Three-Dimensional Vibrations of a Suspension Bridge Under Stochastic Traffic Flows and Road Roughness

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When studying the vibration of a bridge–vehicle coupled system, most researchers mainly focus on the vertical vibration of bridges under moving vehicular loads, while the lateral and longitudinal vibrations of the bridges and the stochastic characteristics of the traffic flows are neglected. However, for long-span suspension bridges, neglecting the bridge’s three-dimensional (3D) vibrations under stochastic traffic flows can cause considerable inaccuracy in predicting the dynamic performance. This study is mainly focused on establishing a new methodology fully considering a suspension bridge’s vertical, lateral, and longitudinal vibrations induced by stochastic traffic flows under varied road roughness conditions. A new full-scale vehicle model with 18 degrees of freedom (DOFs) was developed to predict the longitudinal and lateral vibrations of the vehicle. An improved Cellular Automaton (CA) model considering the influence of the next-nearest vehicle was introduced. The bridge and vehicles in traffic flow coupled equations are established by combining the equations of motion of both the bridge and vehicles using the displacement relationship and interaction force relationship at the patch contacts. The numerical simulations show that the proposed method can rationally simulate the 3D vibrations of the suspension bridge under stochastic traffic flows.

Keywords: Bridge–traffic coupled system; suspension bridge; 3D vibrations; stochastic traffic flows; road surface.

1. Introduction
The bridge–vehicle interaction has attracted much attention over the last two decades due to the significant increase of heavy and high-speed vehicles in highway...
traffic. When studying the vibration of a bridge–vehicle coupled system, most previous researchers focused on the vertical vibration of bridges under moving vehicular loads.\textsuperscript{1–7} Recently, some researchers started to pay attention to the bridge’s lateral vibration under moving vehicles.\textsuperscript{8–13} However, by neglecting the action of the vehicle-induced lateral forces on the bridge vibration, they aimed their studies at the bridge lateral vibrations induced mainly by the strong wind but not by the stochastic traffic flows. In a previous study,\textsuperscript{14} the lateral vibration of high-pier bridges was found to affect significantly the riding comfort. However, the effect of the traffic flows was not considered, and no research efforts are found on bridges’ longitudinal vibration. Therefore, the lateral and longitudinal vibrations of the bridge by the stochastic traffic flows are not reported in the literature. For long-span bridges such as the suspension bridges, neglecting such three-dimensional (3D) vibrations under stochastic traffic flows has been found to possibly cause considerable inaccuracy of the predictions for the dynamic performances.\textsuperscript{15} Therefore, in this study, 3D vibrations of a suspension bridge under stochastic traffic flows are presented.

In most previous studies on the bridge–vehicle interaction, researchers either simplified the stochastic traffic flows with multiple vehicles distributed with assumed patterns\textsuperscript{8} or modeled the traffic flows as a simplified statistical process.\textsuperscript{16,17} For the long-span bridges, this simplification for the actual vehicle fleet has been found to possibly cause considerable inaccuracy of the predictions for the dynamic performance.\textsuperscript{15,18} Recently, based on the CA-based traffic flow simulation, Chen and Wu\textsuperscript{15} developed a general bridge dynamic performance analytical model considering the stochastic traffic flows under normal operation situations. The effect of the stochastic characteristics on the dynamic performance of long-span bridges is significant.\textsuperscript{19,20} However, those studies were all focused on the vertical vibration of the bridge and none were on the lateral and longitudinal vibrations. In addition, the influence of the next-nearest vehicle was not considered in the simulated traffic flows, while this influence does exist in real traffic and cannot be ignored.\textsuperscript{21}

This study mainly focuses on establishing a new methodology to consider the bridge’s lateral and longitudinal vibrations induced by stochastic traffic flows. A new full-scale vehicle model with 18 DOFs was developed to include the longitudinal and lateral vibrations of the vehicle. An improved CA model considering the influence of the next-nearest vehicle was introduced. The bridge and traffic flow coupled equations are established by combining the equations of motion of both the bridge and vehicles using the displacement relationship and interaction force relationship at the patch contacts. The numerical simulations show that the 3D vibration responses including the dynamic displacements, impact factors, and ride comfort of the suspension bridge under stochastic traffic flows and road roughness can be simulated rationally with the proposed method.
2. Methodology of Traffic Flows-Bridge Coupled System

2.1. Equations of motion of a 3D vehicle model

Based on the 3D vehicle model of 12 DOFs in Yin et al.,\textsuperscript{14} in the present study, a new full-scale vehicle model with 18 DOFs was developed including the lateral and longitudinal vibrations of the vehicle (Figs. 1 and 2). The total DOFs include the longitudinal displacements ($x_t$), vertical displacements ($z_t$), lateral displacements ($y_t$), pitching rotations ($\theta_t$), roll displacements ($\varphi_t$), and yawing angle ($\psi_t$) of the vehicle body, and the longitudinal displacement ($x_{a1}$, $x_{a2}$, $x_{a3}$, and $x_{a4}$), vertical displacements ($z_{a1}$, $z_{a2}$, $z_{a3}$, and $z_{a4}$) and lateral displacements ($y_{a1}$, $y_{a2}$, $y_{a3}$, and $y_{a4}$) of the vehicle’s axles, respectively.

To simulate the interaction between the vehicle wheel and road surface, the wheel was modeled as a 3D elementary spring as shown in Fig. 3, and the mass of the wheel was included in the mass of the axle.

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**Fig. 1.** A 18-DOFs full-scale vehicle model.

