in practical systems is the synchronization of signals from the two terminals. For the channel with multiple paths, one viable solution is to combine PLNC with orthogonal-frequency division-multiplexing (OFDM) modulation. The OFDM modulated ANC in radio channels has been considered in [9]–[11], [18]–[20], where relay only performs the amplify-and-forward operation.

C. Our Work

PLNC is an appealing approach for underwater acoustic (UWA) communications which are bandwidth-limited in nature. Consider the scenario in Fig. 2, where two terminals (e.g. AUVs or sensors) need to exchange information but cannot reach each other due to the long distance or obstacles in-between. By incorporating a relay within the communication ranges of both terminals, PLNC enables the message exchange with two time slots. Moreover, relative to the relay node in radio channels, the relay node (e.g. the buoy as shown in Fig. 2) in the underwater acoustic setting can be more capable, such as being equipped with multiple receiving hydrophones and strong processing capability.

Compared with radio channels, the UWA channel is recognized as having a very large delay spread and fast time-variations [21], [22]. In recent years, significant progress on UWA communications has been witnessed for both multicarrier and single-carriers transmissions; see, e.g., [22]–[31], and references therein. However, research on the network coding in UWA channels is still quite limited. Existing works can be found in e.g., [32], [33], which mainly focus on network coding at the network layer.

In this work, we focus on the two-way relay UWA network as shown in Fig. 2, where the relay node can be equipped with multiple receiving hydrophones. Contributions of this paper are as follows.

- We investigate the OFDM-modulated PLNC in the UWA two-way relay network with DF at the relay node. Different from existing research which mainly focuses on PLNC in AWGN channel or the flat-fading channel [8], [15], [16] in terrestrial radio environments, the UWA channel under consideration is usually doubly spread, hence the intercarrier interference (ICI) has to be explicitly considered.
- Note that existing works are all based on non-iterative receiver processing. We propose iterative receiver processing at relay to fully exploit the decoding potential [34]. Under the framework of iterative processing, three decoding approaches are examined, including i) the separate decoding where the XOR-ed codeword is obtained after decoding the codewords from both terminals individually, ii) the XOR-ed PLNC decoding which estimates the XOR-ed codeword directly based on the received signal at the relay node [8], [15], and iii) the generalized PLNC decoding where the XOR-ed codeword is recovered by estimating a composite codeword formed by the codewords from both terminals [16].
- We suggest a Gaussian message passing (GMP)-based channel equalization method for time-varying channels to address the ICI. Compared with the maximum \(a posteriori\) (MAP) equalization method, the proposed method enjoys a good performance and a reasonable complexity.
When the number of receiving hydrophones is more than one, the proposed method becomes a minimum-mean-square-error (MMSE) equalizer [35].

- Extensive simulations and real experimental results are provided to compare the performance of the proposed iterative processing schemes.

Based on the simulations and experimental results, we have observations as follows.

- First, existing works showed that in the flat-fading channel, the XOR-ed PLNC decoding is better than the noniterative separate decoding [8], [15], [16]. Our observation reveals that the XOR-ed PLNC decoding suffers significant performance degradation in the channel with multiple paths.

- Secondly, existing works that mainly focus on noniterative processing and flat-fading channels, showed that the generalized PLNC is much better than the noniterative separate decoding [16]. In this work, the iterative processing results show that when the relay node only has one receiving hydrophone, the generalized PLNC decoding is the best option, and as the number of receiving hydrophones increases, the iterative separate decoding with a lower processing complexity can converge to the generalized PLNC decoding.

**Remark 1:** In this work, we only consider the scenario that two terminals are either stationary or have similar moving velocities. The scenario that two terminals have very different moving velocities is of interest but beyond the scope of this work. Preliminary studies on the receiver design for an OFDM moving velocities is of interest but beyond the scope of this work. Preliminary studies on the receiver design for an OFDM moving velocities is of interest but beyond the scope of this work.

**Notation:** Bold upper case letters and lower case letters denote matrices and column vectors, respectively. $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively. $\propto$ denotes equality of functions up to a scaling factor. $\oplus$ denotes the XOR-ed operation between two coded sequences. $[a]_{m}$ denotes the $m$th element of vector $a$, and $[A]_{m,k}$ denotes the $(m,k)$th element of matrix $A$. $|A|$ denotes the determinant of matrix $A$. $I_N$ denotes an identity matrix of size $N \times N$.

## II. OFDM Modulated PLNC in Underwater Acoustic Channels

We consider a half-duplex two-way relay network in the UWA environment. It consists of two single-antenna terminals $A$ and $B$, and a relay node with $N_r$ receiving hydrophones, as shown in Fig. 2. A system model will be developed in the sequel to build a relationship between measurements at the relay node and transmitted symbols from both terminals.

### A. Transmitted Signal and Channel Model

Assume that the two terminals use an identical parameter set for OFDM modulation with a total of $K$ subcarriers. Denote $X_\mu$ as a channel coded sequence over GF($M$) from the $\mu$th terminal with $\mu = A$ or $B$. Define $T$ as a modulation mapping operator. The transmitted data symbol vector at the $\mu$th terminal is obtained via $s_{\mu} = T(X_\mu)$. The data symbol vector $s_{D,\mu}$ is multiplexed with pilot symbols to form a transmitted symbol vector $s_\mu$ of size $K \times 1$. The transmitted passband waveform $\tilde{s}_\mu(t)$ is then obtained via the OFDM modulation [37].

