Abstract—It is well known to active-sonar engineers that the reflected signal from a target can be highly aspect-dependent, hence in many cases only receivers located in a particular zone determined by the source/target receive-geometry and the target aspect can detect the return signal. Thus, submarines can hide well from traditional sonar systems. For these low-visibility targets, we propose a target localization paradigm based on a distributed sensor network which consists of low complexity sensors that only report binary detection results. Based on binary outputs and the positions of the sensors, we develop optimal maximum likelihood and suboptimal line-fitting based estimators, and derive the Cramer-Rao lower bound on estimation accuracy. We extend our results from single source to multi-source settings, both with and without explicit incorporation of a reflection model that links the target orientation to the propagation direction. Our numerical results verify the feasibility of the proposed estimators. We do not rely on continuous quantities such as signal strength, direction of arrival, time or time-difference of arrival, and instead localize based on discrete detection results which include both false alarms and missed detections.

Index Terms—Active sonar, cross section, multi-static, sensor network, submarine localization

I. MOTIVATION AND CONTEXT

A. Traditional Approach and Low-Visibility Targets

Submarine detection and localization is one major application of sonar systems. We in this paper focus on localization with active sonar systems, as those studied in [3], [4], [9], [15]. Traditional active sonar systems rely on one or a few source-receiver pairs. A system configuration with one source and one co-located receiver is called a monostatic setting, while that with one source and one receiver not co-located is termed bistatic; if multiple source-receiver pairs are present it is multistatic [3], [9], [15]. In these systems, the receiver is typically quite capable in terms of signal processing and communication. The receiver usually consists of an acoustic array, and provides estimates on the direction-of-arrival (DOA), the time-of-arrival (TOA), or the time-difference-of-arrival (TDOA). With the estimated DOA, the receiver determines a line on which the target should be located. With the estimated TOA/TDOA, the receiver infers that the target is on an ellipse with the source and receiver as foci. Combining the DOA and TOA/TDOA information, only one source-receiver pair is capable of source localization [3], [9], [15]. A multistatic configuration further improves estimation accuracy and reduces the sensitivity to the source-target-receiver geometry [3], [9], [15].

Source localization is possible only when the receiver is in the propagation path of the reflected wave from the target. A submarine can effectively hide itself, and thus become “invisible” to the sonar system, if by its orientation it can direct the reflected wave away from the receiver, since most real targets exhibit considerable variation in their sonar cross-section as a function of this aspect; see e.g., the measured data in [17, pp. 310-312]. Hence, when a source emits a probing signal, only receivers located in a particular zone are able to detect the return waves, as depicted in Fig. 1.

If the beamwidth of the detection zone is small, then such targets have low probability of detection by traditional sonar systems. What would be a better approach to deal with these low-visibility targets?

B. The Proposed Approach based on Sensor Network

A distributed sensor network provides unprecedented capabilities for target detection and localization relying on densely deployed cheap sensors. (Overviews on sensor networks can be found in e.g., [1], [5], [12], [14].) We here propose localization solutions tailored to low-visibility targets based on a distributed sensor network. A notional application scenario is depicted in Fig. 2, where a sensor field is established along a coast, with multiple static or mobile sources. When
a source emits a probing signal, the direction of the return wave is uncertain. However, the likelihood of the return wave passing by the sensor field is very high for a long sensor field. Although the position is random, a small zone of the sensor field will be “illuminated” (strobed) by the return wave. This suggests the possibility of target detection and localization.

We assume low-complexity sensors that can only report binary detection results based on threshold comparison at the correlator output. Our objective of this paper is to develop various estimators and assess their performance for target localization based on “detection only” sensors. We first present the propagation model in Section II that specifies the detection probability of each sensor as a function of the propagation direction and distance. We then develop optimal maximum likelihood and suboptimal line-fitting based estimators in Section III, and derive the Cramer-Rao lower bound on the estimation accuracy in Section IV. We extend the results to the multistatic setting, without any assumption on the reflection model in Section V, and with a “mirror” reflection model in Section VI. We also specify in Section VI the conditions on the orientation under which the target becomes visible. Our numerical results in Section VII confirm that accurate location estimation is possible via a network of “detection-only” sensors. The density of the sensor field and the knowledge of the beamwidth turn out to be important factors that determine the estimation accuracy when only binary outputs are available.

Most existing approaches on source localization rely on continuous quantities, such as DOA, TOA, TDOA, received signal strength, or combinations of them [3], [6]–[9], [15], [16]. A sensor-network based approach for source localization has been studied with either binary data [10] or multi-bit quantization of analog quantities [11]. Our proposed method relies on discrete (binary) detection results, which can be viewed as one-bit quantization on the received signal strengths. Sharing many common aspects with [10], [11], our distinctions are: i) we study a submarine localization problem with aspect-dependent reflection, while [10], [11] deals with a land-based wireless sensor network with isotropic propagation; ii) we will consider a Rayleigh fading channel (typical in water), while [10], [11] have considered a non-fading channel (valid for short-range transmissions in land-based sensor networks); iii) we focus on an active sonar scenario, where a multi-static setting can be considered. Also, the submarine is typically outside the sensor field. A passive localization scenario is considered in [10], [11], with a single source usually inside the sensor field.

Our focus in this paper is on the estimation aspect of this problem. Various issues such as communication, signal processing, data fusion, and data collection protocols will not be elaborated here; see e.g., [1], [5], [14] for challenges on these issues.

### II. Propagation Model

We use \((x_s, y_s)\) to denote the position of the source, and \((x_t, y_t)\) that of the target. We consider a sensor field with a total of \(N\) sensors, where the \(n\)th sensor is located at \((x_n, y_n), n = 1, \ldots, N\).

