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AN ELASTIC CONTACT MECHANICS FRACTURE FLOW MODEL

Guillermo E. Franco [1]
Amvrossios C. Bagtzoglou [2], Member, ASCE

ABSTRACT

The study of flow through fractured rock is a challenging research topic. The inherent complexity of the geometry of natural surfaces in contact, the lack of knowledge regarding flow behavior in a small and extremely irregular space and the deformation of the confining solids, make for a difficult landscape of study. In this paper, a classic theory in the area of contact mechanics, first applied to optics and lenses in contact by Hertz, is used to obtain a simple model of a deforming fracture. The model accounts for elastic deformations of the solids in contact and assumes laminar flow in the created voids. Results are presented and compared to experimental tests carried out by and presented in Gale (1975) .

Keywords: Fracture flow, Contact mechanics

FUNDAMENTALS OF HERTZ'S CONTACT MECHANICS THEORY

Hertz formulated his basic theory on Contact Mechanics in the early 1880s, when he studied the distortion of images due to the elastic deformation of lenses in contact. In this work we apply this classic theory to study the deformation of a fracture and its effect on flow.

The fracture is modeled as two rectangular arrays of micro hemispheres, which are set in contact, simulating the porous fracture, as seen in figure 1. Pressure is applied on these layers and the deformation is obtained from Hertz's contact mechanics theory. Each of the hemisphere-pairs (one from the upper and one from the lower layer) in contact are a particular case of Hertz's study (Johnson 1985). Given the normal force applied on each pair (see figure 2), the contact radius and the vertical deformation can be computed from

$$a = \sqrt[3]{\frac{3PR(1-\nu^2)}{4E}} \qquad d = \sqrt[3]{\frac{9P^2(1-\nu^2)}{2E^2R}}, \tag{1}$$

where P is the vertical load applied over the pair in contact, E and ν are the characteristic Young's Modulus and Poisson Ratio of the material, respectively, and R is the radius of the hemispheres.

The force applied over each pair of hemispheres is obtained from

$$P = \frac{P_{Total}}{\left(\frac{w}{2R}\right)\left(\frac{L}{2R}\right)}, \tag{2}$$

[1]Dept. of Civ. Eng. and Eng. Mech., Columbia University, 634A S.W. Mudd Bldg., New York, NY 10027. E-mail: franco@civil.columbia.edu

[2]Dept. of Civ. Eng. and Eng. Mech., Columbia University, 626 S.W. Mudd Bldg., New York, NY 10027. E-mail: abagtzog@civil.columbia.edu

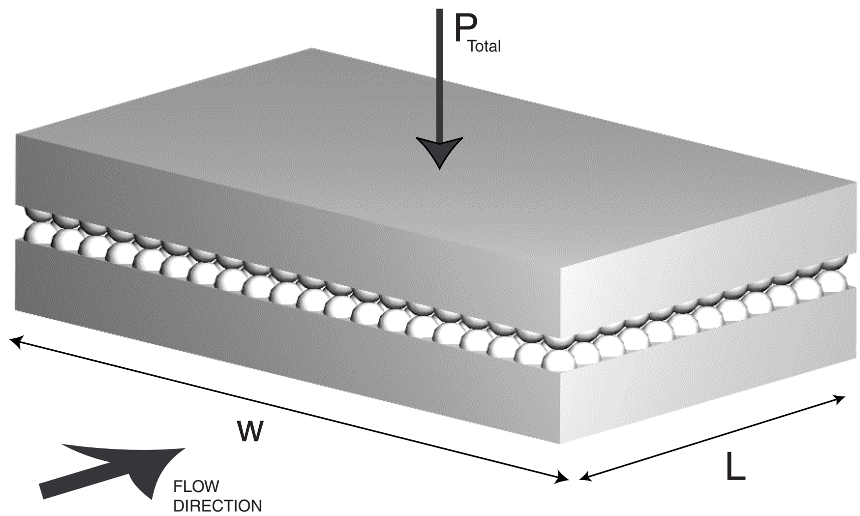

FIG. 1. Fracture geometric model.

where w and L are the width and depth of the contact respectively. Due to the discrete nature of the number of possible contacts, each term in the denominator of equation (2) must be set equal to the nearest integer.