We adopt a path-based model to characterize the channel between each terminal and receiving hydrophone pair [37]. Define $N_{p,\nu,\mu}$ as the number of paths between the $\mu$th terminal and the $\nu$th receiving hydrophone. Within one OFDM block, we assume that i) the path amplitude does not change, and ii) the path delay follows a first order kinematic model with a Doppler rate representing the delay changing rate. The channel impulse response with path-specific Doppler scales can be written as

\begin{equation}
\begin{aligned}
\hat{h}_{\nu,\mu}(t; \tau) = \sum_{p=1}^{N_{p,\nu,\mu}} A_{p,\nu,\mu} \delta(\tau - (\tau_{p,\nu,\mu} - \alpha_{p,\nu,\mu} t))
\end{aligned}
\end{equation}

\(1\)

where $A_{p,\nu,\mu}$ and $\tau_{p,\nu,\mu}$ are the amplitude and initial delay of the $p$th path, respectively, and $\alpha_{p,\nu,\mu}$ is the Doppler rate

1**Despite simplicity of the network under consideration, with slight modifications the proposed receiving algorithms in this work can be applied to complex networks with more terminals or more than one relay node.**
of the $p$th path which is related to the path speed $v_{p,v,\mu}$ via $a_{p,v,\mu} = v_{p,v,\mu}/c$ with $c$ being the sound speed in water.

B. Received Signal at the Relay Node

Assume that both terminals are aware of their distances to the relay node. By adjusting the transmission time at one terminal, signals from the two terminals can be quasi-synchronized at the relay node. The passband signal received at the $v$th hydrophone can be expressed as

$$
\hat{y}_v(t) = \sum_{p=1}^{N_{pa,v,A}} A_{p,v,A} s_A((1 + a_{p,v,A})t - \tau_{p,v,A}) \\
+ \sum_{p=1}^{N_{pa,v,B}} A_{p,v,B} s_B((1 + a_{p,v,B})t - \tau_{p,v,B}) + \tilde{n}(t),
$$

where $\tilde{n}(t)$ is the ambient noise.

At the receiver side, a resampling operation (with a resampling factor $(1 + \hat{\alpha})$) is performed to remove the main Doppler effect in the received signal caused by the platform mobility. After the passband to baseband signal downshifting and lowbandpass filtering, the residual Doppler effect is taken as a carrier-frequency-offset (CFO) and compensated by multiplying a phase rotation term $e^{-j2\pi\hat{\alpha}t}$ with the baseband signal, where $\hat{\alpha}$ denotes the estimated CFO; please refer to [22], [37] for detailed descriptions on the resampling factor and CFO estimation. The channel impulse response in (1) translates into a $K \times K$ channel matrix $H_{v,\mu}$ in the frequency domain with the input-output relationship at the $v$th receiving hydrophone expressed as

$$
z_v = H_{v,\mu}s_A + H_{v,\mu}s_B + w_v
$$

where $z_v$ of size $K \times 1$ is the frequency measurement vector at all subcarriers, and $w_v$ is the ambient noise vector in the frequency domain.

The $(m,k)$th element of the channel matrix $H_{v,\mu}$ is related to channel path parameters via

$$
H_{v,\mu}[m,k] = \sum_{p=1}^{N_{pa,v,\mu}} A'_{p,v,\mu} e^{-j2\pi f_m \tau'_{p,v,\mu}} g_m,k(b_{p,v,\mu}, \hat{\alpha}_v),
$$

with

$$
b_{p,v,\mu} = \frac{a_{p,v,\mu} - \hat{\alpha}_v}{1 + \hat{\alpha}_v}, \quad A'_{p,v,\mu} = \frac{A_{p,v,\mu}}{1 + b_{p,v,\mu}},
$$

$$
\tau'_{p,v,\mu} = \frac{\tau_{p,v,\mu}}{1 + b_{p,v,\mu}}, \quad g_m,k(b, \epsilon) := G\left(\frac{f_m + \epsilon}{1 + b} - f_k\right)
$$

where $G(f)$ is the Fourier transform of the pulse shaping window at the transmitter; for a rectangular window, we have $G(f) = \frac{\sin(\pi T)}{\pi T} e^{-j\pi fT}$. Please see [37], [38] for detailed derivations of (4), which will not be replicated here.

Depending on channel variations, channel matrix $H_{v,\mu}$ has the following properties.

- In the time-invariant channel, i.e., $a_{p,v,\mu} = 0$, $\forall p$, we have $H_{v,\mu}[m,k] = 0$, $\forall m \neq k$.

Hence $H_{v,\mu}$ is a diagonal matrix.

