Minesweeper and NP-Completeness

CSE 3500

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Minesweeper: Introduction

- Each cell: a mine, or a number (how many immediate 8 neighbors with mines), or empty.
- Example: cells with letters unknown.
- Deduce: A, B and C are mines. How about D and E?
Minesweeper Consistency

• In 2000, Richard Kaye shows interesting connection between minesweeper and NP-completeness.

• Problem formulation: is a given configuration of minesweeper game feasible?
  – A decision problem
  – Example: is this feasible?

• Key observation: minesweeper can be used to construct a Boolean circuit
  – Thus, circuit satisfiability problem can be polynomial-time reduced to minesweeper consistency problem.
First Construction: Wiring Needed in Boolean Circuit

• Boolean: a cell with mine is T, without is F.
• A horizontal wire implemented in minesweeper below.
  – Each cell with x has the same value, and x’ has the same but complement value from x: x+x’=1.
  – This allows to propagate T/F to input to a gate.
NOT Gate

- Gate (middle) implemented in Minesweeper
- Two wires: input (left) and output (right), but with different values.
  - Check it yourself
AND Gate

- Input: U, V.
- Output: W.
- t=T: AND gate satisfied, when u and v are both T.
- Check all cells with same labels (e.g. t, t’, u, u’, v, v’)

- If t=T, what is \(a_2\) and \(a_3\)? Then what is \(a_1\)? What is s and s’? Similarly, what is r and r’? What is u and u’?
More Needed for Minesweeper Reduction

• Not done yet, e.g.
  – How to bend wires to go different direction on the board?
  – How to cross wires without interference? And so on...

• Kaye got all figured out

• Minesweeper Consistency Problem is NP-complete.