Estimation of Aerodynamic Forces and Moments on Bridge Decks Based on Two Dimensional Velocity Fields

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ABSTRACT:

In order to predict the structural responses under random wind loads to avoid catastrophe failures, structural dynamic analysis is necessary, which requires a reasonable description of wind loads on structures. Based on the equations of fluid mechanics, equations for aerodynamic forces for structures in motion and coefficients of drag, lift and pitching moment for static structures are derived based on two dimensional velocity fields. Since the pressure terms in the equations are eliminated under 2-D velocity assumptions, the equations can be used to obtain the instantaneous aerodynamic forces based on the velocity information along the segments enclosing the bridge deck. In the present study, the divergence theorem is used to convert the control volume integral to the control surface integral. Similarly, the equations for pitching moments are proposed in a similar approach based on the conservation of the momentum moment. This makes it possible to predict the aerodynamic forces and moment for bridge decks of large span bridges, which are more vulnerable to wind induced vibrations. Therefore, the direct relationship between the loads from wind on structures and the velocity field can be founded. Accordingly, the equations for the coefficient of wind forces can be obtained for static bridge decks. In order to validate the equations at a higher Reynolds number (Re), velocity in the wind field around static bridge decks are obtained from both numerical simulations and wind tunnel Particle Image Velocimetry (PIV) experiments at Re being about 10⁴. Two types of sections including a square cylinder and a twin box girder are used as the prototypes of the bridge decks in the present study. PIV experiments are set up in a wind tunnel and the fluid domain in the numerical simulation is set with the same size of the wind tunnel. Based on the sensitivity analyses, several cases with different sizes of control volumes and space resolutions are used to obtain the wind force coefficient for both of the square cylinder section and the twin-box girder section. Good agreements are found for all the cases for the drag coefficient and acceptable agreements are found for certain cases for the lift coefficient. However, discrepancies are found for some cases for the coefficient of lift and pitch moment. Discussions are made upon the agreements and discrepancies associated with the predicted wind force coefficients. With a
fine resolution both in time and space domain, a possible new method to predict wind loads on bridge decks is suggested.

**Keywords:** aerodynamic force; velocity field; Particle Image Velocimetry; computational fluid mechanics; square cylinder; twin-box girder; wind-bridge interaction

1 **INTRODUCTION**

When structures immerse in wind, the time varying wind pressure might induce the structure to vibrate and the wind pressure might be redistributed along the structure surface and cause a change in the structural kinetics. Possible catastrophic failure might be induced after the interactions between the wind and structures. After the failure of the 1st Tacoma Narrows Bridge due to the wind effects, structural engineers and researchers have conducted various scientific investigations on bridge aerodynamics. In the last several decades, the span length of bridges has reached to thousands of meters with the development of modern materials and construction techniques. Accordingly, due to the decrease of structural rigidity, low critical flutter wind speed or large amplitude of wind induced vibrations are expected. In addition, in order to ensure the safety of the large span bridges during its design life, it is necessary to carry out the structural dynamic analysis to predict stress history and structural maximum response, which needs the input of aerodynamic forces for the dynamic system. Currently, the aerodynamic forces on the bridge are stated as the summation of static, self-excited and buffeting force. Accordingly, wind induced vibrations are categorized as buffeting, flutter, galloping and vortex induced vibrations. Based on the research in the last fifty years, the equations of static wind forces, buffeting forces and self-excited forces are proposed based on the quasi-static linear theory. From the wind tunnel experiments, the coefficients, such as the coefficient of lift, drag and pitching moment, the flutter derivatives and the frequency-dependent aerodynamic admittance functions, can be obtained. Moreover, in the wind tunnel experiments, the aerodynamic forces can be evaluated directly from the external force needed to hold the body. Based on the experiments of a cylinder with large amplitude vortex-induced vibrations under the restoring force of a spring system, Gharib (1999) succeeded in measuring the unsteady forces from the time displacement data.