This theory holds under three basic assumptions: 1) Surfaces are continuous and non-conforming, 2) Strains are small, and 3) Each solid can be considered an elastic half-space. The first holds true from the choice of geometry, that is continuous and non-conforming hemispheres. The last two hold naturally if $a << R$, which is a limitation to the model presented here, that is, the interface cannot be deformed further than up to a certain maximum displacement d, associated with a maximum contact radius a.

The natural material involved in these experiments, like rock, is not smooth and frictionless. As a first approximation and in the absence of lateral stresses, frictionless surfaces are assumed. Hertz's theory can be further developed to account for rough surfaces, although it is not an objective of this first approach to model the surfaces in such detail.

HYDRO-MECHANICAL MODEL

The application of normal load to a fractured specimen leads to the closure of the interface, which constraints the flow in the interior voids. It is generally accepted that the cubic law is not applicable in natural fractured rock, due to the irregular configuration of the surfaces in contact. Some studies (Louis 1969) introduce a roughness factor into the cubic law to account for this. However, at this point of the modeling stage, applicability of the cubic law is accepted.

Many efforts have been invested to model the imperfection of the surfaces at a smaller scale in order to attain a better description of the flow network inside of the fracture. The complex void geometry leads to a great difficulty calculating the flow in the voids. Therefore, it seems reasonable to consider a compromise between accuracy of geometry description and simplicity of flow computation.

The array proposed is far away from an actual configuration since the radii of all the hemispheres are considered equal and the attained compactness is minimal. It is, however, a first attempt to employ a simple theory of contact mechanics and is in line with similar attempts

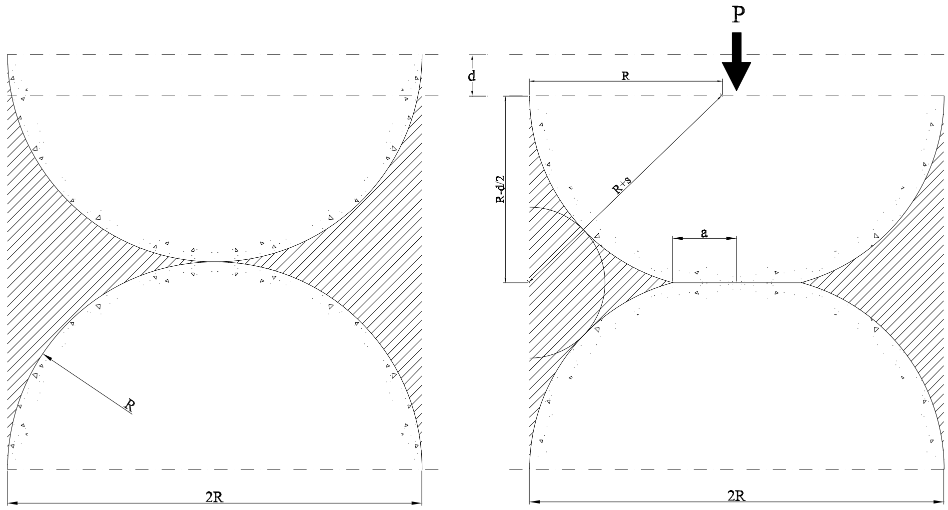

FIG. 2. Typical Compression Scheme.

in the fields of soil mechanics and erosion protection (Cundall and Strack 1979; Gotoh et al. 1996; Worman and Olafsdottir 1992). The configuration of the voids, suggests the search for an equivalent prismatic aperture for application of the cubic law.