**Fig. 2.** Schematic diagram of tire model: longitudinal springs ($x$), lateral springs ($y$), and radial springs ($z$).
The displacement in the radial direction of the wheel spring (see Fig. 3) at the contact position $x$ can be expressed as:

$$U_{txx} = \left\{ z_a + \frac{s}{2} \phi_a - [-r_z(x)] \right\} + \Delta - R(1 - \cos \theta) - z_{bx,\text{contact}} \right\} / \cos \theta,$$

where $U_{txx}$ is the vertical deformation of the wheel at the position $x$; $s$ is the distance between the right and left wheels; and $\cos \theta = \frac{R-\Delta}{\sqrt{(x)^2+(R-\Delta)^2}}$. From Eq. (1), one can observe that $U_{txx}$ is a function of the vehicle axle displacement $z_a$; roll displacement of vehicle axle $\phi_a$; wheel radius $R$; wheel deformation due to the load of vehicle weight $\Delta$; and bridge dynamic vertical deflection $z_{bx,\text{contact}}$ at the contact position $x$. The $r_z(x)$ represents the vertical road roughness profile. Therefore, the vertical interaction forces acting on the road surface through the patch length $l_{ty}$ of the wheel can be written as:

$$F_{tz} = \int_{-l_{ty}/2}^{l_{ty}/2} k_{tz} U_{txx} \cos \theta dx,$$

$$F_{dtz} = \int_{-l_{ty}/2}^{l_{ty}/2} c_{tz} U_{txx} \cos \theta dx,$$

where $F_{tz}$ is the elastic force due to the vertical deformation of the wheel; $F_{dtz}$ is the damping force due to the vertical deformation of the wheel; $k_{tz}$ is the spring stiffness of the wheel in the radial direction; and $c_{tz}$ is the damping coefficient of the wheel in the radial direction.

The interactive vertical force $F_{v-b}$ between the bridge and the wheel can be obtained as:

$$F_{v-b} = -F_{tz} - F_{dtz}.$$

According to Gim and Nikravesh, the lateral force of a pneumatic tire-road surface interaction can be considered as a resultant force composed of three components of force $F_{ys}$, $F_{yo}$, and $F_{y\gamma}$ due to the tire running with an “S” shape, slip angle $\alpha$, and camber angle $\gamma$, respectively. The lateral force can be obtained as (see Fig. 3. Schematic diagram of wheel deformation.)

$$F_{v-b} = -F_{tz} - F_{dtz}.$$
more description and definition in Gim and Nikravesh\textsuperscript{22})

\[ F_y = F_{ya} + F_{y\gamma} + F_{ys}, \]  

\[ F_{ys} = \text{sign}(\alpha) \cdot \left[ C_{\alpha} S_{\alpha} l_n^2 + \left( \mu_y - \frac{C_\gamma S_\gamma}{F_{v-b}} \right) F_z (1 - 3 l_n^2 + 2 l_n^3) \right], \]  

\[ F_{y\gamma} = -\text{sign}(\gamma) \cdot C_\gamma S_\gamma, \]  

\[ F_{ys} = [k_{ty}(y_a + y_{ts} - y_{bs,contact}) + c_{ty}(\dot{y}_a + \dot{y}_{hs} - \dot{y}_{bs,contact})]l_{tr}, \]  

where \( C_\alpha \) is the cornering stiffness; \( S_\alpha \) is the absolute value of the lateral slip ratio \( S_{sy} \); a nondimensional contact patch length \( l_n \) is defined as \( l_n = l_a / l_{ty} \), where \( l_a \) is the length of the adhesion region from the front extremity to the breakaway point for the sliding region of the contact patch, therefore, \( l_a \) is varied from 0 to the value of \( l_{ty} \) is the patch length of the wheel; \( \mu_y \) is the tire-road surface friction coefficient in the slipping region; \( F_z \) is the tire vertical force acting on the road surface and equals \(-F_{v-b}\); \( C_\gamma \) is the camber stiffness; and \( S_\gamma \) is the absolute value of the lateral slip ratio due to the camber angle \( \gamma \) and defined as: \( S_\gamma = |\sin \gamma| \). Based on the studies in Gim and Nikravesh\textsuperscript{22} and Yin \textit{et al.}\textsuperscript{14} the slip angle can vary from \(-10^\circ\) to \(10^\circ\), and the camber angle varies from \(-8^\circ\) to \(8^\circ\). Furthermore, \( k_{ty} \) and \( c_{ty} \) are the tire lateral stiffness and damping coefficients; \( y_a \) is the lateral displacement of the vehicle axle; \( y_{bs} \) is the tire lateral displacement due to tire running with an “S” shape; and \( y_{bs,contact} \) is the bridge dynamic lateral deflection at the contact position \( x \).

The problem of simulating the tire running with an “S” shape is very complex. To simplify the model, the “S” shape was assumed as a “Sine” shape with a random amplitude and random phase angle in Yin \textit{et al.}\textsuperscript{14} and the results shows the simplification can simulate very well for the “S” motion. For more accuracy, the “S” shape was assumed to be a zero-mean stationary Gaussian random process and can be generated through a trigonometric series function\textsuperscript{14} as:

\[ y_{ts}(x) = \sum_{k=1}^{N} A_k \cos(2\pi x / l_s + \varphi_s), \]  

where \( A_k \) is the random amplitude; \( l_s \) is the wavelength; and \( \varphi_s \) is the initial phase angle. Based on the studies,\textsuperscript{23} \( A_k \) can be assumed to follow a symmetrical distribution from 2.5 mm to 5 mm; \( l_s \) can be obtained from a symmetrical distribution from 6.65 m to 10 m; and \( \varphi_s \) can also be assumed to follow a symmetrical distribution from 0 to 2\( \pi \).