In addition to the analytical approach and wind tunnel experiments, the force equations can be derived from the equations of fluid mechanics and evaluated using the data from either computational fluid dynamics (CFD) or PIV wind tunnel experiments. Based on control-volume concepts, Wu (1981) derived equations of the aerodynamic forces for single and multiple bodies at the expense of the domain of integration being infinite. Based on the transformation from volume integrals to surface integrals, Noca et al. (1997, 1999) derived a momentum-based control-volume formulation, which only required the input of the temporal and spatial velocity derivatives on the external surface of the control volume. Their method was tested for self-consistency by using different equations and modifying the domains of integrations. However, no independent force measurements were performed. Unal et al. (1997) prompted similar approach by using a control volume that was fixed to the reference frame of an accelerating body and was used to determine the instantaneous force on a body. In their approach, the instantaneous velocity within the whole control volume and derivatives of velocity along the control surface were needed. Based on the work of Quartapelle and Napolitano (1983), Protas (2000) presented an extension method to compute forces in external flows of viscous incompressible fluids which could separate the contributions of pressure and viscous stresses without using time differentiation. With the development of CFD and Particle Image Velocimetry, it is possible to obtain the velocity and their derivatives both in the control volume
and on their surfaces. Therefore, the derived equations might be validated for possible use for predicting the wind loads on structures.

The term PIV first appeared in the literature in 1984 (Adrian 2005). Based on the light scattering characteristics and the aerodynamic tracking capabilities, certain PIV seeding particles could be added into the fluid of interest (Melling 1997). If the particles have almost the same speed as the fluid, the velocity of the particles can represent the velocity of the fluid. High-energy laser illuminates the particles in the fluid, while charge-coupled device (CCD) captors and fast frame grabbers take pairs of images at the same time in a nanosecond time interval and the images are transferred at a high speed to computers (Raffel et al. 1998). Suitable particle image pattern matching scheme and PIV algorithms are used to obtain the velocity of the fluid. After the development in the last three decades, PIV has become an accurate and quantitative measurement of fluid velocity vectors at a very large number of points simultaneously (Adrian 2005).

In the present study, the divergence theorem is used to convert the control volume integral to control surface integral in order to obtain the equations of aerodynamic forces on lift and drag based on the conservation of momentum with the only input of the velocities of the wind field and their derivatives. Similarly, the equations for pitching moment were proposed in a similar approach based on the conservation of the momentum moment. The input wind field velocity and their derivatives are obtained through numerical simulations and PIV experiments.

2 WIND LOADS ESTIMATION

2.1 Drag and life force

The early attempt to predict the aerodynamic force and moment in viscous flow based on velocity fields was done by Wu (1981). Saffman (1993) derived the impulse equation based on the concept of the fluidic body, which extended the integrations to the whole space including the body. Noca et al. (1999) obtained the flux equation, in which the integrations are carried over the whole fluid domain only. The volume integral part is transformed into surface terms and the two important identities are used in the derivations:

\[
\int_V \mathbf{x} \times \nabla \times \mathbf{u} dV = (N - 1) \int_V \mathbf{u} dV + \oint_S \mathbf{x} \times (\hat{n} \times \mathbf{u}) dS \tag{1}
\]

\[
(N - 1) \oint_S \phi \hat{n} dS = -\oint_S \mathbf{x} \times (\hat{n} \times \nabla \phi) dS \tag{2}
\]

where, \( \hat{n} \) is a unit vector, \( \mathbf{u} \) is the flow velocity vector, \( \mathbf{x} \) is the position vector, \( N \) is the space dimension and \( N=2 \) for two dimensional velocity field, \( V \) is the control volume around the bridge deck bounded externally by the outer surface \( S(t) \) and internally by the body surface \( S_b(t) \) as shown in Fig. 1. The former equation (or an equation similar to it) appeared in the works of Batchelor (1967) and Saffman (1993) and the Green’s theorem was used to transfer the volume integral to surface integral. The latter equation appears in the paper by Wu & Wu (1996).
In the present study, the divergence theorem is used to convert the control volume integral to control surface integral. After introducing the following identities in Eqs. (3), the two volume parts can be stated as Eq. (4) and (5):

\[ \mathbf{x} \times (\nabla \times \mathbf{u}) = (N-1)\mathbf{u} + \nabla (\mathbf{x} \cdot \mathbf{u}) - \nabla \cdot (\mathbf{xu}) \; ; \quad \mathbf{u} = \nabla \cdot (\mathbf{ux}) - (\nabla \cdot \mathbf{x}) \mathbf{u} \]  

\[ \int_{V(t)} \mathbf{x} \times (\nabla \times \mathbf{u}) dV = \int_{V(t)} \{(N-1)\nabla \cdot (\mathbf{ux}) + \nabla \cdot [(\mathbf{x} \cdot \mathbf{u}) \mathbf{I}] - \nabla \cdot (\mathbf{xu})\} dV \]

\[ = \oint_{S(t)} \hat{n} \cdot [(N-1)\mathbf{ux} + (\mathbf{x} \cdot \mathbf{u}) \mathbf{I} - \mathbf{xu}] dS \]

\[ \int_{V(t)} \mathbf{u} dV = \int_{V(t)} [\nabla \cdot (\mathbf{ux}) - (\nabla \cdot \mathbf{u}) \mathbf{x}] dV = \oint_{S(t)} \hat{n} \cdot (\mathbf{ux}) dS \]  