The system works as follows. First, the source emits a waveform \(s(t)\). The propagation is assumed to be omnidirectional, so that both the sensors and the target will receive this signal. The signal arrived at the submarine surface gets reflected. The reflected wave, however, is no longer omnidirectional: it propagates along a certain direction with a small beamwidth. The direct arrival from the source is usually much stronger than the reflected signal. We assume that the sensors are woken up by the direct arrival from the source, and then use a correlator to detect the reflected signal \(s(t)\) within a certain time window\(^1\); the length of the time window is decided by the maximum survey area that a target is visible to the sensor network. Denote \(z_n\) as the detection result for the \(n\)th sensor. If the correlator output is higher than a certain threshold, the \(n\)th sensor declares a “detection” by setting \(z_n = 1\). Otherwise, it declares a “no-detection” by setting \(z_n = 0\). Thanks to the directional propagation of the reflected wave, only sensors within a certain zone are likely to report detections. When sensors outside the zone do report detections, those are likely false alarms due to additive noise in the correlator, although of course the network does not “know” that.

Denote \(\text{SNR}_n\) as the average signal to noise ratio at the \(n\)th sensor corresponding to the return wave from the target. Assuming a Rayleigh fading signal model (see justifications in, e.g., [17, pages 189 and 382]), the detection variable at the correlator output is exponentially distributed with variance proportional to \((1 + \text{SNR}_n)\). With a threshold \(\Gamma_{th}\), the probability of detection is thus

\[
P_{D,n} = p(z_n = 1) = \exp\left(-\frac{\Gamma_{th}}{1 + \text{SNR}_n}\right).
\]

When a sensor is outside the reception zone, the false alarm rate is

\[
P_{fa} = \exp(-\Gamma_{th}),
\]

which can be controlled by adjusting the threshold \(\Gamma_{th}\).\(^1\) The time delay between the direct and the reflected arrivals can be exploited for target localization. This warrants further investigation.
We now specify the dependence of $\text{SNR}_n$ on propagation distance and propagation angle. The latter is also the direction of arrival (DOA) from the sensor field point of view. We assume that the source and the sensors are approximately at the same depth as the submarine. Ignoring small depth differences, the travel distance of the signal from the source to the $n$th sensor via reflection on the target is

$$r_n = \sqrt{(x_s - x_t)^2 + (y_s - y_t)^2 + (x_n - x_t)^2 + (y_n - y_t)^2}. \quad (3)$$

Denote the propagation angle as $\alpha$ and the angle from the $n$th sensor to the target as $\psi_n$. As depicted in Fig. 3, we compute $\psi_n$ as

$$\psi_n = \arctan\left(\frac{x_n - x_t}{y_n - y_t}\right), \quad \forall n \quad (4)$$

There is no ambiguity in determining $\psi_n$ from (4) when $y_n < y_t$ and thus $\psi_n$ belongs to the range of $[-\pi/2, \pi/2]$. Whether the sensor is in the zone or outside the zone depends on the angle difference between $\alpha$ and $\psi_n$. Generically, we can write

$$\text{SNR}_n = c_0 f_1(r_n) f_2(\psi_n, \alpha), \quad (5)$$

where $c_0$ is a constant, $f_1(\cdot)$ describes the dependence on the propagation distance and $f_2(\cdot)$ specifies the dependence on the propagation angle.

What would be good choices for $f_1(\cdot)$ and $f_2(\cdot)$? They might depend on the specific environment. In this work, we assume that the average signal strength is inversely proportional to the propagation distance (corresponding to cylindrical spreading [17, p. 178])

$$f_1(r_n) = r_n^{-1}. \quad (6)$$

We model the angle dependence by a Butterworth filter\(^2\) as

$$f_2(\psi_n, \alpha) = \frac{1}{1 + (\frac{\psi_n - \alpha}{W})^{2K}}, \quad (7)$$

where $2W$ is the 3dB bandwidth, and $K$ is the filter order.

With $f_1(\cdot)$ in (6) and $f_2(\cdot)$ in (7), we re-write (1) as:

$$P_{D,n} = \exp\left(-\frac{\Gamma_{th}}{1 + c_0 f_1(r_n) f_2(\psi_n, \alpha)}\right), \quad (8)$$

\(^2We emphasize that we take as a proxy the Butterworth radiation pattern for the convenience of illustrating the idea. Our analysis can be similarly carried out for other shapes if their close-form expressions are available; but one should not expect that the Butterworth pattern would match well a wide range of patterns. Note that a butterfly pattern has been used in [17, p. 311] to approximate real measurements.

The detection result $z_n$ is a binary random variable with probability mass function (pmf)

$$p(z_n = 1) = P_{D,n}, \quad p(z_n = 0) = 1 - P_{D,n}. \quad (9)$$

Notice that the pmf is different from sensor to sensor. When the sensor is in the center of the zone, the probability of detection $P_{D,n}$ is high; when outside, $P_{D,n}$ becomes essentially the probability of false alarm.

### III. Optimal and Suboptimal Location Estimation

When the source emits its signal, the sensors in the field report binary detection results. Our objective is to estimate the target location $(x_t, y_t)$ and the DOA $\alpha$ based on these binary results.

Communicating all $z_n$’s of the sensor field to the data processing center is time-consuming. We envision a data collection process as follows. The sensors with detections ($z_n = 1$) first check their neighborhood, to see whether a few neighbors also have detections. If so, they form a cluster, and send a reporting request to a data collection unit. The data collection unit then surveys a region that contains the reporting cluster. Inside that region, not only the results of “$z_n = 1$” are collected, but also the results of “$z_n = 0$”. A result of “no-detection” indicates that the corresponding sensor is probably outside the propagation zone, that also contains valuable information about the target position and DOA.

