1 Time analysis

What is the running time of the following algorithm (using big-O notation)? Show your analysis in two ways: (1) a detailed analysis which counts the number of times each line of code executes, (2) using simpler analysis with big-O notation.

1: result ← 1
2: for i = 1 to n do
3:     for j = 1 to i do
4:         result ← result + 1
5:     end for
6: end for

2 Asymptotic Notations

For this problem, you need to base your answer on the definitions of the asymptotic notations (not the common sense rules).

1. Show $8n^3\log n + 14n^2 = \Theta(n^3\log n)$.
2. If $f(n) = O(g(n))$, can we conclude $2f(n) = O(2g(n))$? Justify your answer.
3. Prove for any integer constant $a$ and real constant $b$, $(n + b)^a = \Theta(n^a)$. In fact, this holds even when $a$ is a real constant, but you do not have to prove it.

3 Asymptotic order

The following functions are selected from problem 3-3(a) on page 61 of your textbook (p.58 if you use the 2nd edition). Order these functions asymptotically as explained in the textbook. You do not need to give proof, but you may want to work out yourself for self-study. Note that $\log n$ means $\log_2(n)$.

$n^2$, $n!$, $(\frac{3}{2})^n$, $n^3$, $\log^2 n$, $n \cdot 2^n$, $\ln n$, 1, $2^{\log n}$, $e^n$, $2^n$, $n \log n$, $n$.

4 Sum of two numbers

You are given an array $A[1 \ldots n]$ of arbitray integers and another integer $u$. The problem is to check if the array $A$ has two elements $x$ and $y$ such that $x + y = u$. Present an algorithm to solve this problem. What is the run time of your algorithm?

5 Search in $O(\log n)$ time

Given a sorted array of distinct integers $A[1,n]$, you want to find out whether there is an index $i$ for which $A[i] = i$. Give an algorithm that runs in time $O(\log n)$. Note: you should explain why the algorithm is correct and then analyze the running time.