- In the time-varying channel, i.e., $a_{p,v,\mu} \neq 0$, we have $|H_{v,\mu}[m,k]| > 0$, $\forall m, k$. Hence, $H_{v,\mu}$ becomes a full matrix with off-diagonal values specifying ICI coefficients caused by the path Doppler spread $\max_{p,q} |a_{p,v,\mu} - a_{q,v,\mu}|$. For the sake of computational efficiency, a band-limited ICI-leakage assumption is usually adopted by approximating

$$
H_{v,\mu}[m,k] \simeq 0, \quad \forall |m - k| > D
$$

meaning that at each subcarrier only the ICI from $D$ neighboring subcarriers on each side is considered, and the residual ICI is taken as ambient noise. We term $D$ as the ICI depth, which is a design parameter related to the channel Doppler spread [37].

Putting $\{z_v\}$ at all receiving hydrophones into a long vector yields

$$
\begin{bmatrix}
{z_1} \\
\vdots \\
{z_N_v}
\end{bmatrix} = \begin{bmatrix}
{H}_{1,A} \\
\vdots \\
{H}_{N_v,A}
\end{bmatrix} s_A + \begin{bmatrix}
{H}_{1,B} \\
\vdots \\
{H}_{N_v,B}
\end{bmatrix} s_B + \begin{bmatrix}
{w}_1 \\
\vdots \\
{w}_{N_v}
\end{bmatrix}
$$

which can be rewritten as

$$
z_R = H_A s_A + H_B s_B + w_R,
$$

where the size of $z_R$ is $N_v K \times 1$, and the size of $H_A$ and $H_B$ is $N_v K \times K$.

The primary task of the relay node at the MAC phase is to recover $X_R := X_A \oplus X_B$, with the corresponding modulated symbol vector denoted by $s_R$. After decoding and successfully recovering $X_R$, relay broadcasts the OFDM modulated symbol $s_R$ to both terminals, as shown in Fig. 2. Both A and B will recover $X_R$ from its corresponding received signal. With the estimated $X_R$, the intended message can be extracted by subtracting its own message.

This paper focuses on the MAC phase. Three iterative receiver processing schemes to recover $X_R$ at the relay node is presented in the next section.

Remark 2: Note that quasi-synchronization of signals from two terminals is required in (2) to achieve the frequency-domain synchronization illustrated in (3). To account for the temporal synchronization inaccuracy, a large guard interval $T_g$ between consecutive OFDM symbols is necessary. Denote $T_{ch}$ as the channel delay spread, and $\Delta T$ the difference of time-of-arrivals of signals from two terminals. With a guard interval $T_g \geq T_{ch} + \Delta T$, the frequency-domain synchronization in (3) can be safely obtained from the time-domain signal in (2). The cost incurred with a large $T_g$ is a reduction of the system spectral efficiency, which is linearly proportional to $T/(T + T_g)$ with $T$ being the time duration of each OFDM symbol.

III. THE ITERATIVE OFDM RECEIVER DESIGN

In this section, we consider an overall structure of the iterative processing for OFDM in doubly selective two-way relay channels. We first assume that the channel state information is available via channel estimation based on frequency
measurements at pilot subcarriers [37], hence mainly focus on the iterative symbol detection and channel decoding. We consider three receiver schemes.

The first scheme is the conventional iterative separate detection and decoding (I-SDD). In this scheme, the relay node first tries to recover both $X_A$ and $X_B$ individually. The relay encoded symbol $X_R$ can then be obtained by doing simple XOR. Since the objective at relay is to compute $X_R$, instead of recovering both $X_A$ and $X_B$ which is not necessary, it is acceptable to recover only the XOR-ed signal $X_A \oplus X_B$ directly. The second receiver scheme performs iterative detection and decoding on the XOR-ed codeword $X_A \oplus X_B$. We term this scheme as iterative XOR-ed PLNC detection and decoding (I-XPDD). Ref. [16] shows that instead of performing channel decoding to recover both $X_A$ and $X_B$ separately or to recover $X_A \oplus X_B$, a better way to exploit all potentials of PLNC is to construct a super-code and perform channel decoding on $(X_A, X_B)$. Hence in the third scheme we propose to perform iterative detection and decoding on a super-code over $(X_A, X_B)$ followed by PLNC mapping to recover $X_A \oplus X_B$. We term the third scheme as iterative generalized PLNC detection and decoding (I-GPDD) following [16].

The receiver diagrams for all three schemes are depicted in Fig. 3. In each scheme, there are two kinds of modules: the equalization module and the error control decoding module(s). For complexity consideration, the equalization module is implemented differently depending on whether ICI is present. In the absence of ICI, the equalization module will be based on MAP detection. In the presence of ICI, it will be based on Gaussian message passing (GMP) [39]. The error control decoding modules are all based on the sum-product algorithm (SPA) [40].

A. Iterative Separate Detection and Decoding

The conventional iterative receiver tries to recover both $X_A$ and $X_B$. This is exactly the well-known multiple-access problem.