We assume that the survey region has $N_s$ sensors; $N_s$ is usually much smaller than $N$. Due to the randomness of the detector output, the number of “detections” $L$ is a random variable. To estimate the source location based on the detection-only sensors, we assume that the sensor positions are available at the data center. The sensor positions could be known a priori, or they can be estimated periodically during the network maintenance, see e.g., [8], [13] and references therein. The impact and inclusion of the position errors on the estimation performance will be treated separately in future work.

We next specify two estimators. These estimators differ on performance, complexity, and in their use of “no-detection” results.

#### A. ML estimator

Collect the unknowns in the vector $\theta = (x_t, y_t, \alpha)$. With the detection results collected from $N_s$ sensors, the optimal maximum likelihood estimator is

$$\hat{\theta}_{\text{ML}} = \arg \max_{\theta} \prod_{n=1}^{N_s} p(z_n | \theta), \quad (10)$$

where $p(z_n | \theta)$ is computed from (8) and (9) based on the values from $\theta$. The ML estimator in (10) entails numerical search over a three-dimensional parameter space. We underscore that the ML estimator in (10) is based on the detection statistics, not the underlying analog data.

**Remark:** In the ML formulation of (10) and the CRB analysis in Section IV, we assume perfect knowledge on the model parameters $c_0, W, K$. These values could be estimated...
in practice, based on e.g., a prior data. One alternative way is to treat \( c_0, W, \) and \( K \) as unknowns and formulate an optimization problem with six variables \( f(x, y, \alpha, z_n) \). Then the ML algorithm and the CRB performance bound can be similarly derived. Such an approach has been adopted in [10], [11] where they have an optimization problem with only three unknowns \( f(x, y, z_n) \) due to omni-directional propagation. For this approach to work in our scenario, one has to derive efficient numerical algorithms. Investigating the properties of the objective function and developing low-complexity optimal/near-optimal numerical algorithms are left for our future work.

B. Line fitting based on sensors with detections

The sensors with \( z_n = 1 \) reveals (or illuminates) the detection zone. An intuitive approach is to fit a line in the detection zone so that the DOA can be estimated.

Suppose there are a total of \( L \) sensors reporting “detection” in the survey region, and we want to fit a line across the region. We specify a line by three parameters \( (x_0, y_0, \beta) \), and all points on this line shall satisfy:

\[
(x - x_0) = -(y - y_0) \tan \beta. \tag{11}
\]

The distance of the sensor \( n \) to this line is \( |(y_0 - y_n) \sin \beta - (x_n - x_0) \cos \beta| \), as illustrated in Fig. 4. Our objective is to find \( (x_0, y_0, \beta) \) to minimize the total squared distance

\[
f(x_0, y_0, \beta) = \sum_{n=1}^{L} [(y_0 - y_n) \sin \beta + (x_0 - x_n) \cos \beta]^2. \tag{12}
\]

Letting \( \partial f(x_0, y_0, \beta) / \partial x_0 = 0 \) and \( \partial f(x_0, y_0, \beta) / \partial y_0 = 0 \), we obtain:

\[
\sin \beta \sum_{n=1}^{L} (y_0 - y_n) + \cos \beta \sum_{n=1}^{L} (x_0 - x_n) = 0. \tag{13}
\]

Letting \( \partial f(x_0, y_0, \beta) / \partial \beta = 0 \), we have:

\[
\tan(2\beta) = \frac{2 \sum_{n=1}^{L} (x_0 - x_n)(y_0 - y_n)}{\sum_{n=1}^{L} [(x_0 - x_n)^2 - (y_0 - y_n)^2]}. \tag{14}
\]

We have two equations, and three unknowns. This means that the target position cannot be determined. We can fix \( x_0 \) and solve for \( y_0 \) and \( \beta \) based on (13) and (14).

One smart choice of \( x_0 \) and \( y_0 \) that satisfies (13) regardless of \( \beta \) is

\[
x_0 = \frac{1}{L} \sum_{n=1}^{L} x_n, \quad y_0 = \frac{1}{L} \sum_{n=1}^{L} y_n. \tag{15}
\]

This means that the center of the “detection” sensors has to be on the line to minimize the total fitting error. With \( x_0 \) and \( y_0 \) given in (15), we solve \( \beta \) from (14). Since \( \tan(2\beta + \pi) = -\tan(2\beta) \), there are several feasible solutions of \( \beta \). We just need to choose the one that leads to the smallest cost \( f(x_0, y_0, \beta) \).

The optimal solution \( \beta \) thus serves as an estimate of the DOA \( \alpha \).

The advantages of the line fitting method relative to the ML solution are threefold. First, we assume nothing about the propagation model, and hence have no need for model parameter estimation; second, the computation is simple. Third, it only uses sensors with “detections”, which implies less data to be collected by the data processing center. However, the disadvantage is also obvious: we only have a DOA estimate. In the presence of missed detections and false alarms, the performance of the line fitting method may be considerably worse than the ML solution.

IV. CRB Analysis

In this section, we evaluate the Cramer-Rao bound (CRB), that serves as a performance benchmark for any unbiased estimator. This analysis is carried out for a fixed survey region with \( N_s \) sensors. By varying \( N_s \), one can test the CRB dependence on the size of the survey region, as will be done in Section VII.

We collect the \( N_s \) detection results into a vector \( z = [z_1, \ldots, z_{N_s}] \). For each discrete random variable \( z_n \), we can express the probability density function (pdf) as

\[
p(z_n) = p_{D,n} \delta(z_n - 1) + (1 - p_{D,n}) \delta(z_n), \tag{16}
\]

where \( \delta(\cdot) \) is the Dirac delta function. The joint likelihood function of all variables in \( z \) is

\[
p(z|\theta) = p(z_1|\theta) \cdots p(z_{N_s}|\theta). \tag{17}
\]

The \( 3 \times 3 \) information matrix \( J \) has the \((i, j)\)th element

\[
[J]_{i,j} = E \left[ \frac{\partial \ln p(z|\theta)}{\partial \theta_i} \frac{\partial \ln p(z|\theta)}{\partial \theta_j} \right]. \tag{18}
\]

The CRB matrix is then \( J^{-1} \). The diagonal entries of \( J^{-1} \) specify the lower bound on the estimation accuracy on the corresponding parameters [2].