The longitudinal force of a pneumatic tire-road surface interaction can be considered as a resultant force due to the longitudinal deformation and longitudinal friction of the tire. The longitudinal force can be obtained as:

\[ F_x = -\text{sign}(S_{sz}) \cdot [C_S S_n l_n^2 + \mu_x F_z (1 - 3 l_n^2 + 2 l_n^3)], \]  

where \( C_S \) is the longitudinal stiffness; \( S_n \) is the absolute value of the longitudinal slip ratio \( S_{sz} \); \( l_n \) is the nondimensional contact patch length; \( \mu_x \) is the tire-road surface friction coefficient; \( y_{ts} \) is the tire lateral displacement due to tire running with an “S” shape; and \( y_{bs,contact} \) is the bridge dynamic lateral deflection at the contact position \( x \).
friction coefficient in the longitudinal \((x)\) direction; and \(F_z\) is the tire vertical force acting on the road surface equal to \(-F_{v-b}\).

The vertical displacements of the suspension springs can be written as:

\[
U_{z1} = z_t - z_{a1} + (s_1/2)\phi_t - l_1\theta_t, \quad (7)
\]
\[
U_{z2} = z_t - z_{a2} - (s_1/2)\phi_t - l_1\theta_t, \quad (8)
\]
\[
U_{z3} = z_t - z_{a3} + (s_2/2)\phi_t - l_2\theta_t, \quad (9)
\]
\[
U_{z4} = z_t - z_{a4} - (s_2/2)\phi_t - l_2\theta_t, \quad (10)
\]

where \(l_1\) is the distance between the front and the center of the vehicle, \(l_2\) is the distance between the rear axle and the center of the vehicle, \(s_1\) and \(s_2\) are the distance between the right and left axles, respectively.

The vertical elastic and damping forces of the suspension can be written as:

\[
F_{z1i} = K_{z1i}U_{z1i},
\]
\[
F_{dz1i} = C_{z1i}\dot{U}_{z1i}, \quad i = 1, 2, 3, 4,
\]

where \(K_{z1i}\) and \(C_{z1i}\) are the suspension spring stiffness and damping of the \(i\)th axle, respectively.

The lateral and longitudinal displacements of the suspension springs can be written as:

\[
U_{sy1} = y_t - y_{a1} - l_1\phi_t + h_1\phi_t, \quad U_{sy2} = y_t - y_{a2} - l_1\phi_t - h_1\phi_t,
\]
\[
U_{sy3} = y_t - y_{a3} + l_2\phi_t + h_1\phi_t, \quad U_{sy4} = y_t - y_{a4} + l_2\phi_t - h_1\phi_t, \quad (13a)
\]
\[
U_{sxi} = x_{t1} - x_{ai}, \quad i = 1, 2, \quad U_{sxi} = x_{t2} - x_{ai}, \quad i = 3, 4, \quad (13b)
\]

where \(h_1\) is the height of the vehicle center to the driver seat.

The lateral and longitudinal elastic and damping forces of the suspension can be written as:

\[
F_{s1i} = K_{s1i} \cdot U_{s1i}; \quad F_{ds1i} = C_{s1i} \cdot \dot{U}_{s1i}, \quad (14a)
\]
\[
F_{sxi} = K_{sxi} \cdot U_{sxi}; \quad F_{dxsi} = C_{sxi} \cdot \dot{U}_{sxi}. \quad (14b)
\]

The equations of motion of the full-scale vehicle can be obtained from the Lagrangian formulation, and can be written as:

\[
m_t\ddot{x}_t + F_{x1} + F_{x2} + F_{x3} + F_{x4} + F_{dx1} + F_{dx2} + F_{dx3} + F_{dx4} = m_tg,
\]
\[
m_t\ddot{y}_t + F_{sy1} + F_{sy2} + F_{sy3} + F_{sy4} + F_{dsy1} + F_{dsy2} + F_{dsy3} + F_{dsy4} = 0, \quad (15b)
\]
\[
m_t\ddot{z}_t + F_{sz1} + F_{sz2} + F_{sz3} + F_{sz4} + F_{dsz1} + F_{dsz2} + F_{dsz3} + F_{dsz4} = 0, \quad (15c)
\]
\[
I_{zt}\ddot{\phi}_t + (s_1/2)(F_{sz1} - F_{sz2}) + (s_2/2)(F_{sz3} - F_{sz4}) + (s_1/2)(F_{dsz1} - F_{dsz2}) + (s_2/2)(F_{dsz3} - F_{dsz4}) = 0, \quad (15d)
\]
\[ I_{1} \ddot{\theta}_{1} + l_{1}(F_{s1} + F_{s2}) - l_{2}(F_{s3} + F_{s4}) + l_{1}(F_{d1} + F_{d2}) - l_{2}(F_{d5} + F_{d4}) = 0, \]  
\[ I_{2} \ddot{\theta}_{2} + l_{1}(F_{s1} + F_{s2}) - l_{2}(F_{s3} + F_{s4}) + l_{1}(F_{d1} + F_{d2}) - l_{2}(F_{d5} + F_{d4}) = 0, \]

\[ m_{ai} \ddot{z}_{ai} - F_{si} + F_{tzi} - F_{dxi} + F_{hti} = m_{ai}g \quad i = 1, 2, 3, 4, \]

\[ m_{ai} \ddot{y}_{ai} - F_{sji} - F_{sjy} + F_{jy} = 0 \quad i = 1, 2, 3, 4, \]

\[ m_{ai} \ddot{x}_{ai} - F_{dxi} - F_{sxi} + F_{xi} = 0 \quad i = 1, 2, 3, 4, \]

where \( m_t \) and \( m_{ai} \) represent the mass of the vehicle body and the \( i \)th axle, respectively.