Following the turbo equalization principle [34], extrinsic information can be exchanged between the equalization and error control decoding. Transforming the extrinsic information $P_e^{(a)}\{X_A\}$ and $P_e^{(a)}\{X_B\}$ from the two channel decoders to the a priori information $P_t^{(a)}\{s_A\}$ and $P_t^{(a)}\{s_B\}$ for symbol detection, the MAP detection in the absence of ICI or the GMP based detection over a factor graph in the presence of ICI is used to derive the extrinsic information, $P_t^{(e)}\{s_A, s_B\}$, which is then transformed into the a priori information, $P_t^{(a)}\{X_A \oplus X_B\}$. Then channel decoding is applied on $X_A \oplus X_B$. After channel decoding, the decoder outputs extrinsic message to the detector and also make a tentative decision for $X_A$ and $X_B$. The iteration between detection and decoding goes on until the two decoder succeed or the number of iterations reaches a predetermined threshold.

Calculation of the extrinsic information $P_t^{(e)}\{s_A\}$ and $P_t^{(e)}\{s_B\}$ in the MAP and GMP-based symbol detection methods will be discussed in Section IV. Please refer to, e.g., [17] for detailed calculation of the extrinsic information $P_t^{(e)}\{X_A\}$ and $P_t^{(e)}\{X_B\}$ from the channel decoder.

B. Iterative XOR-ed PLNC Detection and Decoding

I-XPDD tries to recover the XOR-ed codeword $X_A \oplus X_B$ directly. The receiver diagram for this scheme is shown in Fig. 3. The MAP detection in the absence of ICI or the GMP based symbol detection [39] in the presence of ICI, and the SPA based channel decoding [40] are adopted.

With the a priori information, $P_t^{(a)}\{(s_A, s_B)\}$ which is transformed from the extrinsic information $P_t^{(e)}\{(X_A \oplus X_B)\}$ from the channel decoders, GMP over a factor graph is used to derive the extrinsic information, $P_t^{(e)}\{(s_A, s_B)\}$, which is then transformed into the a priori information, $P_t^{(a)}\{X_A \oplus X_B\}$. Then channel decoding is applied on $X_A \oplus X_B$. After channel decoding, the decoder outputs extrinsic message to the detector and also make a tentative decision for $X_A \oplus X_B$. Since the mapping from $(s_A, s_B)$ to $s_R$, i.e., from $(X_A, X_B)$ to $X_A \oplus X_B$ is non-invertible, the transformation from $P_t^{(e)}\{X_A \oplus X_B\}$ to $P_t^{(e)}\{(s_A, s_B)\}$ is done by equally splitting the probability of each value of $X_A \oplus X_B$ to all corresponding pairs of $(s_A, s_B)$. The transformation from $P_t^{(e)}\{(s_A, s_B)\}$ to $P_t^{(a)}\{X_A \oplus X_B\}$ is done by merging the probability of all pairs of $(s_A, s_B)$ corresponding to $X_A \oplus X_B$.

C. Iterative Generalized PLNC Detection and Decoding

I-GPDD aims to recover $X_A \oplus X_B$ by performing detection and decoding on the pair $(X_A, X_B)$ followed by PLNC mapping. The receiver diagram for this scheme is shown in Fig. 3. The MAP detection in the absence of ICI or the GMP based symbol detection [39] in the presence of ICI, and the SPA based channel decoding [40] are adopted.

With the a priori information, $P_t^{(a)}\{(s_A, s_B)\}$, transformed from the extrinsic information, $P_t^{(e)}\{(X_A, X_B)\}$ exported from the channel decoders, GMP over a factor graph in the presence of ICI or MAP in the absence of ICI is used to derive the extrinsic information, $P_t^{(e)}\{(s_A, s_B)\}$, which is then transformed into the a priori information, $P_t^{(a)}\{(X_A, X_B)\}$. Then channel decoding is applied on $(X_A, X_B)$. After channel decoding, the decoder outputs extrinsic message to the detector and also make a tentative decision for $(X_A, X_B)$. The iteration between detection and decoding goes on until the decoder succeeds or the number of iterations reaches a predetermined threshold.

Remark 3: In practical systems, the channel state information is unknown, an iterative receiver including the channel estimation in the iteration loop, as discussed in [41], [42], is capable of refining the estimated channel matrix by utilizing the soft or hard decisions of transmitted symbols from two terminals. When dealing with the experimental data sets in Section VI, a sparse channel estimator developed in [37] will be used to estimate the channel matrices $\{H_{v, A}\}$ and $\{H_{v, B}\}$.

IV. Symbol Detection

A. MAP Detection for ICI-Ignorant Processing

In the time-invariant channel, the channel mixing matrix between each terminal and receiving hydrophone pair is diagonal, meaning that ICI is absent. The MAP detection
with the *a priori* information can be applied. Based on (8), the frequency measurement at the $k$th subcarrier of the $\nu$th receiving hydrophone can be represented as

$$z_\nu[k] = H_{\nu,A}[k, k]s_A[k] + H_{\nu,B}[k, k]s_B[k] + w_\nu[k]. \quad (9)$$

For simplicity, we assume a complex white Gaussian ambient noise with $w_\nu[k] \sim \mathcal{CN}(0, \sigma^2)$. The likelihood function of the symbol pair $(s_A[k], s_B[k])$ is

$$\Pr(z_\nu[k]|(s_A[k], s_B[k])) \propto \exp\left\{-\frac{1}{\sigma^2} |z_\nu[k] - H_{\nu,A}[k, k]s_A[k] - H_{\nu,B}[k, k]s_B[k]|^2\right\}. \quad (10)$$