We now evaluate each entry of \( J \). Based on (16) and (17), we obtain

\[
\frac{\partial \ln p(z|\theta)}{\partial \theta_i} = \sum_{n=1}^{N_s} \frac{1}{p(z_n)} \left[ \delta(z_n - 1) - \delta(z_n) \right] \frac{\partial p_{D,n}}{\partial \theta_i}. \tag{19}
\]

Since \( \frac{\partial p_{D,n}}{\partial \theta_i} \) is deterministic and the \( z_n \)'s are independent, we
carry out the expectation in (18) to obtain
\[
[J]_{i,j} = \sum_{n=1}^{N_s} \left\{ \int \frac{1}{p_f(z_n)} [\delta(z_n - 1) - \delta(z_n)]^2 p(z_n) dz_n \right\}
\cdot \frac{\partial P_{D,n}}{\partial \theta_i} \frac{\partial P_{D,n}}{\partial \theta_j}.
\]
Using (21)–(29) into (20), we obtain each entry of
\[
\sum_{n=1}^{N_s} \left( \frac{1}{P_{D,n}} + \frac{1}{1 - P_{D,n}} \right) \frac{\partial P_{D,n}}{\partial \theta_i} \frac{\partial P_{D,n}}{\partial \theta_j}.
\]
\[
(20)
\]
As \(\theta_i\) is either \(x_t\) or \(y_t\), or \(\alpha\), we obtain the needed partial derivatives from (3) and (4) as:
\[
\frac{\partial r_n}{\partial x_t} = \frac{x_t - x_s}{\sqrt{(x_t - x_s)^2 + (y_t - y_s)^2}} + \frac{x_t - x_n}{\sqrt{(x_t - x_n)^2 + (y_t - y_n)^2}},
\]
\[
\frac{\partial r_n}{\partial y_t} = \frac{y_t - y_s}{\sqrt{(x_t - x_s)^2 + (y_t - y_s)^2}} + \frac{y_t - y_n}{\sqrt{(x_t - x_n)^2 + (y_t - y_n)^2}},
\]
\[
(24)
\]
\[
(25)
\]
\[
(26)
\]
\[
(27)
\]
\[
(28)
\]
Substituting (21)–(29) into (20), we obtain each entry of \(J\), which leads to the CRB matrix \(J^{-1}\).

Conventional approaches rely on continuous quantities such as DOA, TOA, TDOA, or signal strengths, with CRB analysis carried out in [6]–[8], [15], [16]. As shown in [10], [11] and this paper, CRB is also readily available for localization based on discrete results.

V. MULTI-STATIC SETTING

So far we have considered the multi-receiver bistatic scenario with single source. We now extend the results to multiple sources. We assume that different sources use different signature waveforms, so that the sensors know how to associate the detection results to the sources; or, if the sources use the same waveform, they could be separated by time-division multiplexing. For example, a mobile source could emit signals at different positions. In this case, we assume that the transmission time difference is moderate so that the target position does not change (we assume a static or slowly moving submarine target). In accordance with the terminology in [3], [9], [15], we denote the setting with multiple sources multi-static.

With little loss of generality, we consider two sources. The variables corresponding to two sources are differentiated by the superscript (1) and (2). For example, \(z_n^{(1)}\) denotes the detection variable for the first source, and \(z_n^{(2)}\) for the second.

A. The ML detector

Denote \(\alpha_i\) as the propagation angle corresponding to source \(i, i = 1, 2\). For the collected detection variables \(\{z_n^{(1)}\}_{n=1}^{N_s}\), the unknown parameters are \(\theta_1 = (x_t, y_t, \alpha_1)\). For the collected detection variables \(\{z_n^{(2)}\}_{n=1}^{N_s}\), the unknown parameters are \(\theta_2 = (x_t, y_t, \alpha_2)\). Now, we pursue joint estimation, and the unknown parameters are \(\theta = (x_t, y_t, \alpha_1, \alpha_2)\).

The propagation angles \(\alpha_1\) and \(\alpha_2\) are, of course, related. However, in this section, we treat them as independent; explicit modeling of the relationship between \(\alpha_1\) and \(\alpha_2\) will be done in Section VI.

With independent \(\alpha_1\) and \(\alpha_2\), the optimal ML estimator is
\[
\hat{\theta} = \arg \max_{\theta} \prod_{n=1}^{N_s} p(z_n^{(1)}|\theta_1) \prod_{n=1}^{N_s} p(z_n^{(2)}|\theta_2).
\]
\[
(30)
\]
This is an extension of (10) to the case with two sources.