Equations (15a)–(15i) can be rewritten in a matrix form as:

\[
\begin{bmatrix}
M_{v} & C_{v} & K_{v}
\end{bmatrix}
\begin{bmatrix}
\dot{U}_{v}
\end{bmatrix}
+
\begin{bmatrix}
F_{G}
\end{bmatrix}
+
\begin{bmatrix}
F_{v-b}
\end{bmatrix},
\]

where \( [M_{v}] \), \( [C_{v}] \), and \( [K_{v}] \) are the mass, damping, and stiffness matrices of the vehicle, respectively; \( \{U_{v}\} \) is the vector including the displacements of the vehicle; \( \{F_{G}\} \) = gravity force vector of the vehicle; and \( \{F_{v-b}\} \) = vector of the wheel-road contact forces acting on the vehicle.

### 2.2. Equations of motion of bridge model

The equation of motion of a bridge can be written as:

\[
\begin{bmatrix}
M_{b}
\end{bmatrix}
\begin{bmatrix}
\dot{U}_{b}
\end{bmatrix}
+
\begin{bmatrix}
C_{b}
\end{bmatrix}
\begin{bmatrix}
\ddot{U}_{b}
\end{bmatrix}
+
\begin{bmatrix}
K_{b}
\end{bmatrix}
\begin{bmatrix}
U_{b}
\end{bmatrix}
=
\begin{bmatrix}
F_{b-v}
\end{bmatrix},
\]

where \( [M_{b}] \), \( [C_{b}] \), and \( [K_{b}] \) are the mass, damping, and stiffness matrices of the bridge, respectively; \( \{U_{b}\} \) is the displacement vector for all DOFs of the bridge; \( \{\dot{U}_{b}\} \) and \( \{\ddot{U}_{b}\} \) are the first and second derivative of \( \{U_{b}\} \) with respect to time, respectively; and \( \{F_{b-v}\} \) is a vector containing all external forces acting on the bridge.

### 2.3. Equations of motion for traffic flows-bridge vibration system

In the present study, in order to simplify the vehicular models in traffic flows, \(^{18}\) all the vehicles are classified as three types: (1) \( v_1 \)-heavy multi-axle trucks; (2) \( v_2 \)-light trucks and buses; and (3) \( v_3 \)-sedan car. Only heavy trucks are modeled with 18 DOFs 3D vehicle models, light trucks and sedan cars are simulated with the single DOF vehicle model to be computationally efficient. The 18 DOFs and single DOF vehicle models are shown in Figs. 1 and 4, respectively. Using the displacement relationship and the interaction force relationship at the patch contacts, the traffic flows-bridge coupled system can be established by combining the equations of motion of both the bridge and vehicles. \(^{14}\) Equations (16) and (17) can be combined and rewritten in a
matrix form as:

\[
\begin{bmatrix}
M_b & M^N \\
\end{bmatrix}
\begin{bmatrix}
\dot{\gamma}_b \\
\dot{\gamma}^N
\end{bmatrix}
+ \begin{bmatrix}
C_b + C_{b-vb}^N & C_{b-v}^N \\
C_{v-b}^N + C_{v-v}^N & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\gamma}_b \\
\dot{\gamma}^N
\end{bmatrix}
= \begin{bmatrix}
\dot{U}_b \\
\dot{U}^N
\end{bmatrix}
+ \begin{bmatrix}
K_b + K_{b-vb}^N & K_{vb}^N \\
K_{v-b}^N + K_{v-v}^N & 0
\end{bmatrix}
\begin{bmatrix}
\gamma_b \\
\gamma^N
\end{bmatrix}
= \begin{bmatrix}
F_{b-vb}^N + F_G^N
\end{bmatrix},
\]  
(18)

where \( N \) is the number of vehicles traveling on the bridge, \( M_b, C_b, \text{ and } K_b \) are mass, damping, and stiffness matrices for the traffic, respectively; \( C_{b-vb}^N \text{ and } K_{b-vb}^N \) are damping and stiffness contribution to the bridge structure due to the coupling effects between the \( N \) vehicles in the traffic flows and the bridge system, respectively; \( C_{v-b}^N \text{ and } K_{v-b}^N \) are the coupled stiffness and damping matrices contributing to bridge vibrations from the \( N \) vehicles in the traffic flows, respectively; \( C_{v-v}^N \text{ and } K_{v-v}^N \) are the coupled damping and stiffness matrices contributing to the vibration of the \( N \) vehicles, respectively; \( C_{v-b}^N \text{ and } K_{v-b}^N \) are the coupled damping and stiffness matrices induced by other vehicles in the traffic flows, respectively. Equation (18) can be solved by the Newmark method in the time domain.

2.4. Traffic flows simulation considering the influence of the next-nearest vehicle

The simulation of the CA traffic model can capture the basic features of probabilistic traffic flows by adopting the realistic traffic rules such as car-following and lane-changing, as well as actual speed limits. One of the most important CA models is Nagel–Schreckenberg (NaSch) model proposed in Nagel and Schreckenberg.\(^{24}\) Though NaSch model is simple, it can describe some traffic phenomena in reality, such as phase transition etc. In recent years, the CA based traffic flow simulation model was introduced and verified to study the vibration of bridges under the traffic flows with great accuracy.\(^{15,19}\) However, all those studies used the CA models did not take into account the influence of the next-nearest vehicle, though this influence exists in real traffic and cannot be ignored.\(^{21}\) In this study, an improved CA model
considering the influence of the next-nearest vehicle in Kong et al.\textsuperscript{21} is introduced and used to simulate the traffic flows.