The equation for the aerodynamic force including the drag and lift forces can be obtained as the following, which is the same as the equation referred as "flux equation" by Noca et al. (1999):

\[ \mathbf{F} = \oint_{S(t)} \hat{n} \cdot \mathbf{Y}_f dS - \oint_{S_b(t)} \hat{n} \cdot (\mathbf{u} - \mathbf{u}_s) \mathbf{I} u dS - \frac{d}{dt} \oint_{S_b(t)} \hat{n} \cdot (\mathbf{ux}) dS \]  

where,

\[ \mathbf{Y}_f = \frac{1}{2} \mathbf{u}^2 \mathbf{I} - \mathbf{uu} - \frac{1}{N-1} \mathbf{u}(\mathbf{x} \times \omega) + \frac{1}{N-1} \omega (\mathbf{x} \times \mathbf{u}) + \mathbf{T} \]

\[ + \frac{\mathbf{x} \cdot (\nabla \cdot \mathbf{T}) \mathbf{I} - \mathbf{x}(\nabla \cdot \mathbf{T})}{(N-1)} - \frac{\partial (\mathbf{ux})}{\partial t} - \frac{1}{N-1} \left[ (\mathbf{x} \cdot \frac{\partial \mathbf{u}}{\partial t}) \mathbf{I} - \mathbf{x} \frac{\partial \mathbf{u}}{\partial t} \right] \]

where, \( \omega = \nabla \times \mathbf{u} \) is the vorticity vector, and \( \mathbf{T} \) is the viscous stress tensor, \( \mathbf{T} = \lambda (\nabla \cdot \mathbf{u}) \mathbf{I} + \mu \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \), for non-compressible fluid, \( \nabla \cdot \mathbf{u} = 0 \) and \( \mathbf{T} = \mu \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \).

Based on the flux equation, the aerodynamic forces for the bridge deck can be stated as the function of the velocity in the fluid domain, and the displacements and velocities of the bridge decks. In addition, two assumptions are made for the typical bridge decks of box girders, namely, the no through flow condition as shown in Eq. (8) and the rigid body motion assumption as shown in Eq. (9).

\[ (\mathbf{u} - \mathbf{u}_s) \cdot \hat{n} = 0 \]  

\[ \mathbf{u}_b = \mathbf{u}_c + \alpha \times (\mathbf{x}_c - \mathbf{x}_b) \]  

where, \( \mathbf{u}_c \) is the velocity vector of the structure body, \( \alpha \) is the rotational vector, \( \mathbf{x}_b \) is the location vector, \( \mathbf{x}_c \) is the location vector of the rotational center of the body.
Accordingly, the second term of Eq. (6) turns zero and the third term can be turned back to the structural motion using the divergence theorem, which leads to the equations for the lift and drag forces (Zhang and Ge 2009a):

\[ D = \rho A \ddot{\rho} + \rho \dot{\rho} \dot{S}_x + \tilde{F}_x \]  
(10)

\[ L = \rho A \ddot{h} - \rho \dot{\rho} \dot{S}_y + \tilde{F}_y \]  
(11)

where, \( h, \ p \) and \( \varphi \) are the three components of the displacement in the vertical direction, lateral direction and rotation; \( S_x \) and \( S_y \) are the static moment to the local coordinates \( X'O'Y' \) of \( x \) and \( y \) as shown in Fig. 2; \( \tilde{F}_x \) and \( \tilde{F}_y \) are the first term of Eq. (6) and can be determined by the velocity and their derivatives of the fluidic body outer surface \( S(t) \).