For the schemes I-GPDD and I-XPDD in channels without ICI, iterative processing is not necessary, where symbol-by-symbol data detection on each OFDM subcarrier is optimal. The posterior probability for I-GPDD is

$$\Pr\{(s_A[k], s_B[k])|z_\nu[k]\} \propto \Pr(z_\nu[k]|(s_A[k], s_B[k])) \quad (11)$$

and the posterior probability for I-XPDD is

$$\Pr\{s_B[k]|z_\nu[k]\} \propto \Pr(z_\nu[k]|s_B[k]) = \sum_{s_A[k]} \Pr(z_\nu[k]|(s_A[k], s_B[k])) \quad (12)$$

where $A(s_B[k])$ denotes a set formed by $\{(s_A[k], s_B[k])\}$ satisfying $X_B[k] = X_A[k] \oplus X_B[k]$. Here, an equal *a priori* probability of each pair is assumed.

The MAP detection for I-SDD with the *a priori* information $\Pr^a\{s_A[k]\}$ and $\Pr^a\{s_B[k]\}$ from the decoder is

$$\Pr\{(s_A[k], s_B[k])|Y_B[k]\} \propto \Pr\{z_\nu[k]|(s_A[k], s_B[k])\}\Pr^a\{s_A[k]\}\Pr^a\{s_B[k]\} \quad (13)$$

In reality, the ambient noise in the UWA environment is most likely colored. A pre-whitening operation proposed in [38] can be performed to change the colored noise to be white. However, in this paper, given the lack of noise statistics in the experiment discussed in Section VI, we keep the white noise assumption in the experimental data decoding.

The *extrinsic* information to be fed to the decoder for terminals A and B in I-SDD can be calculated as

$$\Pr^e\{s_A[k]\} \propto \sum_{s_B[k]} \Pr\{z_\nu[k]|(s_A[k], s_B[k]\})\Pr^a\{s_B[k]\}$$

$$\Pr^e\{s_B[k]\} \propto \sum_{s_A[k]} \Pr\{z_\nu[k]|(s_A[k], s_B[k]\})\Pr^a\{s_A[k]\}$$

**B. GMP Detection for ICI-Aware Processing**

In the time-varying channel with the presence of ICI in the received signal, it is necessary to recover each transmitted symbol based on frequency measurements at all subcarriers. Even with the band-limited assumption of channel matrices (c.f. (6)), the computational complexity of MAP equalizer is still prohibitive for the UWA system with a large number of subcarriers (e.g., $K = 1024$ in some practical OFDM systems [43]). In this paper, we adopt GMP over a factor graph with the *a priori* information for detection. The factor graph representation corresponding to (8) in the MAC phase is shown in Fig. 4 where matrices $\{H_{\nu,A}\}$ and $\{H_{\nu,B}\}$ with one off-diagonal on each side are used for illustration.

**2**In reality, the ambient noise in the UWA environment is most likely colored. A pre-whitening operation proposed in [38] can be performed to change the colored noise to be white. However, in this paper, given the lack of noise statistics in the experiment discussed in Section VI, we keep the white noise assumption in the experimental data decoding.
Fig. 4. Factor graph representation with one receiving hydrophone where the channel mixing matrices $H_{1:A}$ and $H_{1:B}$ have one off-diagonal on each side. The rectangle with a plus inside represents LDPC encoding constraint and the filled rectangle represents the relationship defined in (8).

Fig. 5. Performance comparison of different receivers for OFDM modulated PLNC without ICI in the additive white Gaussian noise Channel, one receiving hydrophone.

and I-XPDD schemes (see Fig. 3). The means and covariances of these Gaussian distributions are calculated based on the feedback information from the LDPC channel decoder.

V. SIMULATION RESULTS

We consider an OFDM transmission with center frequency $f_c = 13$ kHz, bandwidth $B = 9.77$ kHz, symbol duration $T = 104.86$ ms, and guard time $T_g = 24.6$ ms. Out of 1024 subcarriers, there are $|S_N| = 96$ null subcarriers with 24 on each edge of the signal band for band protection and 48 evenly distributed in the middle for the carrier frequency offset estimation; there are $|S_P| = 256$ pilot subcarriers uniformly distributed among the 1024 subcarriers, and the remaining are $|S_D| = 672$ data subcarriers for delivering information symbols. The pilot symbols at each terminal are drawn independently from a QPSK constellation. The data within each OFDM symbol is encoded by a rate-1/2 GF(4) nonbinary near-regular LDPC code of length 672 symbols [17] and modulated using a QPSK constellation, leading to a data rate

$$R = \frac{1}{2} \cdot \frac{|S_D|}{T + T_g} \log_2 4 = 5.2 \text{ kb/s/user}. \quad (14)$$

We use a channel model depicted in (1). Unless specified, we simulate the channel between each terminal and receiving hydrophone pair with 10 discrete paths. In each instantiation of the channel, we assume that the inter-arrival times of paths are exponentially distributed with inter-arrival mean of 1 ms. The amplitudes of paths are Rayleigh distributed with the average power decreasing exponentially with delay, where the difference between the beginning and the end of the guard time is 20 dB. In the time-invariant channel, the Doppler rate of each path is set zero. To simulate the time-varying channel, the Doppler rate of each path is drawn independently from a zero-mean uniform distribution according to the standard deviation of path speed $\sigma_v \text{ m/s}$.