In the car-following model, most researchers usually consider the influence of the vehicle ahead using the following equation,\textsuperscript{21}

\begin{equation}
\ddot{x}_n(t + T) = \lambda (\dot{x}_{n+1} - \dot{x}_n),
\end{equation}

where $T$ is a response time lag, $\lambda$ is the sensitivity coefficient, $\ddot{x}_n$ is the acceleration of the vehicle, and $\dot{x}_n$ is the velocity of the vehicle. The model shows that the response of the following vehicle is in direct proportion to the stimulus received from the leading vehicle. Considering the influence of the next-nearest vehicle, Eq. (19) can be changed to:

\begin{equation}
\ddot{x}_n = \lambda_1 (\dot{x}_{n+1} - \dot{x}_n)_{t-T_1} + \lambda_2 (\dot{x}_{n+2} - \dot{x}_n)_{t-T_2},
\end{equation}

where $T_1$ is a response time lag of the nearest ahead, $T_2$ is a response time lag of the next-nearest ahead, $\lambda_1$ and $\lambda_2$ are the sensitivity coefficients, respectively, and both of them are confined between 0 and 1.

According to Eq. (20), the sensitivity coefficients of the nearest and next nearest vehicle are $\lambda_1$ and $\lambda_2$, respectively, and $\lambda_1 > \lambda_2$. The acceleration of the vehicle is written as:

\begin{equation}
\ddot{x}_n(t+1) = \tilde{\lambda}(\Delta \dot{x}_{n+1}(t), \Delta \dot{x}_{n+2}(t-1)),
\end{equation}

where \(\tilde{\lambda} = \lambda_1 (\dot{x}_{n+1}(t) - \dot{x}_n(t)) + \lambda_2 (\dot{x}_{n+2}(t-1) - \dot{x}_n(t-1))\). Using Eq. (21), the rule of the acceleration/deceleration can be changed in the NaSch model. Using the above simulation equations (20) and (21) considering the influence of the next-nearest vehicle, the two-lane cellular automata model with the influence of the next-nearest vehicle was established for the tested highway bridge. As mentioned earlier, the vehicles are classified as three types: (1) $v_1$-heavy multi-axle trucks; (2) $v_2$-light trucks and buses; and (3) $v_3$-sedan car. In the present study, only heavy trucks are modeled with the 3D vehicle dynamic models. To simplify the vehicular model, light trucks and sedan cars are modeled with the quarter vehicle model to have a better computational efficiency. A similar bridge has been selected as the prototype bridge in several previous studies.\textsuperscript{18} The approaching roadway at each end of the bridge is assumed to be 1005 m. The speed limit of the highway system is assumed as 135 (km/h), which is converted to the maximum velocity of vehicles in CA model as 5 cell/s. The sensitivity coefficients of the nearest and next nearest vehicle are $\lambda_1 = 0.2$ and $\lambda_2 = 0.05$.\textsuperscript{21} The traffic flow simulation results with the CA model usually become stable after a continuous simulation with a period which equals to 10 times the cell numbers of the traffic simulating system.\textsuperscript{18,24} For the purpose of comparison, two different vehicle occupancies $\rho$ are considered: median traffic ($\rho = 0.15$ corresponding to 32 vehicles/mile/lane) and busy traffic flows ($\rho = 0.3$ corresponding to 64 vehicles/mile/lane). It can be found from Fig. 5 that the $x$-axis and $y$-axis represent the coordinates in both the spatial and time domains, respectively; each dot on
the figures represents a vehicle; and with the increase of the traffic occupancy, local congestions may be formed at some locations as indicated by the black belts in Fig. 5.

2.5. Modeling of progressive deterioration for road surface

The road surface condition is an important factor that affects the dynamic responses of both the bridge and vehicles. The road surface profile is usually assumed to be a zero-mean stationary Gaussian random process and can be generated through an inverse Fourier transformation based on a power spectral density (PSD) function\(^{14}\) as:

\[
\begin{align*}
    r(x) = \sum_{k=1}^{N} \sqrt{2\varphi(n_k)} \Delta n \cos(2\pi n_k x + \theta_k),
\end{align*}
\]  

(22)

where \(\theta_k\) is the random phase angle uniformly distributed from 0 to \(2\pi\); \(\varphi()\) is the PSD function (m\(^3\)/cycle) for the road surface elevation; and \(n_k\) is the wave number (cycle/m). In the present study, the following PSD function\(^{14}\) has been used:

\[
\varphi(n) = \varphi(n_0) \left( \frac{n}{n_0} \right)^{-2} \quad (n_1 < n < n_2),
\]  

(23)

where \(n\) is the spatial frequency (cycle/m); \(n_0\) is the discontinuity frequency of \(1/2\pi\) (cycle/m); \(\phi(n_0)\) is the roughness coefficient (m\(^3\)/cycle) whose value is chosen depending on the road condition; and \(n_1\) and \(n_2\) are the lower and upper cut-off

<table>
<thead>
<tr>
<th>Classifications</th>
<th>Ranges of (\varphi(n_0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very good</td>
<td>(2 \times 10^{-6}) to (8 \times 10^{-6})</td>
</tr>
<tr>
<td>Good</td>
<td>(8 \times 10^{-6}) to (32 \times 10^{-6})</td>
</tr>
<tr>
<td>Average</td>
<td>(32 \times 10^{-6}) to (128 \times 10^{-6})</td>
</tr>
<tr>
<td>Poor</td>
<td>(128 \times 10^{-6}) to (512 \times 10^{-6})</td>
</tr>
<tr>
<td>Very poor</td>
<td>(512 \times 10^{-6}) to (2048 \times 10^{-6})</td>
</tr>
</tbody>
</table>
frequencies, respectively. The International Organization for Standardization\textsuperscript{25} has proposed a road roughness classification index from A (very good) to H (very poor) according to different values of $\phi(n_0)$ shown in Table 1.