B. The joint CRB

The CRB on the joint estimation of \(\theta_1\) and \(\theta_2\) can be easily obtained. Denote \(J^{(i)}\) as the information matrix on \(\theta_i\), based on the detection variables \(\{z_n^{(i)}\}\). Compute \(J^{(1)}\) and \(J^{(2)}\) using the results in Section IV. Assuming independence of \(\alpha_1\) and \(\alpha_2\), the information matrix on the estimation of \(\theta\) is simply:
\[
J^{\text{joint}} = J^{(1)} + J^{(2)}.
\]
\[
(31)
\]
C. The estimator based on line fitting

With one source, we only obtain a DOA estimate that specifies a line where the target should be located. With two sources, the crossing point of two lines serves as a good estimate for the target location. Specifically, corresponding to the first source, the target position should satisfy:
\[
x_t - x_0^{(1)} = -(y_t - y_0^{(1)}) \tan \beta^{(1)},
\]
while with the second source, we have
\[
x_t - x_0^{(2)} = -(y_t - y_0^{(2)}) \tan \beta^{(2)}.
\]
\[
(32)
\]
\[
(33)
\]
Based on (32) and (33), we obtain the location estimate as:

\[
\begin{bmatrix}
\tilde{x}_t \\
\tilde{y}_t
\end{bmatrix} = \begin{bmatrix}
1 & \tan\beta^{(1)} \\
1 & \tan\beta^{(2)}
\end{bmatrix}^{-1} \begin{bmatrix}
x_0^{(1)} + y_0^{(1)} \tan\beta^{(1)} \\
x_0^{(2)} + y_0^{(2)} \tan\beta^{(2)}
\end{bmatrix}.
\]  

(34)

VI. LINKING PROPAGATION ANGLE WITH TARGET ORIENTATION

In our original development, we treated the propagation angle as one free parameter. When dealing with multiple sources, we did not model the dependence between the propagation angles corresponding to multiple sources. Instead, we assumed independence between \(\alpha_1\) and \(\alpha_2\) in Section V.

In this section, we explicitly model how the propagation angle is related to the source-target geometry and the target orientation. Specifically, we assume a specular reflection model [17], where the submarine surface acts as a reflecting mirror. For the \(i\)th source, we define the source-to-target angle as

\[
\psi_{s,t}^{(i)} = \tan^{-1} \left( \frac{x_s^{(i)} - x_t}{y_s^{(i)} - y_t} \right).
\]

(35)

The angle \(\psi_{s,t}^{(i)}\) is well defined in \((-\pi/2, \pi/2)\), as \(y_s^{(i)} < y_t\) in practice. Now, denote \(\phi_t\) as the orientation of the submarine, as depicted in Fig. 3, which belongs to the interval \([-\pi/2, \pi/2]\).

As the submarine surface acts like a mirror, we have

\[
\alpha_t = 2\phi_t - \psi_{s,t}^{(i)},
\]

(36)

as illustrated in Fig. 3. This way, all the propagation angles are decided by the target orientation together with the geometry of sources and target. Hence, we only need to deal with three unknowns \(\hat{\theta} := (x_t, y_t, \phi_t)\), irrespective of how many sources are present.

With two sources, the ML estimator in (30) is now

\[
\hat{\theta}_{\text{ML}} = \arg \max_{\theta} \left[ P(z_n^{(1)}|\tilde{\theta}) \cdot P(z_n^{(2)}|\tilde{\theta}) \right].
\]

(37)

Compared with (30), the search in (37) is done in a three-dimensional space, rather than four. This leads to complexity reduction due to the added reflection model information.

We next specify the joint CRB with the reflection model imposed.

A. CRB with the mirror reflection model

With the unknowns in \(\hat{\theta} = (x_t, y_t, \phi_t)\), the information matrix \(\tilde{J}\) is of size \(3 \times 3\). As the information matrix is additive, we have:

\[
\tilde{J}_{\text{joint}} = \tilde{J}^{(1)} + \tilde{J}^{(2)},
\]

(38)

where \(\tilde{J}^{(1)}\) and \(\tilde{J}^{(2)}\) corresponds to the information matrix for source 1 and 2, respectively.

We now compute \(\tilde{J}^{(1)}\) separately following the steps in Section IV. To simplify the notation, we omit the superscript in \(\tilde{J}^{(i)}\). Recall that we are evaluating the information matrices one at a time; based on (36), we first to change \(f_2(\psi_n, \alpha)\) to \(f_2(\psi_n, \psi_{s,t}, \phi_t)\) as

\[
f_2(\psi_n, \psi_{s,t}, \phi_t) = \frac{1}{1 + \left(\frac{\psi_n + \psi_{s,t} - 2\phi_t}{\psi_n}ight)^2}.
\]

(39)

The information matrix has the \((i, j)\)th entry as

\[
[J]_{i,j} = \sum_{n=1}^{N} \left( \frac{1}{P_{D,n}} + \frac{1}{1 - P_{D,n}} \right) \frac{\partial P_{D,n}}{\partial \theta_i} \frac{\partial P_{D,n}}{\partial \theta_j}.
\]

(40)

where

\[
\frac{\partial P_{D,n}}{\partial \theta_i} = \frac{c_0}{\Gamma_{\text{th}} P_{D,n}} P_{D,n}(\ln P_{D,n})^2 \left[ \frac{\partial f_1(r_n)}{\partial \theta_i} f_2(\psi_n, \psi_{s,t}, \phi_t) + f_1(r_n) \frac{\partial f_2(\psi_n, \psi_{s,t}, \phi_t)}{\partial \theta_i} \right].
\]

(41)

The partial derivative of \(f_1(r_n)/\partial \theta_i\) is available in (22). The partial derivative of \(f_2(\cdot)\) is

\[
\frac{\partial f_2(\psi_n, \psi_{s,t}, \phi_t)}{\partial \theta_i} = -\frac{2K}{W^2 \psi_n} f_2^2(\psi_n, \psi_{s,t}, \phi_t) \cdot (\psi_n + \psi_{s,t} - 2\phi_t)^{2K-1} \left( \frac{\partial \psi_n}{\partial \theta_i} + \frac{\partial \psi_{s,t}}{\partial \theta_i} - 2 \frac{\partial \phi_t}{\partial \theta_i} \right).
\]

(42)

