1 MST

Let $G$ be a connected, undirected graph, where the edge weights are all distinct. You are also given a specific edge $e$ in $G$. You want to find out whether $e$ is contained in some minimum spanning tree.

1. First prove that $e = (u, v)$ does not belong to any MST iff there is a path between $u$ and $v$ with edges all cheaper than $e$.
2. Give an $O(|V| + |E|)$ algorithm for this problem.

2 Shortest path

Do Exercise 24.1-1 (on p. 654). I did not explain the $\pi$ values in class, but they are just like the trace-back we encountered before and record from where each node gets their shortest distance value.

3 Shortest path, again

Do Exercise 24.3-10 (on p.664).

4 More on shortest path

One useful property of the shortest paths is that the shortest paths can be obtained by following a tree (called shortest path tree), which is rooted at source $s$. Read Chapter 24 if you do not understand what is shortest path tree (e.g. Figure 24.4 on p.652).

Now, we have a directed weighted graph $G(V, E)$ (possibly with negative edges but no negative cycles). We are also given a tree $T(V, E')$ where $E' \subseteq E$, that is rooted at node $s$. Now give an $O(V + E)$ time algorithm to check whether $T$ is a shortest path tree.

5 FFT

Now let us practice the polynomial multiplication by FFT. Suppose that you want to multiply the two polynomials $1 + x + 2x^2$ and $2 + 3x$ using the FFT. Choose an appropriate power of two, find the FFT of the two polynomials, and then multiply the results componentwise. You only need to give the point evaluation form for the resulting polynomial (i.e. you do not need to perform interpolation to obtain the coefficient representation).