

# VLSI Design Verification and Testing

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## Built-In Self-Test (BIST)

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## Overview

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- **Motivation and economics**
- **Definitions**
- ***Built-in self-testing (BIST) process***
- ***BIST pattern generation (PG)***
- ***BIST response compaction (RC)***
- ***Aliasing* definition and example**
- **Summary**

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## BIST Motivation

- **Useful for field test and diagnosis (less expensive than a local automatic test equipment)**
- **Software tests for field test and diagnosis:**
  - **Low hardware fault coverage**
  - **Low diagnostic resolution**
  - **Slow to operate**
- **Hardware BIST benefits:**
  - **Lower system test effort**
  - **Improved system maintenance and repair**
  - **Improved component repair**
  - **Better diagnosis at component level**

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## Costly Test Problems Alleviated by BIST

- **Increasing chip logic-to-pin ratio – harder observability**
- **Increasingly dense devices and faster clocks**
- **Increasing test generation and application times**
- **Increasing size of test vectors stored in ATE**
- **Expensive ATE needed for GHz clocking chips**
- **Hard testability insertion – designers unfamiliar with gate-level logic, since they design at behavioral level**
- **Shortage of test engineers**
- **Circuit testing cannot be easily partitioned**

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## Benefits and Costs of BIST with DFT

Level	Design and test	Fabri-cation	Manuf. Test	Maintenance test	Diagnosis and repair	Service interruption
<b>Chips</b>	+ / -	+	-			
<b>Boards</b>	+ / -	+	-		-	
<b>System</b>	+ / -	+	-	-	-	-

- + Cost increase**
- Cost saving**
- +/- Cost increase may balance cost reduction**

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## Economics – BIST Costs

- **Chip area overhead for:**
  - Test controller
  - Hardware pattern generator
  - Hardware response compacter
  - Testing of BIST hardware
- **Pin overhead -- At least 1 pin needed to activate BIST operation**
- **Performance overhead – extra path delays due to BIST**
- **Yield loss – due to increased chip area or more chips In system because of BIST**
- **Reliability reduction – due to increased area**
- **Increased BIST hardware complexity – happens when BIST hardware is made testable**

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## BIST Benefits

- **Faults tested:**
  - Single combinational / sequential stuck-at faults
  - Delay faults
  - Single stuck-at faults in BIST hardware
- **BIST benefits**
  - Reduced testing and maintenance cost
  - Lower test generation cost
  - Reduced storage / maintenance of test patterns
  - Simpler and less expensive ATE
  - Can test many units in parallel
  - Shorter test application times
  - Can test at functional system speed

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## Definitions

- **BILBO** – *Built-in logic block observer*, extra hardware added to flip-flops so they can be reconfigured as an LFSR pattern generator or response compacter, a scan chain, or as flip-flops
- **Concurrent testing** – Testing process that detects faults during normal system operation
- **CUT** – *Circuit-under-test*
- **Exhaustive testing** – Apply all possible  $2^n$  patterns to a circuit with  $n$  inputs
- **LFSR** – *Linear feedback shift register*, hardware that generates pseudo-random pattern sequence

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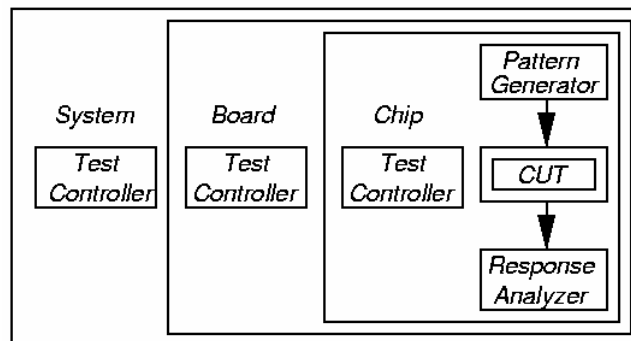
## More Definitions

- **Primitive polynomial** – Boolean polynomial  $p(x)$  that can be used to compute increasing powers  $n$  of  $x^n$  modulo  $p(x)$  to obtain all possible non-zero polynomials of degree less than  $p(x)$
- **Pseudo-exhaustive testing** – Break circuit into small, overlapping blocks and test each exhaustively
- **Pseudo-random testing** – Algorithmic pattern generator that produces a subset of all possible tests with most of the properties of randomly-generated patterns
- **Signature** – Any statistical circuit property distinguishing between bad and good circuits
- **TPG** – Hardware *test pattern generator*

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## BIST Process

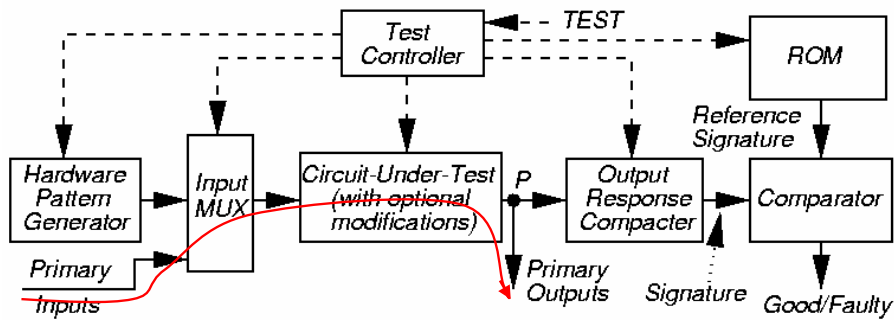


- **Test controller** – Hardware that activates self-test simultaneously on all PCBs
- Each board controller activates parallel chip BIST Diagnosis effective only if very high fault coverage

