Estimation Bias Tutorial

These slides are a subset from a Dissertation Presentation

Converted Measurement Trackers for Systems
with
Nonlinear Measurement Functions

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• $r$ is known (1000m)
• $\alpha_i$ is a normal random variable with mean 45 deg and standard deviation 5 deg ($\sigma_\alpha^2 = 0.0076$)
• N samples ($i=1\ldots N$)
• Examine the conversion $x_i = r \cos \alpha_i$

![Graph illustrating the conversion $x_i = r \cos \alpha_i$. The graph shows a scatter plot of sampled points $(r \cos \alpha_i, r \sin \alpha_i)$ for $i=1\ldots N$. The points are shown with a line connecting them, indicating the relationship between $r$, $\alpha_i$, and $x_i$. The coordinates $(1000 \cos 45, 1000 \sin 45)$ are marked on the graph.]
Conversion bias is when the expected value of the conversion does not equal the truth.

\[ E[1000 \cos(\alpha)] \neq 1000 \cos(45) \]
• In order to use a converted measurement in an estimator (e.g. a Kalman filter or LLSE), an estimate of the converted measurement error variance is needed
  – Often requires the truth to calculate
  – Traditionally, in practice, it is based on the measurement
  – In the example, an approximate variance is:

\[
\sigma_i^2 \approx r^2 \sigma_\alpha^2 \sin^2 \alpha_i
\]

– What happens when we use this in an estimator?

\[
\hat{x} = \frac{\sum_i r \cos(\alpha_i) / \sigma_i^2}{\sum_i 1 / \sigma_i^2}
\]
If the converted error covariance is based on the **measurement** the estimator will wrongly “trust” some measurements more than others.

Weighted low

Weighted high
• Results in a bias in the estimate (even if the conversion of the measurements is unbiased)

\[1000 \cos(45)\]

\[E[\hat{x}]\]