Two Equal Infinite Sets  
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Abstract  

An example was presented on 1/16/17, where this formalizes that discussion.

1 Two Infinite Sets  

Let $X = \{x \in \mathbb{R} : \exists r \in \mathbb{R}, x = r^2 \}$ and let $Y = \{y \in \mathbb{R} : y \geq 0 \}$.

Show $X = Y$  

First, show $X \subset Y$.

Let $x \in X$. Since $x = r^2$ for some $r \in \mathbb{R}$, then $r^2 \in \mathbb{R}$ and $r^2 \geq 0$, so $x \in Y$.

Second, show $Y \subset X$.

Let $y \in Y$, so $y \in \mathbb{R}$ and $y \geq 0$. Every non-negative real number has a real square root. Let $\sqrt{y}$ denote the real square root of $y$. But, then $y = (\sqrt{y})^2$, so $y \in X$.

Therefore, $X = Y$. 