3. Numerical Studies

3.1. Description of a suspension bridge

There are many existing suspension bridges in valley areas or wide water areas. With small stiffness in the lateral and longitudinal directions, the lateral and longitudinal vibrations of the long span suspension bridge together with the vertical vibration can easily be excited by external dynamic loads such as traffic flows and/or wind loads. As shown in Fig. 6, a suspension bridge was completed in 2012, located in Sichuan Province, China. The geometrical characteristics of the bridge are as: a total length of 1295.00 m, the longest span of 820 m, and a bridge width of 29.78 m.

3.2. Numerical model of the bridge

Based on the configuration of the bridge, a finite element (FE) model was created for this bridge, as shown in Fig. 7. Before the numerical simulation, the FE bridge model was updated with the results of an onsite modal test performed using the ambient vibration method. The details for the experimental setup and model updating technique can be found in Yin \textit{et al.}\textsuperscript{14}

![Fig. 6. A suspension bridge.](image-url)
3.3. Parameters of traffic vehicles

As mentioned earlier, both the 3D vehicle dynamic model with 18 DOFs and the single DOF vehicle model are used in the simulation. The mechanical and geometric properties are listed in Tables 2–3 and can be obtained from Yin et al., and Chen and Cai.

Table 2. Parameters of the 3D vehicle.

<table>
<thead>
<tr>
<th>Truck parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of truck body $m_t$</td>
<td>26,745 kg</td>
</tr>
<tr>
<td>Pitching moment of inertia of truck body $I_{zt}$</td>
<td>172,160 kg · m²</td>
</tr>
<tr>
<td>Rolling moment of inertia of truck body $I_{xt}, I_{yt}$</td>
<td>61,496 kg · m²</td>
</tr>
<tr>
<td>Mass of truck front axle $m_{a1}, m_{a2}$</td>
<td>710 kg</td>
</tr>
<tr>
<td>Mass of truck rear axle $m_{a3}, m_{a4}$</td>
<td>800 kg</td>
</tr>
<tr>
<td>Suspension spring vertical stiffness of the first axle $K_{sz}^1, K_{sz}^2$</td>
<td>242,604 (N/m)</td>
</tr>
<tr>
<td>Suspension spring longitudinal/lateral stiffness of the first axle $K_{sy}^1, K_{sy}^2, K_{sx}^1, K_{sx}^2$</td>
<td>102,302 (N/m)</td>
</tr>
<tr>
<td>Suspension vertical damper of the first axle $C_{sz}^1, C_{sz}^2$</td>
<td>2,190 (N·s/m)</td>
</tr>
<tr>
<td>Suspension longitudinal/lateral damper of the first axle $C_{sy}^1, C_{sy}^2, C_{sx}^1, C_{sx}^2$</td>
<td>1,690 (N·s/m)</td>
</tr>
<tr>
<td>Suspension spring vertical stiffness of the second axle $K_{sz}^3, K_{sz}^4$</td>
<td>1,903,172 (N/m)</td>
</tr>
<tr>
<td>Suspension spring longitudinal/lateral stiffness of the second axle $K_{sy}^3, K_{sy}^4, K_{sx}^3, K_{sx}^4$</td>
<td>1,003,031 (N/m)</td>
</tr>
<tr>
<td>Suspension vertical damper of the second axle $C_{sz}^3, C_{sz}^4$</td>
<td>7,882 (N·s/m)</td>
</tr>
<tr>
<td>Suspension longitudinal/lateral damper of the second axle $C_{sy}^3, C_{sy}^4, C_{sx}^3, C_{sx}^4$</td>
<td>5,869 (N·s/m)</td>
</tr>
<tr>
<td>Radial direction spring stiffness of the tire $k_{ty}$</td>
<td>276,770 (N/m)</td>
</tr>
<tr>
<td>Radial direction spring damper coefficient of the tire $c_{ty}$</td>
<td>1,990 (N·s/m)</td>
</tr>
<tr>
<td>Length of the patch contact</td>
<td>345 mm</td>
</tr>
<tr>
<td>Width of the patch contact</td>
<td>240 mm</td>
</tr>
<tr>
<td>Distance between the front and the center of the truck $l_1$</td>
<td>3.73 m</td>
</tr>
<tr>
<td>Distance between the rear axle and the center of the truck $l_2$</td>
<td>1.12 m</td>
</tr>
<tr>
<td>Distance between the right and left axles $s_1$</td>
<td>2.40 m</td>
</tr>
</tbody>
</table>
Three-Dimensional Vibrations of a Suspension Bridge Under Stochastic Traffic

3.4. Comparison with and without considering the effect of next-nearest vehicles

The Transportation Research Board classifies the “level of service” from a driving operation under a desirable condition to an operation under forced or breakdown conditions. Three traffic occupancies are computed: \( \rho = 0.07 \) (15 vehicles/mile/lane) corresponding to smooth traffic; \( \rho = 0.15 \) (32 vehicles/mile/lane) corresponding to median traffic; and \( \rho = 0.3 \) (64 vehicles/mile/lane) corresponding to busy traffic.

In following sections, three traffic flow occupancies, including the smooth traffic \( \rho = 0.07 \), median traffic \( \rho = 0.15 \) and busy traffic flow \( \rho = 0.3 \), are also used to study the 3D dynamic responses of the bridge under the good classification of surface roughness. The 3D dynamic responses of the bridge are shown in Figs. 8–12, whose \( x \)-axis is the position of the first vehicle in the traffic flows.