Now we specify the derivatives with respect to each \(\hat{\theta}_i\). Eqs. (24)–(27) have already provided \(\partial \hat{r}_n/\partial \theta_i, \partial \hat{r}_n/\partial y_t, \partial \psi_n/\partial x_t, \partial \psi_n/\partial \theta_i\). Other necessary partial derivatives are:

\[
\frac{\partial \psi_{s,t}}{\partial x_t} = \frac{y_s - y_t}{(x_s - x_t)^2 + (y_s - y_t)^2},
\]

(43)

\[
\frac{\partial \psi_{s,t}}{\partial y_t} = \frac{x_s - x_t}{(x_s - x_t)^2 + (y_s - y_t)^2},
\]

(44)

\[
\frac{\partial \hat{r}_n}{\partial \theta_i} = 0, \frac{\partial \hat{r}_n}{\partial \phi_t} = 0, \frac{\partial \psi_{s,t}}{\partial \theta_i} = 0,
\]

(45)

\[
\frac{\partial \phi_t}{\partial \theta_i} = 0, \frac{\partial \phi_t}{\partial y_t} = 0, \frac{\partial \phi_t}{\partial \phi_t} = 1.
\]

(46)

After computing \(\tilde{J}^{(1)}\) and \(\tilde{J}^{(2)}\), we obtain \(\tilde{J}_{\text{joint}}\) in (38). The CRB lower bound is simply \(\tilde{J}_{\text{joint}}^{-1}\). With the additional information provided by the mirror reflection model, one may reasonably expect that \(\tilde{J}_{\text{joint}}\) dictate better estimation for the target location than \(\tilde{J}_{\text{joint}}\) in (31). We will compare the CRBs with and without the mirror reflection model numerically.

Remark 1: The reflection model helps to reduce the number of unknowns, thus improving the estimation accuracy in a multi-static setting. On the other hand, we have verified numerically that if we have one source, then treating \(\phi_t\) as unknown or treating \(\alpha\) as unknown does not make a difference on the estimation accuracy of the source location. We verified this conclusion numerically. The reason is that for any \(\psi_{s,t}\), we can find a \((\alpha, \phi_t)\) pair that satisfies (36), which implies that the reflection model is just a linear transformation between \(\phi_t\) and \(\alpha\).

Remark 2: Although imposing a reflection model helps to improve the localization accuracy, the estimator in Section V that has no model has its own value. First, the reflection model may not match exactly the practical situation. Second, the target orientation may change for multiple sources, especially when the sources work in a time-division multiplexed fashion (e.g., a moving source).
B. When is the target visible?

In a conventional sonar setup, a target can be detected only when the receiver is in the propagation zone of the return wave. Assuming that the propagation angle is uniformly distributed within $[-\pi/2, \pi/2]$, and the beamwidth of the reflected transmission is $B$, the probability of detecting the target is $B/\pi$. This probability could be small if $B$ is small.

We now examine the visibility issue in our sensor-network based approach. For simplicity, we use the mirror reflection model in (36), and assume that the submarine orientation is random. A target is visible to a source if the reflected wave originating from this source passes through the sensor field. Depending on the position of the target and the sensor field, the target is visible only when the propagation direction is in a particular range. Let us denote the interval as $(\alpha_{\text{low}}^{(i)}, \alpha_{\text{high}}^{(i)})$ for the $i$th source. For example, if the target is inside the sensor field, then the reflected wave always passes through a particular zone of sensors, such that $\alpha_{\text{low}}^{(i)} = -\pi$ and $\alpha_{\text{high}}^{(i)} = \pi$. In this case, the target is always visible. If the target is not inside the sensor field, but the sensor field is infinitely long, then we have $\alpha_{\text{low}}^{(i)} = -\pi/2$, $\alpha_{\text{high}}^{(i)} = \pi/2$, a situation as depicted in Fig. 2.

For a given source-target geometry, the target is visible to source $i$ (i.e., $\alpha_i \in (\alpha_{\text{low}}^{(i)}, \alpha_{\text{high}}^{(i)})$) only when the target orientation satisfies

$$0.5\psi_{s,t}^{(i)} + 0.5\alpha_{\text{low}}^{(i)} < \phi_t < 0.5\psi_{s,t}^{(i)} + 0.5\alpha_{\text{high}}^{(i)}.$$  \hspace{1cm} (47)

If the system has $Q$ sources, then the target is visible to all sources simultaneously when its orientation belongs to the intersection of the individual visible regions, i.e.,

$$\phi_t \in \bigcap_{i=1}^{Q} \left(0.5\psi_{s,t}^{(i)} + 0.5\alpha_{\text{low}}^{(i)}, 0.5\psi_{s,t}^{(i)} + 0.5\alpha_{\text{high}}^{(i)}\right).$$ \hspace{1cm} (48)

On the other hand, the target is visible to at least one source when the orientation belongs to the union of the individual visible regions as

$$\phi_t \in \bigcup_{i=1}^{Q} \left(0.5\psi_{s,t}^{(i)} + 0.5\alpha_{\text{low}}^{(i)}, 0.5\psi_{s,t}^{(i)} + 0.5\alpha_{\text{high}}^{(i)}\right).$$ \hspace{1cm} (49)

We next calculate the visible regions for one example setup.

Example 1: We consider two sources, located at $(0,0)$ and $(2000,0)$, with an infinitely long sensor field in the $x$ direction. We assume a moving target that is always outside the sensor field. With the target moving from $(0,1000)$ to $(2000,1000)$ along a straight line, we plot in Fig. 5 the visible orientation in degrees corresponding to these two sources.

The intersection of two visible regions has a minimum span of $45^\circ$. Hence, in this setup, the probability of observation by two sources simultaneously is $25\%$, as the target orientation is in the interval of $(-\pi/2, \pi/2)$. The probability of observation by at least one source is much higher (about $70\%$).