The deformation of the array of hemispheres in contact leads to a shrinkage of the voids inside the matrix. The deformed shape of the voids can be easily approached considering two losses of volume, one due to the vertical displacement d, the other due to the formation of a contact disk on each hemisphere's tip. The deformed void volume within a cell of eight hemispheres can be expressed as:

$$V_{Def} = (2R - d)(2R)^2 - \frac{4\pi}{3}\sqrt{(R^2 - a^2)^3} - 2\pi a^2 \sqrt{R^2 - a^2}. \tag{3}$$

Once the deformed void volume has been obtained, the cubic law can be applied assuming laminar flow

$$Q_{Cell} = C b_e^3 \Delta h, \tag{4}$$

where the effective aperture, b_e, is obtained from a geometric similarity, maintaining the total void volume and assuming a prismatic flow tube of section $b_e \times 2R$ and length $2R$ as

$$b_e = \frac{V_{Def}}{4R^2}. \tag{5}$$

The constant present in the cubic law corresponds to flow between parallel plates and is given by

$$C = \frac{b_e}{L}\left(\frac{\gamma}{12\mu}\right). \tag{6}$$

Multiplying Q_{Cell} by the number of cells along the fracture axis, an approximation of the fracture flow-through is obtained from

$$Q_{Total} = Q_{Cell}\left(\frac{w}{2R} - 1\right). \tag{7}$$

The choice of b_e preserves the volume of flow, but it is probably an overstatement of the actual equivalent aperture, since flow will exhibit stagnation regions and associated energy losses inside the complex fracture volume.

An alternate choice of flow geometry is the inscribed tube of radius s between the deformed hemispheres, as seen in figure 2, in which case, the constant C in the cubic law is given by

$$C = \frac{s}{L}\left(\frac{\gamma}{8\mu}\right), \tag{8}$$

where s is obtained from simple geometrical relationships as:

$$s = \sqrt{\left(R - \frac{d}{2}\right)^2 + R^2} - R. \tag{9}$$

Numerical results show that the choice of this reduced volume implies a higher value of radius of the hemispheres to simulate the experimental results. This might be an indication of an understatement of the actual flow volume, but could provide a lower limit calculation. However, all numerical results presented in this paper are obtained considering a rectangular section flow with conservancy of the total void volume.

The two geometries mentioned could, therefore, be interpreted as two limit cases of flow geometry. A detailed description of the flow behavior inside such a complex volume is beyond the scope of this study and is currently a major research topic in the field of fracture flow.

NUMERICAL RESULTS

Gale (1975) published results on his investigation on fracture flow through a granite core. These results, often referred to in the literature, have been used to design a numerical experiment to compare numerical results obtained using our model.

Gale's original experiment was conducted with a sawed cylindrical granite core. Due to the geometry considered for a simple model of hemispheres in contact, a rectangular version of that problem will be studied here, maintaining the flow external perimeter, area and material properties as constant. Two rectangular granite blocks, $3.03m$ wide (w) and $0.24m$ deep (L), are placed in contact, with a preset difference in pressure Δh of $276kPa$, considering horizontal flow, between the inflow and the outflow perimeters. A Young's Modulus of $10,335MPa$ is assumed for granite with a Poison's ratio of 0.2. The fluid used is water with a specific weight of $9.81Nm^{-3}$ and a viscosity of $1.005 \times 10^{-3}Nsm^{-2}$. The results[1] obtained for different hemisphere radius values are shown in figure 3. Radii vary between $100\mu m$ and $250\mu m$.

In the high range of applied pressure, it can be seen that the values for radii around $200\mu m$ approach the experimental results by Gale quite nicely. However, in the low range of pressure, the model is not able to reproduce the experimental results. This is a clear consequence of the fact that according to the model, from the beginning of the numerical experiments, the granite surfaces are supposed to be in perfect contact, whereas reality is different. During the first part of Gale's experiment, in contrast to the proposed model, both surfaces undergo an adaptation process by crushing small grains in between and gradually achieving a high level of

[1] Units used in the graphs presented herein are expressed in the Imperial system for ease of comparison with Gale's published results, although the international system is used throughout the rest of the paper.