As discussed earlier, the effects of the next-nearest vehicles were not taken into account in some simplified traffic flow models. The time histories of the vertical displacements at the mid-span and 1/4 span of the stiffening girder under two typical traffic flows with and without considering the next-nearest vehicles are presented in Fig. 8. The maximal vertical displacement for the traffic flow without considering the next-nearest vehicles are found larger than that if the effect of the next-nearest

Table 3. Parameters of the single DOF vehicle model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Sedan car</th>
<th>Light truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprung mass</td>
<td>kg</td>
<td>1,611</td>
<td>4,870</td>
</tr>
<tr>
<td>Stiffness of suspension system ((K_{1x}, K_{1y}, K_{1z}))</td>
<td>N/m</td>
<td>434,920</td>
<td>500,000</td>
</tr>
<tr>
<td>Damping ((C_{1x}, C_{1y}, C_{1z}))</td>
<td>N/(m/s)</td>
<td>5,820</td>
<td>20,000</td>
</tr>
<tr>
<td>Radial direction spring stiffness of the tire</td>
<td>N/m</td>
<td>138,365</td>
<td>150,000</td>
</tr>
<tr>
<td>Radial direction spring damper coefficient of the tire</td>
<td>N/(m/s)</td>
<td>995</td>
<td>1,000</td>
</tr>
</tbody>
</table>

(a) Vertical displacements of mid-span (b) Vertical displacements of 1/4 span

Fig. 8. Bridge vertical responses under two different traffic flow models.
vehicles is considered. In the present study, for example, the maximal vertical dis-
placements of the mid-span are equal to 12.11 cm and 15.02 cm corresponding to the
cases without and with considering the next-nearest vehicles, respectively. This
phenomenon could be explained that the characters of the traffic, such as speed of
vehicles and distance of each vehicle, can be changed due to considering the effects of
the next-nearest vehicles.

3.5. 3D dynamic responses of the suspension bridge under three traffic flows occupancies

3.5.1. Vertical displacements of stiffening girder

It is found from Fig. 9 that the vertical displacements at the mid-span generally
increase with the increase of the vehicle occupancies. For example, the maximum
vertical displacements at the mid-span (and 1/4 span) increase from 12.11 (21.72) cm
to 29.85 (42.03) cm when the vehicle occupancy increases from the smooth traffic to
the busy traffic. Compared with Figs. 8(a) and 8(b), it can be found that the vertical
displacements at the 1/4 span are larger than those at the mid-span under the same
vehicle occupancies.

3.5.2. Lateral displacements of stiffening girder

The lateral displacements at the mid-span of bridge under three types of traffic flow
occupancies are presented in Fig. 10. It is found that the lateral displacements at the
mid-span increases significantly with the increase of vehicle occupancies. The max-
mum lateral displacement in the mid-span increases from 12.49 cm to 18.21 cm in the
smooth and busy traffic scenarios, respectively. Comparing Figs. 10(a) and 10(b),
one can find that the lateral displacements at the 1/4 span are smaller than those at
the mid-span under the same vehicle occupancies. However, Fig. 9 shows that the
vertical displacements at the 1/4 span are larger than those at the mid-span under
the same vehicle occupancies. Therefore, in general, those results in Figs. 9 and 10 may show that the critical cross-sections for the suspension bridge could be multiple cross-sections other than a single cross-section in the design.

3.5.3. Longitudinal displacements of bridge

It is found from Fig. 11 that the longitudinal displacements at the mid-span and the top of tower increases with the vehicle occupancies. The maximum longitudinal displacement of the mid-span increases from 2.42 cm for the smooth traffic to 5.33 cm for the busy traffic, and the maximum longitudinal displacement of the top of tower increases from 9.57 cm to 12.03 cm.

From Figs. 9–11, it can be found that the effect of vehicle occupancies on the 3D vibrations are significant and cannot be neglected. For example, the longitudinal displacement of the main beam can reach 5.33 cm, which may accelerate the damage
for the expansion joints at both ends of the bridge; and the displacements for the top of tower of 12.03 cm could add extra forces in the cable and the tower and accelerate their possible damages. However, both the two vehicle-induced longitudinal displacements of the main beam and tower are usually neglected and are not considered in the design.

3.6. 3D dynamic displacements of the suspension bridge under three road roughness

In the previous studies in Yin et al.\textsuperscript{14} and Deng and Cai,\textsuperscript{26} the road surface condition was found as an important factor that affects the dynamic responses of bridges. Thus, in this section, the effects of the road roughness on the 3D dynamic displacements are discussed. As shown in Fig. 12, the 3D vibration displacements of the bridge increase when the road roughness condition changes from good to poor. The maximum vertical displacements of the mid-span change from 20.61 cm to 24.35 cm for a good and a poor roughness condition. Therefore, the road surface condition has proven to have a large influence on the vibration, and regular maintenance of the road surface is a very effective way of reducing vehicle-induced vibration and maintaining the safety for the suspension bridge.

3.7. Comparison of the impact factor under three types of traffic flow occupancy and road surface roughness

In the bridge design, the dynamic effects of moving vehicles on a bridge are usually considered with the impact factor. However, the dynamic impact factors in the design codes do not usually include the effect of traffic flows and road surface roughness. Deng and Cai\textsuperscript{26} proposed a function of impact factor for the bridges with respect to bridge span length and bridge surface roughness. However, their study was based on simply supported bridges, and more theoretical support was also needed to study the impact for long-span suspension bridges. In this study, the impact factor is defined as follows:

\[
IM = \frac{R_d(x) - R_s(x)}{R_s(x)},
\]

(24)

where \(R_d(x)\) and \(R_s(x)\) are the maximum dynamic and static response of the bridge at location \(x\), respectively.

To compare the impact factor obtain from design codes and the present method, the Chinese highway bridge design code (CHBDC)\textsuperscript{27} was given as an example, whose \(IM\) is defined as a function of the natural frequency of the bridge as shown below:

\[
\begin{cases}
IM = 0.05, & \text{when } f < 1.5 \text{ Hz}; \\
IM = 0.1767\ln(f) - 0.0157, & \text{when } 1.5 \text{ Hz} \leq f \leq 14 \text{ Hz}; \\
IM = 0.45, & \text{when } f > 14 \text{ Hz},
\end{cases}
\]

(25)

where \(f\) is the natural frequency of the bridge.
Three-Dimensional Vibrations of a Suspension Bridge Under Stochastic Traffic

Fig. 12. Bridge responses under three types of road roughness.