VII. NUMERICAL RESULTS

In this section, we present some numerical results. We consider a rectangular sensor field confined in the region $\{(x,y)\mid 200 \leq x \leq 2200, 0 \leq y \leq 500\}$, with an area of $1.2 \text{ km}^2$. We consider three different sensor placement scenarios. The first is a regular sensor field with sensors uniformly spaced both horizontally and vertically on the grid of $(i \cdot r_{\text{min}}, 50 + j \cdot r_{\text{min}})$, where $i,j$ are integers and $r_{\text{min}}$...
is the minimum distance between sensors. With $r_{\text{min}} = 100$, the regular sensor field has 5 rows and 25 columns of sensors, as depicted in Fig. 6. The sensor density corresponding to $r_{\text{min}} = 100$ is 104.2 units-per-km$^2$. With $r_{\text{min}} = 50$, the regular sensor field has 10 rows and 49 columns, with the density of 408.3 units-per-km$^2$. The second scenario has a random sensor field with sensors randomly placed inside the region with a uniform distribution. However, the number of sensors is pre-determined. The third option is a Poisson sensor field, which differs from the random sensor field in that the number of sensors is randomly chosen following a Poisson distribution for different realizations. A realization of random and Poisson sensor fields is shown in Fig. 7.

We set the false alarm rate to $P_{\text{fa}} = 0.01$, which decides the threshold $\Gamma_{\text{th}}$. We define the constant $c_0$ through $c_0 = r_{\text{ref}} \cdot \text{SNR}$, such that the specified SNR is achieved at the reference distance $r_{\text{ref}}$. We set $r_{\text{ref}} = 2000$ in our tests.

A. Single source, regular sensor field

We first consider a regular sensor field with a single source placed at the position $(0, 0)$. Unless otherwise specified, we assume a target at $(1000, 1000)$, and set $r_{\text{min}} = 100$, SNR=20dB, $K = 4$, $W = 5\pi/180$. Thus the 3dB beamwidth of the Butterworth filter $(2W)$ is 10 degrees.

Test Case 1: How much data should we collect? Due to practical constraints, the survey region contains only a finite number of sensors. It is interesting to investigate how the estimation accuracy depends on the size of the data set.

We assume a rectangular data collection window (or survey region) as depicted in Fig. 8. This region covers five rows of sensor within a horizontal window of length $L_{\text{win}}$, hence the detection results from a total of $5L_{\text{win}}$ sensors are used for estimation. Fig. 8 illustrates one realization of the illuminated sensor field with $\alpha = \pi/6$ (or $30^\circ$), where the stars stand for $z_n = 1$, and the circles stand for $z_n = 0$. The data collection window is properly centered around sensors with $z_n = 1$. With $L_{\text{win}} = 8$, the CRB ellipse is also plotted in Fig. 8. It shows that a good estimate of the target position is indeed feasible using simple sensors with only detection capabilities.

We now change the length of the data collection window. Fig. 9 shows that using a very large window size does not add much information; we have similar observations with other DOA values. This confirms the intuition that only sensors within and near the detection zone contribute to target localization. This result is encouraging, as the data collection unit only needs to survey a small region of interest. In our following numerical testings, we will use $L_{\text{win}} = 8$ unless specified otherwise.

Test Case 2: Maximum likelihood estimator. In this test, we assume that all the parameters $c_0, W, K$ are exactly known at the receiver side for the ML estimator. Fig. 10 plots the CRB ellipse with $\alpha = \pi/6$. In addition, we plot the ML estimates for 100 Monte-Carlo trials. We see that the CRB ellipse matches well with the ML estimates.

Test Case 3: CRB dependence on parameters. We now
check the CRB for different parameter values. To simplify the plot, we use the root mean square error (RMSE) as the performance measure, defined as
\[
\text{RMSE} = \sqrt{E[(\hat{x}_t - x_t)^2 + (\hat{y}_t - y_t)^2]}.
\] (50)

The CRB specifies a lower bound on RMSE as
\[
\sqrt{J^{-1}_{1,1} + J^{-1}_{2,2}}.
\]

With SNR=20dB and \(K = 4\), Fig. 11 shows the CRB dependence on the propagation angle, where different propagation beamwidths are used. We observe that the CRB fluctuates as the propagation angle changes. When the propagation beamwidth is small, e.g. \(W = 5\pi/180\), very poor estimation accuracy shows up for certain angles due to the discrete nature of the sensor field. As the beamwidth increases, the dependence on the propagation angle tends to decrease. Fig. 12 shows the CRB dependence on the SNR. This plot shows that increasing SNR does not necessarily translate to the estimation accuracy improvement when the SNR is above a certain threshold. Intuitively, once the SNR is “high”, all the relevant sensors will detect, so further gain in SNR is not effective.

An intuitive explanation of the behavior in Figs. 11 and 12 is that the estimation accuracy is mainly decided by the minimum sensor distance. We increase the sensor density by 4 and 16 times, by reducing the sensor distance to \(r_{\text{min}} = 50\) and \(r_{\text{min}} = 25\), respectively. The results are also shown in Figs. 11 and 12. First, we see that the CRB decreases considerably. Second, the dependence on angle and SNR is much reduced. Therefore, the sensor field density is one key factor that greatly affects the performance. This is intuitively sound because our localization task is carried out based on discrete “detection” results.

**Test Case 4: ML performance with parameter mismatch.** The ML estimation in (10) assumes the exact knowledge of the model parameters \(c_0, K, W\). These parameters have to be estimated in practice subject to a certain estimation accuracy. We now test the performance degradation of ML estimation with mismatched parameters. The true parameters are SNR=20dB, \(K = 4\), \(W = 7.5\pi/180\). We test both \(r_{\text{min}} = 100\) and \(r_{\text{min}} = 50\).