For a rectangular fluidic body around the bridge decks, the two forces are simplified as the summations of the integrations of the velocity terms and their derivatives (Zhang and Ge 2009a).

\[ \tilde{F}_x = \int_{DA+BC} \left[ -u_1 u_2 - u_2 y \omega + \frac{\mu}{\rho} \left( \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} \right) - \frac{\mu}{\rho} y \left( 2 \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_2}{\partial xy} \right) - x \frac{\partial u_2}{\partial t} + y \frac{\partial u_1}{\partial t} \right] \, dx 
+ \int_{AB+CD} \left[ -\frac{1}{2} u_1^2 + \frac{1}{2} u_2^2 - u_1 y \omega + \frac{\mu}{\rho} \left( \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} \right) - \frac{\mu}{\rho} y \left( 2 \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_2}{\partial xy} \right) - x \frac{\partial u_2}{\partial t} - y \frac{\partial u_1}{\partial t} \right] \, dy \]  
(12)

\[ \tilde{F}_y = \int_{DA+BC} \left[ \frac{1}{2} u_1^2 - \frac{1}{2} u_2^2 - u_2 x \omega + \frac{\mu}{\rho} \frac{\partial u_2}{\partial y} + \frac{\mu}{\rho} x \left( 2 \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_2}{\partial xy} \right) - y \frac{\partial u_2}{\partial t} - x \frac{\partial u_1}{\partial t} \right] \, dx 
+ \int_{AB+CD} \left[ -u_1 u_2 - u_1 x \omega + \frac{\mu}{\rho} \left( \frac{\partial u_2}{\partial x} + \frac{\partial u_1}{\partial y} \right) - \frac{\mu}{\rho} x \left( 2 \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_2}{\partial xy} \right) - y \frac{\partial u_1}{\partial t} + x \frac{\partial u_2}{\partial t} \right] \, dy \]  
(13)

where, \( u_1 \) and \( u_2 \) are the velocity components in the \( x \) and \( y \) directions.

For a fixed bridge deck, both of the first two terms Eqs. (10) and (11) are zero and the static coefficient of drag and lift forces can be stated as:

\[ C_d = \tilde{F}_x / \left( \frac{1}{2} \rho U_{\infty}^2 H \right), \quad C_l = \tilde{F}_y / \left( \frac{1}{2} \rho U_{\infty}^2 B \right) \]  
(14)

where, \( B \) and \( H \) are the width and height of the bridge deck.

### 2.2 Pitch moment

In a similar approach, the equation for the pitch moment can be obtained based on the conservation of momentum moment in light of the derivation of force equations from the conservation of momentum. The same configuration of control volume and surface is chosen as
shown in Fig. 1. Given an arbitrary, time-dependent control volume $V(t)$ bounded externally by a surface $S(t)$ and internally by the body surface $S_b(t)$, the integral form of the momentum moment equation is:

$$
\frac{d}{dt} \int_{V(t)} x \times \rho \mathbf{u} dV = \oint_{S(t) \cup S_b(t)} x \times (\hat{n} \cdot \mathbf{P}) dS - \oint_{S_b(t)} x \times (\hat{n} \cdot \mathbf{P}) dS - \oint_{S(t)} x \times (\hat{n} \cdot \mathbf{P}) dS
$$

where $\mathbf{P} = -p \mathbf{I} + \mathbf{T}$ is the stress tensor.

The moment from the fluid acting on the body can be stated as the following:

$$
\mathbf{M} = -\oint_{S_b(t)} x \times (\hat{n} \cdot \mathbf{P}) dS \mathbf{M} - \oint_{S(t)} x \times (\hat{n} \cdot \mathbf{P}) dS - \rho \frac{d}{dt} \int_{V(t)} x \times \mathbf{u} dV
$$

After introducing the non-slipping condition and Navier Stokes equation, the following equations could be obtained (Zhang and Ge 2009a):

$$
\mathbf{M} = \oint_{S(t)} \left( -\rho x \hat{n} + x \times\hat{n} \cdot \mathbf{T} \right) dS + (N-1) \oint_{S(t)} \rho x \hat{n} dS - \rho \oint_{S(t)} \hat{n} \cdot \mathbf{u} \left[ \nabla \cdot (x \times \mathbf{u}) x \right] dS - \oint_{S(t)} \hat{n} \left[ \frac{1}{2} x \nabla \rho u^2 x + \rho x \times \mathbf{u} \times \omega \mathbf{x} + x \nabla \cdot \mathbf{T} x \right] dS - \rho \frac{d}{dt} \oint_{S_b(t)} \hat{n} \cdot \left[ (x \times \mathbf{u}) x \right] dS - \rho \frac{d}{dt} \int_{V(t)} (x \cdot \omega) x dV
$$

where, $p$ is the air pressure scalar.