To explore the benefit of iterative processing between channel equalization and decoding, we assume that the channel estimate is available prior to the symbol detection. As each OFDM symbol is encoded separately, we use the block-error-rate (BLER) of the XOR-ed symbol as the performance merit. We next examine the PLNC decoding algorithms in three channel settings.

A. The Single-Path Time-Invariant Channel

We first consider a single-path time-invariant channel described by (1) with $N_{p,a,v,\mu} = 1$, where the path has a unit amplitude, a random delay and a zero Doppler rate. Fig. 5 demonstrates the BLER performance of three receivers with a MAP equalizer. One can see that I-GPDD has the best performance, I-XPDD is in the middle, and I-SDD has the worst performance. This is consistent with the observation in [16]. Despite the performance improvement with the iterative processing, there is still a considerable gap between I-SDD and the other two schemes.

B. The Multipath Time-Invariant Channel

Corresponding to the multipath time-invariant channel described by (1) with $N_{p,a,v,\mu} = 10$ and $a_{p,a,v,\mu} = 0$, Fig. 6 shows performance comparison of three receivers with different the number of receiving hydrophones. Note that in this case the iterative processing for I-XPDD and I-GPDD is not necessary. As a performance benchmark, the lower and upper bounds of the outage probability with single receiving hydrophone are provided; please refer to the Appendix for details about the outage probability bounds.

We can see from Fig. 6 that the iterative processing can improve the performance of I-SDD significantly. When the number of receiving hydrophone is one, I-SDD performs worse than I-GPDD. This is reasonable because the I-SDD scheme tries to recover both $X_A$ and $X_B$, which is under-determined when the relay node has only one receiving
hydrophone. When the number of receiving hydrophone is larger than one, the iterative I-SDD receiver can catch up with the I-GPDD scheme with only one iteration. Note that the decoding complexity of I-GPDD is much higher than that of I-SDD.

Meanwhile, we can see from Fig. 6 that performance of the scheme I-XPDD degrades significantly in the multipath channel relative to that in the AWGN channel shown in Fig. 5. Hence, we will not consider the scheme I-XPDD for the following simulations.

C. The Multipath Time-Varying Channel

Now, we consider the doubly selective channel with path-specific Doppler scales, as described by (1) with $N_{pa,p,\nu,\mu} = 10$ and $|a_{p,\nu,\mu}| \geq 0$, $\forall p$. Fig. 7 shows the performance comparison of I-SDD and I-GPDD with different number of receiving hydrophones and different standard deviations of path speed. For GMP-based detection we assume that the channel mixing matrix between each terminal and receiving hydrophone pair is banded with one and three off-diagonals on each side for $\sigma_v = 0.1$ m/s and $\sigma_v = 0.2$ m/s, respectively [37].

We can see from Fig. 7 that the iterative processing improves the performance of I-SDD significantly for both values of $\sigma_v$. For I-GPDD, iterative processing also boosts its performance when $\sigma_v = 0.2$ m/s as shown in Fig. 7 whereas not much improvement is seen when $\sigma_v = 0.1$ m/s. With single receiving hydrophone, one can see that I-GPDD outperforms I-SDD considerably. With more than one receiving hydrophones, the iterative I-SDD receiver with a lower decoding complexity can catch up with the non-iterative I-GPDD scheme by using more iterations.

D. The Two-Way Relay Channel with Power-Level Difference

When two terminals have similar transmission power levels but are located at different distances from the relay node, signals from the two terminals have different power levels when they reach the relay node due to attenuation. With the multipath channel setting described in Section V-C, Fig. 8 demonstrates the decoding performance of two receivers with different values of the power-level difference.

Comparing Fig. 7(b) with Fig. 8, one can see that the overall decoding performance decreases as the power-level difference increases. Nonetheless, observations on the decoding performance of two receivers in the time-varying multipath channel in Section V-C are still applicable to the scenario that signals from two terminals have different power levels at the relay node.

VI. EMULATED RESULTS

For the PLNC decoding performance evaluation, we use the data set collected in a two-input multiple-out (TIMO) experimental setting to emulate the data set collected in a two-way relay channel. The surface processes and acoustic communications experiment (SPACE08) was held off the coast of Martha’s Vineyard, Massachusetts, from Oct. 14 to Nov. 1, 2008. The water depth was about 15 meters. The source and a total of six receivers were anchored at different locations of the sea bottom. In this work, we consider the data collected by receivers labeled as S1 and S3 which were 60 meters and 200 meters away from the source, respectively. Each receiver was equipped with twelve receiving hydrophones.