We first fix the target at \((1000, 1000)\) and \(\alpha = 30\pi/180\). Each time, we change one parameter to deviate from the true value when using the ML estimator. With 100 Monte-Carlo runs based on a data collection window of \(L_{\text{win}} = 10\), the biases of \(\hat{x}_t, \hat{y}_t\) and the RMSE are listed in Table I for ML estimators with various mismatched parameters. To remove the dependence of our results on a particular source-target-receiver geometry, we have also tested two other setups. One is to keep \(\alpha = \pi/6\), but with the target uniformly distributed over \(x_t, y_t \in [900, 1100]\) for each Monte-Carlo trial. The other

<table>
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<tr>
<th>(x_t) bias</th>
<th>(y_t) bias</th>
<th>RMSE</th>
<th>(x_t) bias</th>
<th>(y_t) bias</th>
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<td>88.2</td>
</tr>
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</table>

**TABLE I**

**THE ML ESTIMATOR WITH VARIOUS MISMATCHED PARAMETERS**
is to fix the target at (1000, 1000), but with the orientation $\alpha$ uniformly distributed within $[0, \pi/6]$. The results are omitted due to lack of space.

We observe that the estimation performance is most sensitive to $W$ (the “beamwidth” of the target return), among all parameters we tested. Hence, an accurate estimation of $W$ is required. The reason is that the sensors are sparsely distributed, and a large bias might show up when a zone with inaccurate width is formed to fit the detection results. This is more evident when $W$ is under-estimated than over-estimated.

**Test Case 5: More complex aspect dependence.** Given the previous observation, it seems appropriate to test using an aspect-angle return-strength dependence model that is more complex than the Butterworth pattern in (7). As shown in Fig. 13, the “butterfly” pattern of [17] is less sharp than the Butterworth pattern. (It may need a mixture of Butterworth (or Gaussian) patterns to approximate a butterfly pattern; we use an aspect-angle return-strength dependence model that is more complex than the Butterworth pattern in (7).) As shown in Fig. 13, the “butterfly” pattern of [17] is less sharp than the Butterworth pattern. (It may need a mixture of Butterworth (or Gaussian) patterns to approximate a butterfly pattern; we use a Butterworth (or Gaussian) pattern to approximate a butterfly pattern; we leave such a issue for future work.) At SNR=20dB, $\alpha = \pi/6$, we plot 60 ML estimates with the butterfly pattern in Fig. 14, where the detection results from all sensors in the network are used. As expected, estimation with the butterfly pattern shows some loss of fidelity with respect to the “searchlight” Butterworth pattern of Fig. 10; however, the results are promising even in this unfavorable situation. We note that the pattern of “fit” sensors may also be used for target recognition.

**B. Single source, regular/random/Poisson sensor fields**

We now compare ML estimation results for regular, random, and Poisson sensor fields (with the Butterworth propagation pattern). For the regular sensor field, we test both $r_{\min} = 100$ and $r_{\min} = 50$. For each setup of the regular sensor field, we establish the random and Poisson sensor fields with the same density, one deterministically and the other on average. The survey region corresponds to a rectangular region of size 800 × 500. The results are obtained with 100 Monte-Carlo runs, where the target is uniformly distributed with $x_t, y_t \in [900, 1100]$. The propagation angle is fixed as $\alpha = \pi/6$. Other parameters are SNR=20 dB, $K = 4$, and $W = 7.5\pi/180$.

The ML estimation results are listed in Table II. We observe that regular sensor field leads to better average performance than random and Poisson sensor fields, which is expected. Presumably the latter two are more realistic, however.

**C. Multiple sources, regular sensor field**

In this subsection, we consider a multi-static setting with two sources at (0, 0) and (2000, 0). The sensor field is the one depicted in Fig. 6. The data collection window has size $L_{\text{win}} = 8$. Other parameters are SNR=20 dB, $K = 4$, and $W = 7.5\pi/180$.

We assume a target at (1000, 1000) with orientation $\phi_t = 0$. This leads to $\alpha_1 = \pi/4$ and $\alpha_2 = -\pi/4$. Fig. 15 compares four different CRBs: the CRB with source 1 only, the CRB with source 2 only, the joint CRB without the reflection model, and the joint CRB with the reflection model. First, we observe that multi-static setting considerably improves the estimation accuracy relative to the single source case. Not unexpectedly, incorporating the reflection model explicitly leads to better estimation accuracy relative to the counterpart without the model. Notice that the improvement is mainly on the y coordinate in this setting.

With multiple sources, the line fitting based estimator can yield a source location estimate. Fig. 16 shows one realization of the line fitting results. We now compare the performance of the optimal ML estimator of (37) with the suboptimal estimator based on line fitting. The results based on 100 Monte-Carlo simulations are shown in Table III. We observe that the suboptimal estimator has a large bias in the y estimate, while the x estimate is very accurate in this setting.

**VIII. CONCLUSIONS**

In this paper, we proposed a submarine localization paradigm that relies on a network of low-complexity “detection only” sensors, instead of the traditional setup with a few complex sensors. Our sensor network based approach
can effectively deal with low-visibility targets that can reflect the return wave along a certain direction within a narrow beamwidth. We proposed the optimal ML estimator and a suboptimal estimator based on line fitting. We analyzed the CRB in both single source and multi-source settings. Our numerical results verified the feasibility of target location with a network of “detection only” sensors. It is found that the density of the sensor field and the knowledge of the beamwidth are important factors that determine the estimation accuracy.

Our work herein has only relied on detection results, which can be viewed as one-bit quantization on the received signal strengths. It is of great interest to study how the estimation accuracy approaches to the counterpart with analog data as the number of quantization bits increases; this can be pursued along the lines of [11]. Also, in practice the system has only noisy sensor position estimates. Sensitivity with respect to sensor position inaccuracy warrants further investigation.

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