In order to eliminate the pressure term in Eq. (17), the 2D assumption is made in the present study, which leads to $N=2$. The following equation can be derived after introducing the rigid body motion assumption:

$$
\mathbf{M} = \oint_{S(t)} (x \times \hat{n}) \cdot \mathbf{Y}_m dS - 3 \rho \frac{d}{dt} \int_{V(t)} [x \cdot \mathbf{u}_b] dV
$$

where,

$$
\mathbf{Y}_m = \mathbf{T} - \frac{\rho}{2} \nabla u^2 x + \rho \mathbf{u} \times \omega \mathbf{x} + \nabla \cdot \mathbf{T} x
$$

For small rotational angles, the coordinate vector of the point in the structure body can be simplified as:

$$
\begin{align*}
x_i &= p + x \cos \varphi - y \sin \varphi = p + x - y \varphi \\
y_i &= h + x \sin \varphi + y \cos \varphi = h + x \varphi + y
\end{align*}
$$

Finally, the equation for the pitch moment can be stated as:

$$
\begin{align*}
M_\varphi &= 3 \rho (\dot{p} \ddot{p} + \dot{p}^2 + \dddot{h} + \dddot{h}^2) + 3 \rho (\dot{\varphi} \ddot{\varphi} + \dot{\varphi} \dddot{h} + \dddot{h} \dddot{\varphi} - \dddot{h} \dddot{\varphi} - \dddot{h} \dddot{\varphi}) S_y \\
&+ 3 \rho (\dddot{h} - \dot{\varphi} \dddot{p} - \dddot{h} \dddot{p} - \dddot{p} \dddot{\varphi}) S_x - 6 \rho \dot{\varphi} I_{xy} - 3 \rho (\dot{\varphi}^2 + \ddot{\varphi}) I_{y} + 3 \rho (\dot{\varphi}^2 + \ddot{\varphi}) I_{x} + \dddot{M}_\varphi
\end{align*}
$$

where, $I_x$ and $I_y$ are the moment of inertia of the section; and $\dddot{M}_\varphi$, the last term of Eq. (21), can be determined by the velocity and their derivatives of the fluidic body outer surface $S(t)$. For a rectangular fluidic body around the bridge decks, it can be obtained by:

$$
\dddot{M}_\varphi = \int_{DA+CB} \left[ 2 \mu_x \frac{\partial u_x}{\partial y} - \mu_x \left( \frac{\partial u_z}{\partial y} + \frac{\partial u_z}{\partial x} \right) \right] dx + \int_{AB+DC} \left[ -\mu_x \left( \frac{\partial u_z}{\partial x} + \frac{\partial u_z}{\partial y} \right) + 2 \mu_y \frac{\partial u_z}{\partial x} \right] dy
$$

If the bridge deck is fixed, the first four terms of Eqs. (21) are zero and the static coefficient of pitch moment can be stated as:

$$
C_M = \dddot{M}_\varphi / \left( \frac{1}{2} \rho U^2 B^2 \right)
$$
3 ACQUISITION OF WIND VELOCITY AROUND BRIDGE DECKS

3.1 Numerical simulations

Wind velocity around bridge deck can be obtained from either computational fluid dynamics (CFD) method or PIV experiments in the wind tunnel. However, before the PIV experiments are available, numerical calculation is the only method to obtain the whole flow map around bluff bodies simultaneously. However, an accurate simulation of the flow, which is superposed by turbulent fluctuations and the periodic vortex shedding motions, is not easy. Two well-known experimental studies of the flow around a square cylinder at $Re = 22000$ conducted by Lyn et al. (1995) and at $Re = 14000$ conducted by Durão (1988) are used in the literature to validate the numerical simulation techniques (Bouris and Bergeles 1999, Lubcke et al 2001). After validating the numerical simulation results by the two well-known experiments, the velocity data from the wind field obtained by the CFD calculations and PIV experiments are used to validate the equations derived in the present study.

In the CFD calculations, due to the large computational cost of solving the Navier-Stokes equations, Reynolds averaged Navier-Stokes (RANS) equations using a statistical turbulence model, large eddy simulation (LES) or detached eddy simulation (DES) are typically used. Numerical simulations are carried out to obtain the flow maps around a square cylinder in $Re = 22000$ (Zhang and Ge 2009b). A good match is found between the results from the numerical simulations and the two experimental studies done by Lyn et al. (1995) and Durão et al (1988), including the velocity profiles, turbulent kinetic energy profiles, Strouhal number and the drag coefficient. Based on the results from all the compared models, the SST $k-\omega$ Reynolds-averaged Navier-Stokes (RANS) model is found best among the models including Realizable $k-\varepsilon$ model, $k-\varepsilon$ model and Reynolds stress model in the aerodynamic parameter identification and flow field simulations (Zhang and Ge 2009b). The comparison results suggest that the numerical simulations could be used to provide reasonable velocity information in the fluid domain.