To emulate a two-way relay channel, we take two co-located transmitters at the source in the SPACE08 experiment as two terminals in a two-way relay network, and receivers labeled as S1 and S3 in the SPACE08 experiment are taken as two individual relay nodes at different distances to the terminals. Although in a two-way relay network the two terminals do not necessarily have the same distance to the relay node, the TIMO experimental setting in the SPACE08 experiment captures the main feature of UWA channels in the two-way relay network.\(^3\)

\(^3\)Although the channels between two terminals and the relay node could be correlated, it has been shown in [46] that the spatial correlation is low especially when the element distance is large.
During the SPACE08 experiment, there are two periods of tough weather conditions, one around Julian date 297 and the other around Julian date 300, when the wave height and wind speed were larger than those in the other days [37]. The later period was more severe. For each day, there are ten recorded files, each consisting of twenty OFDM blocks. However, some data files recorded in the afternoon on Julian data 300 were distorted, which are excluded for the performance test.

The zero-padded (ZP)-OFDM parameters, the subcarrier distribution and pilot-symbol generation are identical to the simulation setting, except that out of the $|S_P| = 256$ pilot subcarriers, terminal A only transmits non-zero pilot symbols at the even indexed subcarriers, while terminal B transmits the non-zero pilot symbols at the odd indexed subcarriers. With a rate-1/2 nonbinary LDPC code and a QPSK constellation for information bit encoding and mapping, the data rate of each terminal is identical to (14).

During the experiment, the waveform height and wind speed in Julian date 298 are relatively low than other days. Hence, we use the data set collected in this day to test the decoding performance of three schemes in the time-invariant scenario. Different from the simulation, the channel state information is unknown to the receiver. When processing this data set, the sparse channel estimator proposed in [37] is used for schemes I-XPDD and I-GPDD, and channel estimation is included in the iterative operation in the scheme I-SDD [41].

Fig. 9 demonstrates the BLER performance of three receivers. Similar to the simulation results, one can see that I-XPDD has the worst performance due to multiple paths in the channel, and in the channel setting of S1, the performance of I-SDD gradually approaches that of I-GPDD after several iterations.

The data set collected on Julian data 300 is used to test the decoding performance of I-SDD and I-GPDD in the time-varying channel with large Doppler spreads. For the sake of receiver complexity, an ICI depth of one is assumed. To
estimate the ICI coefficients with regularly distributed pilots, a compressive-sensing based ICI-progressive channel estimation method is adopted. With an iterative processing, the receiver firstly assumes the absence of ICI and gets initial estimates of the transmitted information symbols. Then, an ICI-aware channel estimation [37] is carried out based on both pilot symbols and estimated information symbols. Please refer to [37], [42] for details on the ICI-progressive channel estimation.

Fig. 10 demonstrates the BLER performance of two decoding schemes. One can see that the iterative operation improves the decoding performance of I-SDD and I-GPDD considerably. Although I-GPDD outperforms I-SDD overall, in some scenarios the performance of I-SDD can approach the performance of I-GPDD by using more iterations.

Note that the emulated results correspond to data sets collected in shallow water multipath channels. The deep water acoustic channels might have different characteristics, however, it is commonly believed that deep water acoustic channels are more stable than shallow water channels [47], and the simulations results in Section V-B could be applicable.

VII. CONCLUSIONS

In this paper, we investigated PLNC in a two-way relay underwater acoustic network, where the OFDM modulation and LDPC channel coding are adopted. Tailored to the doubly-selective underwater acoustic channels, three iterative processing schemes at the relay node were examined. Both simulation and experimental results revealed that the iterative operation improves the decoding performance considerably. For the relay node with single receiving hydrophone, the generalized PLNC decoding yields the best performance; otherwise, the separate decoding scheme can catch up the generalized PLNC...
decoding after a number of iterations. Moreover, the XOR-ed PLNC decoding suffers severe performance degradation in the presence of multiple paths.

ACKNOWLEDGEMENT

We would like to thank Dr. James Preisig and his team for conducting the SPACE08 experiment.

APPENDIX: OUTAGE PROBABILITY BOUNDS IN TIME-INARIANT CHANNELS

We focus on the outage probability analysis with one receiving hydrophone. In the time-invariant scenario, both $\mathbf{H}_A$ and $\mathbf{H}_B$ are diagonal matrices, meaning that there is no ICI. For this simplest case, it is possible to derive a tighter lower bound and a loose upper bound on the achievable rate for successful decoding at the relay node.

Note that we have considered the case where the two nodes are transmitting at the same rate $R$, although this may be relaxed. Define

$$H_m[k] = \min(|H_A[k]|, |H_B[k]|), \quad k \in S_D,$$

which $S_D$ denotes the set of data subcarriers, and $H_\mu[k]$ denotes the $k$th diagonal element of matrix $\mathbf{H}_\mu$ with $\mu = A$ or $B$.