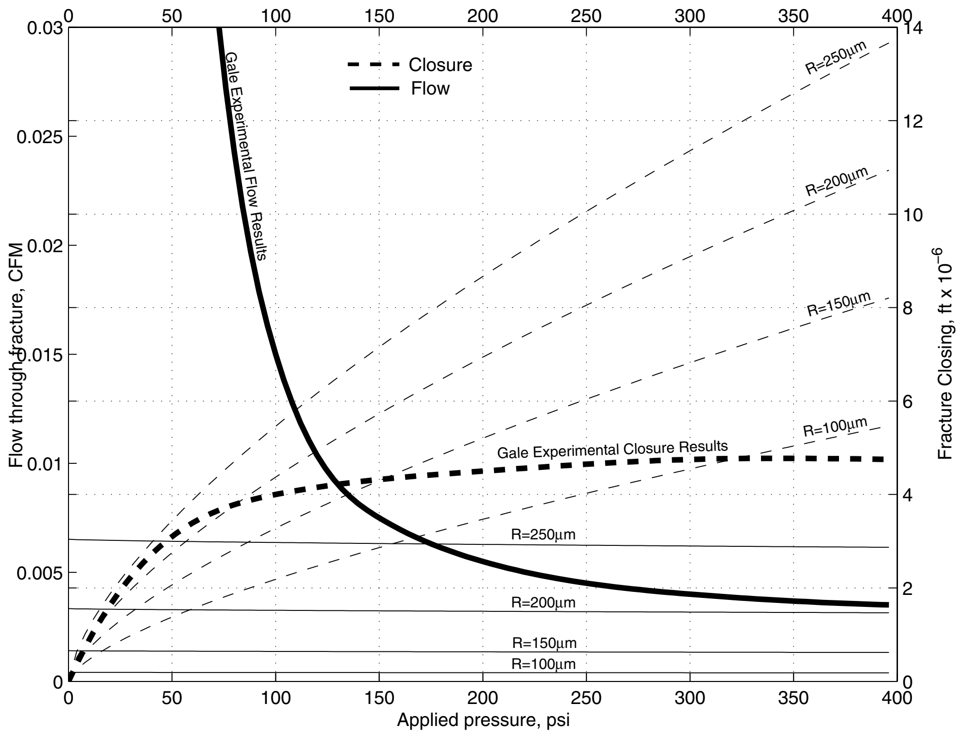

FIG. 3. Flow and fracture closing as a function of applied pressure and hemisphere radius for Young's Modulus of 10,335Mpa

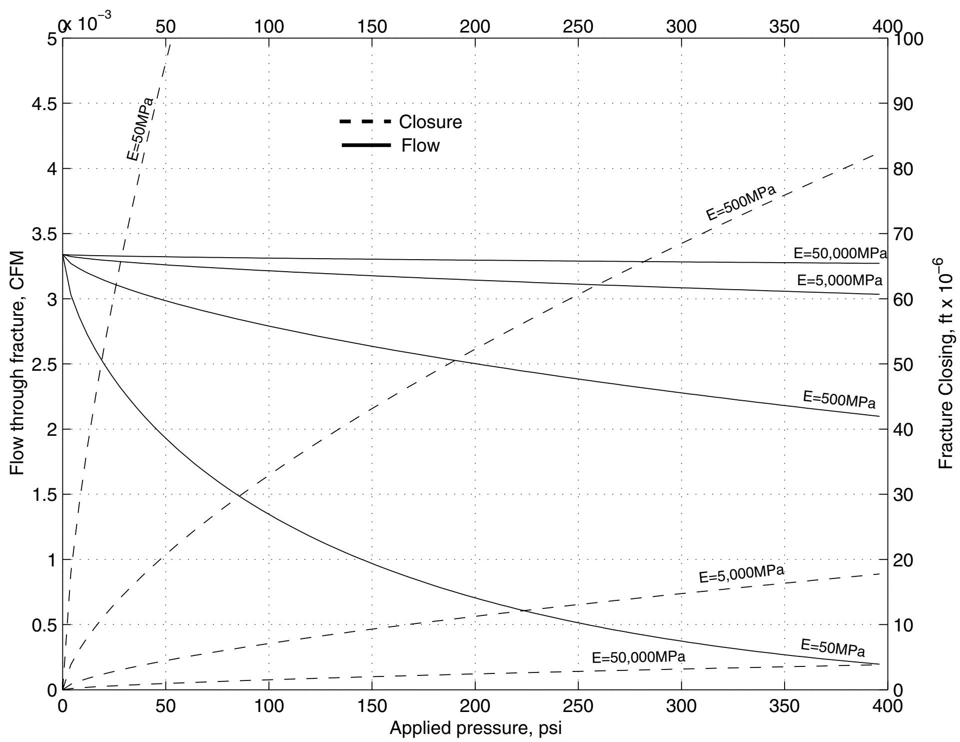

FIG. 4. Flow and fracture closing as a function of applied pressure and Young's Modulus for a hemisphere radius of $200\mu m$

compactness. This is also evident by observing how quickly the experimental closure results deviate from the numerical for pressures greater than 10psi (figure 3).