In the present study, the flow maps obtained by the numerical simulation are used for aerodynamic force estimation of a square cylinder and a twin-box girder. The configurations of numerical simulations are the same as that of the square cylinder in the literature, where the SST $k-\omega$ RANS turbulence model is preferred (Zhang and Ge 2009b). The 2$^{nd}$ order upwind discretization schemes are used. The calculation domain is taken as the same size of the wind tunnel where the PIV experiments are carried out. The domain is 3 m long and 0.8m high and is meshed block by block with different sizes in different blocks. The minimum mesh size near the wall is 0.0002m. Velocity inlet and pressure outlet are defined and the turbulence intensity is defined as 0.5%. The 10m/s inlet velocity and the side length of 0.033m are chosen to ensure $Re = 22000$. The twin-box section is based on the bridge deck of a self-anchored suspension bridge. The dimensions of the twin box girder are shown in Fig. 3.

![Figure 3 Section of twin box girder (unit: mm)](image-url)
3.2 Particle Image Velocimetry experiment

PIV instruments are set up in a closed circuit wind tunnel at Tongji University. The working section is 0.8m wide, 0.8m high and 3.6m long. Liquid tracer particles are emitted from a fog generator in the contraction section of the wind tunnel. PIV experiments start after particles have been dispensed in the whole wind tunnel homogenously. Two neodymium-doped yttrium aluminum garnet (Nd: YAG) laser and one camera are set at the outside of the working section. The four sides of the working section are made by using transparent glass plates for the convenience of taking images from the outside of the wind tunnel. The Nd: YAG laser emits 532nm wavelength light at a five nanoseconds pulse width and a 15 Hz pulse rate. After passing through one cylinder and sphere lens, round light beams are changed to a light sheet. When the laser has illuminated the liquid particles, a monochrome digital charge-coupled device (CCD) camera takes pairs of the sequential images from a perpendicular angle. A synchronizer synchronizes the laser emission and the image taking of the camera. The maximum resolution of the images is 1600 by 1200 pixels and the maximum gray level is 12 bits. FFT based cross-correlation analysis is used to obtain the velocity using an interrogating window of 32 by 32 pixels from two sequential frames. The patterns formed by several particles in an interrogating window are used to calculate the velocity.

The impulse energy and corresponding time delay between two subsequent laser pulses are adjusted by the synchronizer based on the wind speed and the area of the field of view of the CCD camera. In the present study, the time of exposure is set as 30 microseconds that are large enough to meet the one-quarter rule even at the wind speed of 33 m/s that is the maximum speed for the wind tunnel. Since the interrogating window is 32 by 32 pixels, one-quarter rule requires that the maximum displacement of the particles in the image could not exceed eight pixels to have a good correlation (Raffel et al. 1998). The field of view is adjusted considering the dimension of the model. Images are taken on an object with a known length to determine the scale of pixel and millimeter.

The velocity is obtained in pixels after the FFT based cross-correlation analysis and needed calibration. Both of the models of square cylinder and twin-box girders for the PIV experiments are made of acrylic plates. Similar to the settings in the numerical simulations, the length of the sides for the square cylinder is 0.033m and the velocity in the wind tunnel is set as 10m/s to ensure $Re = 22000$. In order to use the velocity data from the low frequency PIV, the equations for the coefficient of lift force, drag force and pitch moment are simplified by ignoring the time derivatives terms.

4 RESULTS AND CONCLUDING REMARKS

4.1 Square cylinder

Based on the derived equations and the velocity obtained from numerical simulations and PIV experiments, the coefficient of wind forces can be obtained. After conducting a sensitivity study by changing the dimensions of the wind field and the mesh size, the dimensions of the wind field around the square cylinder are used in the present study as shown in Table 1.
Table 1 Dimensions for the wind field around the square cylinder