To derive the achievable rate for successful relay decoding, we note that using lattice encoding at both transmitters and lattice decoding at the relay node, it would be possible to achieve a rate of \cite[Theorem 3]{IEEE11}

$$R[k] = \log_2 \left( \frac{|H_m[k]|^2}{|H_A[k]|^2 + |H_B[k]|^2 + |H_m[k]|^2 \bar{\gamma}} \right)$$

at subcarrier $k$, where $P$ is the power per subcarrier at the transmitter and $\bar{\gamma} = \frac{\gamma}{2}$ is the average SNR per subcarrier. The single-user bound dictates that

$$R[k] \leq \log_2 \left( 1 + |H_m[k]|^2 \bar{\gamma} \right)$$

for subcarrier $k$. This bound is valid because if $R[k]$ is larger than the bound, the relay node will not be able to decode the XOR-ed symbol, even given the information of one of the two transmitters (side information). Given such information, the decoding becomes a single user decoding problem.

Combining the achievable rate and upper bound, and summing over the data subcarriers $S_D$, we have

$$R \leq \frac{1}{|S_D|} \sum_{k \in S_D} \log_2 \left( 1 + |H_m[k]|^2 \bar{\gamma} \right)$$

(18)

$$R \geq \frac{1}{|S_D|} \sum_{k \in S_D} \log_2 \left( \frac{|H_m[k]|^2}{|H_A[k]|^2 + |H_B[k]|^2 + |H_m[k]|^2 \bar{\gamma}} \right).$$

(19)

Note that here we have assumed that no power loading is performed at the transmitter. If channel state information is available at the transmitters, and power loading is performed on different subcarriers, it is possible to increase the rate. With the upper and lower bounds on the achievable rate, a loose lower bound and a tight upper bound for the outage probability are obtained

$$P_{out}(R) \geq \frac{1}{|S_D|} \sum_{k \in S_D} \log_2 \left( 1 + |H_m[k]|^2 \bar{\gamma} \right) < R \right),$$

$$P_{out}(R) \leq \frac{1}{|S_D|} \sum_{k \in S_D} \log_2 \left( \frac{|H_m[k]|^2}{|H_A[k]|^2 + |H_B[k]|^2 + |H_m[k]|^2 \bar{\gamma}} \right) < R \right).$$

REFERENCES


Zhaohui Wang (S’10) received the B.S. degree in 2006, from the Beijing University of Chemical Technology (BUCT), and the M.Sc. degree in 2009, from the Institute of Acoustics, Chinese Academy of Sciences (IACAS), Beijing, China, both in electrical engineering. She is currently working toward the Ph.D degree in the Department of Electrical and Computer Engineering at the University of Connecticut (UCONN), Storrs, USA.

Her research interests lie in the areas of communications, signal processing and detection, with recent focus on multicarrier modulation algorithms and signal processing for underwater acoustic communications.

Jie Huang was born in Jiangling, Hubei, P. R. China in 1981. He received the B.S. degree in 2001 and the Ph. D. degree in 2006, from the University of Science and Technology of China (USTC), Hefei, both in electrical engineering and information science. He was a research assistant professor from July 2009 to May 2011, working with the Department of Electrical and Computer Engineering (ECE) at the University of Connecticut, Storrs. Now he is a staff DSP design engineer with Marvell Semiconductor, Santa Clara, CA.

Mr. Huang has 18 journal papers and 25 conference papers published in the area of wireless communications and digital signal processing. His recent focus is on the design and implementation of the UMTS Long-Term-Evolution (LTE) system. Mr. Huang has served as a reviewer for the IEEE Transactions on Information Theory, the IEEE Transactions on Communications, the IEEE Transactions on Wireless Communications, the IEEE Transactions on Signal Processing, the IEEE Journal on Selected Areas in Communications, and the IEEE Communications Letters.

Shengli Zhou (SM’11) received the B.S. degree in 1995 and the M.Sc. degree in 1998, from the University of Science and Technology of China (USTC), Hefei, both in electrical engineering and information science. He received his Ph.D. degree in electrical engineering from the University of Minnesota (UMN), Minneapolis, in 2002.

He has been an assistant professor with the Department of Electrical and Computer Engineering at the University of Connecticut (UCONN), Storrs, 2003-2009, and now is an associate professor. He holds a United Technologies Corporation (UTC) Professorship in Engineering Innovation, 2008-2011. His general research interests lie in the areas of wireless communications and signal processing. His recent focus is on underwater acoustic communications and networking.


Zhengdao Wang (S’00-M’02-SM’08) received his B.S. degree in Electronic Engineering and Information Science from the University of Science and Technology of China (USTC), 1996, the M.Sc. degree in Electrical and Computer Engineering from the University of Virginia, 1999, and Ph.D. in Electrical and Computer Engineering from the University of Minnesota, 2002. He is now with the Department of Electrical and Computer Engineering at the Iowa State University. His interests are in the areas of signal processing, communications, and information theory. He served as an associate editor for IEEE Transactions on Vehicular Technology from April 2004 to April 2006, and has been an Associate Editor for IEEE Signal Processing Letters between August 2005 and August 2008. He was a co-recipient of the IEEE Signal Processing Magazine Best Paper Award in 2003 and the IEEE Communications Society Marconi Paper Prize Award in 2004, and the EURASIP Journal on Advances in Signal Processing Best Paper Award, in 2009.