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## BIST Architecture

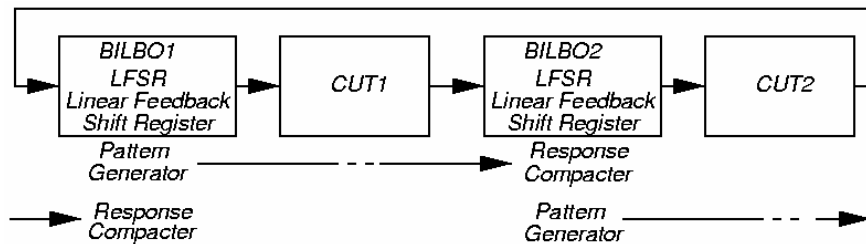


- **Note: BIST cannot test wires and transistors:**
  - From PI pins to Input MUX
  - From POs to output pins

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## BILBO – Works as Both a TPG and a RC

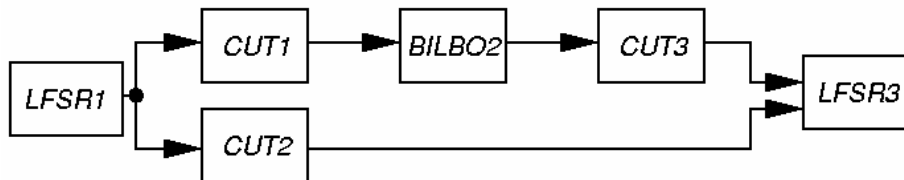


- **Built-in Logic Block Observer (BILBO) -- 4 modes:**
  1. Flip-flop
  2. LFSR pattern generator
  3. LFSR response compacter
  4. Scan chain for flip-flops

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## Complex BIST Architecture

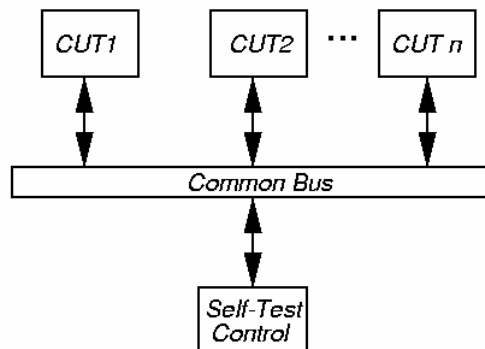


- **Testing epoch I:**
  - LFSR1 generates tests for CUT1 and CUT2
  - BILBO2 (LFSR3) compacts CUT1 (CUT2)
- **Testing epoch II:**
  - BILBO2 generates test patterns for CUT3
  - LFSR3 compacts CUT3 response

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## Bus-Based BIST Architecture



- **Self-test control** broadcasts patterns to each CUT over bus – parallel pattern generation
- Awaits bus transactions showing CUT's responses to the patterns: serialized compaction

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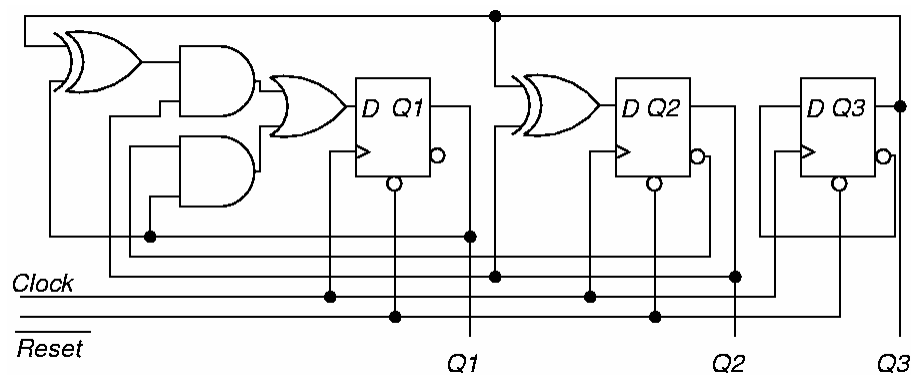
## Pattern Generation

- Store in ROM – too expensive
- *Exhaustive*
- *Pseudo-exhaustive*
- *Pseudo-random* (LFSR) – Preferred method
- Binary counters – use more hardware than LFSR
- Modified counters
- Test pattern *augmentation*
  - LFSR combined with a few patterns in ROM
  - *Hardware diffracter* – generates pattern cluster in neighborhood of pattern stored in ROM

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## Exhaustive Pattern Generation (A Counter)