(a) Vertical displacements of mid-span  
(b) Vertical displacements of 1/4 span

(c) Lateral displacements of mid-span  
(d) Lateral displacements of 1/4 span

(e) Longitudinal displacements of mid-span  
(f) Longitudinal displacements at the top of tower
Figure 13 shows the impact factors obtain by the CHBDC and the present study under four classifications of road roughness. It can be found that the impact factors obtain from the CHBDC are much smaller than the real value of the impact factors at the mid-span and 1/4 span. The impact factor of 1/4 span can reach 0.107 when the road roughness has deteriorated to the very poor classification.

3.8. Method of evaluating ride comfort

Ride comfort problems of the moving vehicles mainly arise from vibrations of the vehicle body, especially for the vehicles moving on long-span bridges. Though the time duration for the vehicle running through a bridge lasts only a few minutes, the short-term ride discomfort could seriously cause the fatigue and affect handling performance of the driver, which may lead to catastrophic vehicle accidents. Thus, the ride comfort has attracted much attention over the past two decades. In most existing studies of the bridge–vehicle interaction, researchers modeled the traffic flows as a simplified statistical process\textsuperscript{14,16} and were not consider the longitudinal vibration of vehicles.

To evaluate the ride comfort, the ISO2631-1(1997)\textsuperscript{28} specifies the root-mean-square (RMS) magnitudes of the vibration acceleration as the standard for ride comfort as shown in Table 4.

For vibrations in more than one direction, the weighted RMS acceleration $a_w$ determined from the vibrations in the orthogonal coordinates is calculated as:

$$a_w = (k_{ax}^2a_{wx}^2 + k_{ay}^2a_{wy}^2 + k_{az}^2a_{wz}^2)^{\frac{1}{2}},$$

where $a_{wx}$, $a_{wy}$, and $a_{wz}$ are the weighted RMS accelerations with respect to the orthogonal axes $x$, $y$, and $z$, respectively; $k_{ax}$, $k_{ay}$, and $k_{az}$ are multiplying factors with...
the orthogonal axes \( x, y, \) and \( z \), respectively, and

\[
a_{wj}^{j=x,y,z} = \left[ \frac{1}{T} \int_{t=0}^{t=T} a_{wj}(t)^2 \right]^{\frac{1}{2}},
\]

where \( a_{wj}(t)^{j=x,y,z} \) is the acceleration as a function of time \( \text{m/s}^2 \) in the \( x, y, \) and \( z \) axle directions; and \( T \) is the duration of the measurement \( \text{s} \).

From the relationship of the weighted RMS accelerations \( a_w \) and the comfort standard, it can be seen that, in addition to the vertical accelerations, the lateral and longitudinal accelerations of the driver seat will also affect the ride comfort.

### 3.9. Comparison of the ride comfort under vehicle occupancies

In this section, a 3D vehicle model including the longitudinal and lateral vibrations, as stated earlier, was used to study the ride comfort. The total values of the weighted RMS accelerations \( a_w \) of the vehicle are calculated directly using different vehicle occupancies and road roughness. The corresponding ride comfort is given in Fig. 14 and Tables 5–6. It can be seen that the vertical, lateral, and longitudinal vibrations of the detailed vehicle model can significantly affect the drive comfort with the factors including the different vehicle occupancies and road roughness. For example,
the classification of ride comfort changes from a little uncomfortable for the good roughness condition to very uncomfortable for the very poor roughness condition.

4. Conclusions

This study is mainly focused on establishing a new methodology considering the bridge’s lateral and longitudinal vibrations induced by stochastic traffic flows. A new full-scale vehicle model with 18 DOFs was developed including the longitudinal and lateral vibrations of the vehicle. An improved CA model considering the influence of the next-nearest vehicle and the road-roughness were introduced. The bridge and traffic flow coupled equations are established by combining the equations of motion of both the bridge and vehicles using the displacement relationship and interaction force relationship at the patch contacts. The proposed method can rationally simulate the 3D vibration of suspension bridges under stochastic traffic flows. The numerical simulations show that:

1. The improved CA model that considers the influence of the next-nearest vehicles can be introduced to study the vibration of bridge–traffic coupled systems and

Fig. 14. Vehicle accelerations under smooth traffic flows with $\rho = 0.07$. 

(a) Vertical acceleration  
(b) Lateral acceleration  
(c) Longitudinal acceleration
the bridge displacements for the traffic flow without considering the next-nearest vehicles are larger than bridge displacements for the considering the next-nearest vehicles.

(2) The longitudinal displacements of the main beam and tower under vehicle occupancies are significant and should be considered in the design; the longitudinal displacement of the main beam can reach 5.33 cm, which may accelerate the damage for the expansion joints at the end of bridge, and the displacements at the top of the tower may accelerate damage development for the main cable or tower. The vertical, lateral, and longitudinal vibrations of the detailed vehicle model can significantly affect the drive comfort with the factors including the different vehicle occupancies.

(3) The impact factors obtained from CHBDC are much smaller than the predicted value of the impact factors at the mid-span and 1/4 span. The impact factor of the 1/4 span can reach 0.107 when the road roughness has deteriorated to a very poor classification.

(4) The road surface condition has proven to have a large influence on the vibration including the displacement, impact factor, and ride comfort. Therefore, regular maintenance of the road surface is a very effective way of reducing vehicle-induced vibration and maintaining the safety for the suspension bridge.

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References


