For $n = 2, 3, 4, \ldots$, let $H(n)$ denote the $n \times n$ Hilbert matrix.

Let $m$ denote a natural number.

Create an algorithm to do matrix inversion.

a. For $n = 3, 4, 5$, produce an exact inverse of each Hilbert matrix, $H(n)$. Show that their product is the identity.

b. For values that you choose for $m$, round the entries of each of the above Hilbert matrices to have $m$ entries to the right of the decimal point. Run your implementation on the modified data and show the resultant products.

c. For each of the matrices created by b., create a measure for error on the resultant product, justifying your choice of the error measure.

d. How is the error related to both $n$ and $m$? Develop a comprehensive conjecture and provide both experimental evidence and an analysis to support that conjecture. Present a formal proof of your conjecture, showing how it applies to any values for $n$ and $m$. 