It is also interesting to observe in figure 4 how flow and fracture closing are affected by the choice of the elastic properties of the material. If the behavior of the first phase of crushing for the small grains could be described as a phase characterized by a reduction in Young's Modulus, it is apparent that flow changes significantly. If Young's Modulus were considered a function of applied pressure, it would be possible to obtain more accurate results, adapting the fracture elastic properties to the crushing process. Although, it seems clear that the ultimate limit would be the actual material Young's Modulus, it is left for further study to identify what reduction should be considered at the beginning of the experiment and what would be the change law. It is hypothesized that this could be an exponential function of the smoothness of the surfaces in contact.

Aperture values appear in a reasonable range at low pressure values, diverging towards higher pressure values from experimental results. This is due to the elastic nature of the model. In fact, normal Hertzian contact mechanics theory ceases to be valid for high values of applied load. The contact radius cannot grow beyond a certain threshold without violating the main hypotheses presented in the first section of this paper. An elastic-plastic model can be developed to capture this effect probably leading to more accurate results in the higher pressure range. Some approaches to consider plastic deformation of bodies in contact are presented in Johnson's work (Johnson 1985).

The accuracy of the model, presented here, is measured in terms of the choice of the initial fracture aperture, that is the hemisphere radius. The choice of a hemisphere radius of $200\mu m$ implies a fracture aperture of $0.4mm$ for Gale's experimental results with granite. Comparing to field studies (Caine and Forster 1999), these values seem somewhat high, but still in very reasonable range.

CONCLUSIONS

A simple model of fracture flow has been presented. Surfaces in contact have been modeled as two layers of hemispheres, arranged in a rectangular array. Once the void within this geometry has been calculated, a cubic law of flow has been assumed to model the movement of the fluid in a volume-preserving prismatic geometry. Numerical experiments have been carried out to observe the evolution of flow under applied pressure on the surfaces in contact. The results, although not accurate enough with this simple formulation, show that this model could be of use as a first estimating tool. Values of flow, aperture closing and initial fracture aperture are in line with those values obtained in actual experiments or in the field.

Some changes are proposed on this model, which could yield better results. One obvious development is to account for plastic behavior of the material, at high pressure ranges. A second proposed development is to study the evolution of the elastic properties of the material as a mean to model the first phases of the load application, without having to change the regular layout of the hemispheres.

Many models are being developed that are trying to account for flow in a complex geometry, determined by surface deformation. It was the objective of the authors to use a not so commonly applied theory of contact mechanics to give some insight into alternate ways of modeling the contact surfaces.

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NOTATION

The following symbols are used in this paper:

a	=	contact radius (m);
b_e	=	effective aperture for flow between parallel plates (m);
C	=	cubic law geometry coefficient ($m^{-1}s^{-1}$);
d	=	fracture closure (m);
E	=	Young's elastic Modulus of material in contact (MPa);
ν	=	Poison's ratio (1);
γ	=	specific weight of fluid (kgm^{-3});
μ	=	viscosity of fluid (Nsm^{-2});
s	=	radius of inscribed tube of flow (m);
w	=	width of fracture (perpendicular to flow direction) (m);
L	=	depth of fracture (along flow direction) (m);
P_{Total}	=	total normal force applied on fracture (N);
P	=	normal force applied on each hemisphere pair (N);
Q_{Total}	=	total flow through fracture (m^3s^{-1});
Q_{Cell}	=	flow through one of the 8-hemispheres cell (m^3s^{-1});
Δh	=	difference in hydraulic head (m);
V_{Def}	=	volume of deformed void space (m^3);