<table>
<thead>
<tr>
<th>Case No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>0.06m</td>
<td>0.06m</td>
<td>0.06m</td>
<td>0.04m</td>
</tr>
<tr>
<td>L2</td>
<td>0.1m</td>
<td>0.09m</td>
<td>0.09m</td>
<td>0.08m</td>
</tr>
<tr>
<td>H</td>
<td>0.06m</td>
<td>0.06m</td>
<td>0.06m</td>
<td>0.04m</td>
</tr>
<tr>
<td>Mesh size</td>
<td>0.002m</td>
<td>0.003m</td>
<td>0.0015m</td>
<td>0.002m</td>
</tr>
</tbody>
</table>

Table 2 Coefficient of wind forces for the square cylinder

<table>
<thead>
<tr>
<th>Coefficient of wind forces</th>
<th>Cd</th>
<th>Cl</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDV experiment(Lyn et al. 1995)</td>
<td>2.05-2.23</td>
<td>---</td>
</tr>
<tr>
<td>CFD result</td>
<td>2.09</td>
<td>0.0081</td>
</tr>
<tr>
<td>Velocity field integration</td>
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<td></td>
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<tr>
<td>Case 1</td>
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<td>0.13</td>
</tr>
<tr>
<td>Case 2</td>
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<tr>
<td>Case S2</td>
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<td>0.91</td>
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<tr>
<td>Case S3</td>
<td>2.00</td>
<td>0.93</td>
</tr>
<tr>
<td>Case P1</td>
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<td>Case P2</td>
<td>2.54</td>
<td>0.13</td>
</tr>
<tr>
<td>Case P3</td>
<td>2.76</td>
<td>0.33</td>
</tr>
</tbody>
</table>

The meaning of each parameter in Table 1 is the same as those shown in Figure 4. L1 is the distance from upstream boundary segment AB to the center O, while L2 is the distance from the center O to the downstream boundary segment CD. At least forty velocities are suggested for L1 and L2 of the rectangular fluid domain (Zhang and Ge 2009a). In addition, the sizes of the PIV images are considered to ensure the velocity along the fluid domain to be obtained through the same image pair. The coefficient of wind forces are listed in Table 2, including the results from wind tunnel experiments and numerical simulations. Since a large numerical calculation error is expected for the small coefficient of pitch moment, only the drag and lift coefficients are calculated for comparisons. As one of the two well-known experimental studies of the flow around a square cylinder, the drag coefficient listed in the first row is based on the Laser Doppler Velocimetry (Lyn et al. 1995). Based on the numerical simulations, the wind force coefficients listed in the second row are obtained by directly integrating the surface pressure on bridge decks. The last ten rows are obtained by using the velocity field and the derived equations in the present study. The first four cases from Case 1 to Case 4 are based on the velocity from numerical simulations, which correspond to the four cases listed in Table 1. The cases from Case S1 to S3 and Case P1 to P3 are based on the velocity obtained from numerical simulations and PIV experiments. Their dimensions of the wind field and mesh size are the same as the Case 1 through Case 3. In the last six cases, the time derivatives terms are neglected.
The drag coefficient obtained from numerical simulations in the four cases matches well with the result from the LDV experiment and the CFD results. Case 1, Case 2 and Case 3 have almost the same drag coefficient as the CFD result and falls in the range of Lyn et al.’s experiment results, while Case 4 only has a difference of 5%. Since the time derivatives terms are neglected for the cases S1 to S3 and P1 to P3, the drag coefficients are found to have a larger difference from a 4% to 20% for S1 to S3 comparing with the corresponding cases with the same wind field dimension and mesh size. Since the lift coefficient is not available for Lyn et al.’s experiment, comparisons are only made between the results from velocity filed integration and the CFD calculations. Lift coefficients are largely over estimated comparing with the CFD results and they differ by one or two orders of magnitude.

4.2 Twin box girder

In order to increase the flutter critical wind speed of the large span bridges, twin-box girder or stabilizers are proposed to mitigate the large amplitude vibrations. The prototype of the twin box girder is used in Daguhe Bridge, which is a self-anchored suspension bridge with a single tower. The coefficient of wind forces are measured by a bottom supported five-component strain-gauge balance. Based on the sensitivity study results, the dimensions of the wind field and the mesh size adopted for the twin box girder are listed in Table 3.

Table 3 Dimensions for the wind field around the twin-box girder

<table>
<thead>
<tr>
<th>Case No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>0.40m</td>
<td>0.40m</td>
<td>0.40m</td>
<td>0.40m</td>
<td>0.40m</td>
<td>0.54m</td>
</tr>
<tr>
<td>L2</td>
<td>0.40m</td>
<td>0.40m</td>
<td>0.40m</td>
<td>0.56m</td>
<td>0.56m</td>
<td>0.60m</td>
</tr>
<tr>
<td>H</td>
<td>0.06m</td>
<td>0.06m</td>
<td>0.06m</td>
<td>0.06m</td>
<td>0.06m</td>
<td>0.06m</td>
</tr>
<tr>
<td>Mesh size</td>
<td>0.008m</td>
<td>0.006m</td>
<td>0.004m</td>
<td>0.008m</td>
<td>0.006m</td>
<td>0.006m</td>
</tr>
</tbody>
</table>

The coefficient of wind forces are listed in Table 4, including the results from force balance measurements and numerical simulations. The results from force balance measurements are listed in the first row. The results for second row are obtained through directly integrating the surface wind pressure on the bridge decks based on the numerical simulations. The results for the six cases are based on the velocity from numerical simulations, which correspond to the cases listed in Table 3. Differences are found between the results for the force balance measurement and the CFD results, especially in the coefficient of lift and pitch moment. Since the velocities in
the fluid domain around the bridge decks are used to obtain the coefficient of wind forces in the present study, the comparisons are made mainly between the six cases and the CFD results instead of the results from force balance measurement. For all the six cases, only 4% to 12% percent difference in the drag coefficients is found compared with the CFD results. For the coefficient of lift, all the results in the six cases are in the same order of magnitude. However, large differences are found for the coefficient of pitch moment from the CFD results and the coefficients of pitch moment are underestimated.

Table 4 Coefficient of wind forces for the twin-box girder

<table>
<thead>
<tr>
<th>Coefficient of wind forces</th>
<th>$C_d$</th>
<th>$C_l$</th>
<th>$C_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force balance measurement</td>
<td>0.86</td>
<td>-0.13</td>
<td>-2.80E-2</td>
</tr>
<tr>
<td>CFD result</td>
<td>0.93</td>
<td>-0.63</td>
<td>-1.01E-2</td>
</tr>
<tr>
<td>Case 1</td>
<td>0.92</td>
<td>-0.93</td>
<td>-1.19E-4</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.92</td>
<td>-0.79</td>
<td>-1.01E-4</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.97</td>
<td>-0.68</td>
<td>-5.64E-5</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.81</td>
<td>-0.99</td>
<td>-1.44E-4</td>
</tr>
<tr>
<td>Case 5</td>
<td>0.84</td>
<td>-0.89</td>
<td>-1.13E-4</td>
</tr>
<tr>
<td>Case 6</td>
<td>0.82</td>
<td>-0.10</td>
<td>-1.29E-4</td>
</tr>
</tbody>
</table>

4.3 Concluding Remarks

In the present study, attempts are made to predict the aerodynamic forces on bridge decks based on the two dimensional velocity fields. Different from the empirical expressions for the aerodynamic forces, equations are derived based on the equations of fluid mechanics. With the elimination of the pressure terms in the equations, the aerodynamic forces could be obtained only based on the flow velocity around bridge decks. Different from the perfect self-consistency validation done by Noca et al. (1999) at low Reynolds number of 100, drag coefficients agree well with the results from CFD results at $Re$ of about $10^4$, based on the integration of wind pressure on the bridge decks, for both the square cylinder and the twin-box girders. Acceptable lift coefficients are obtained for the twin-box girders in certain cases while the coefficient of pitch moment has large differences. It is noteworthy that only self-consistent validation is carried out by Noca et al. by using different equations and modifying the domains of integrations. In the present study, force balance is used in the present study to obtain the force from wind for static bridge deck and comparisons results also suggest good match in drag but poor match in lift and pitch moment. With the increased Reynolds number of being $10^4$, drastic changes in the fluid field and interpolations of velocities are believed as the source of errors. In addition, the assumption of the vorticity being perfectly perpendicular to the section surface in 2D might bring more errors. Smaller mesh sizes and time steps for numerical simulations for the velocity histories in the fluid are expected to improve the results by capturing the fine change of velocity both in the time and space domains. High resolution time-resolved PIV can provide a better wind field velocity for the fluid domain around bridge decks. However, more validations need to be carried out with the velocity maps around bridge decks either from numerical simulations or PIV experiments. Based on the proposed equations for lift force, drag force, and pitch moment, aerodynamic forces could be calculated based on the flow field around, and sensitivity studies can be carried out to derive a simpler equation for the aerodynamic force as the input for the structural dynamic analysis.
5 REFERENCES