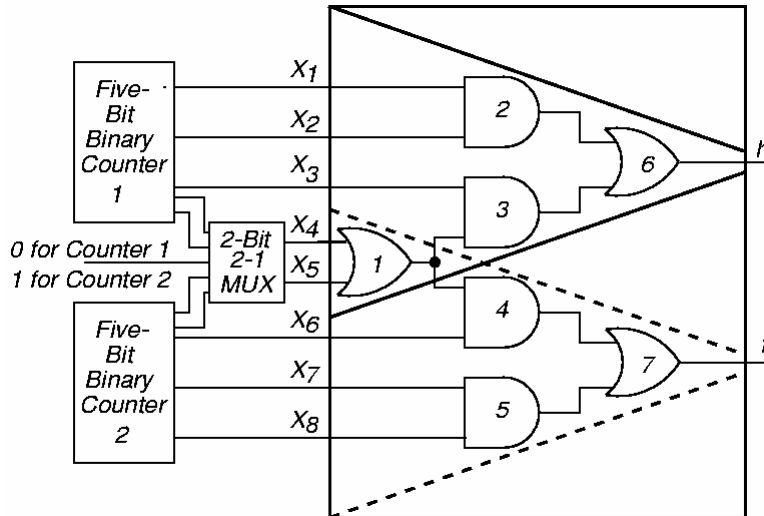
- Shows that every state and transition works
- For  $n$ -input circuits, requires all  $2^n$  vectors
- Impractical for large  $n$  ( $> 20$ )

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# Pseudo-Exhaustive Pattern Generation

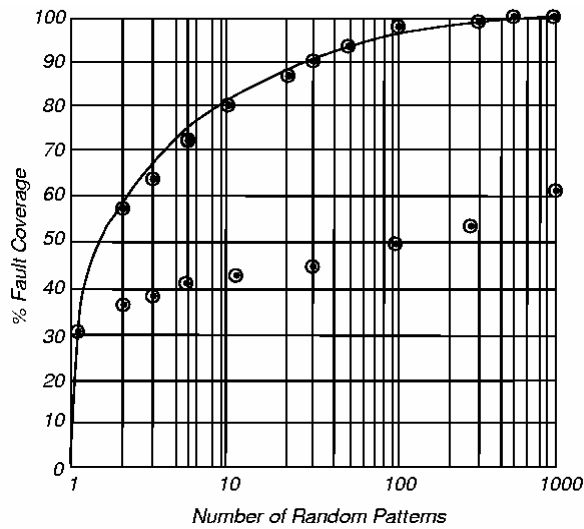


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# Random Pattern Testing

**Bottom:  
Random-  
Pattern  
Resistant  
circuit**

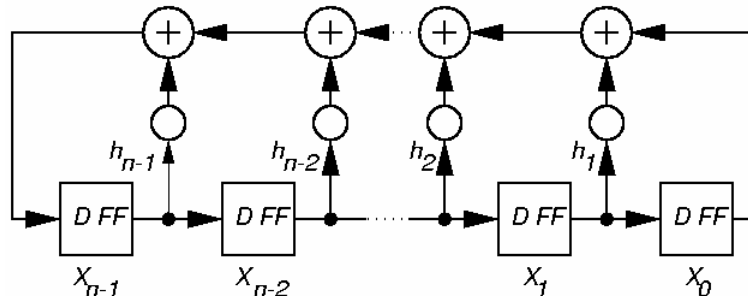


(a) Top curve -- random pattern testing with acceptable fault coverage.  
(b) Bottom curve -- unacceptable random pattern testing.

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## Pseudo-Random Pattern Generation



- **Standard Linear Feedback Shift Register (LFSR)**
  - Normally known as *External XOR* type LFSR
  - Produces patterns algorithmically – repeatable
  - Has most of desirable random # properties
- Need not cover all  $2^n$  input combinations
- Long sequences needed for good fault coverage

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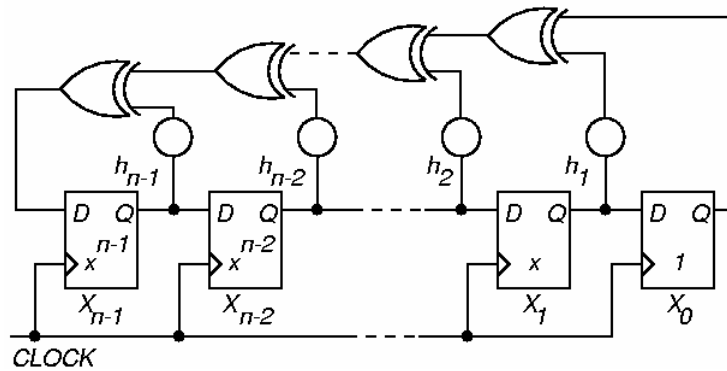
## Theory: LFSRs

- **Galois field (mathematical system):**
  - Multiplication by  $x$  same as right shift of LFSR
  - Addition operator is XOR ( $\oplus$ )
- **$T_s$  companion matrix for a standard (external XOR type) LFSR:**
  - 1<sup>st</sup> column 0, except  $n$ th element which is always 1 ( $X_0$  always feeds  $X_{n-1}$ )
  - Rest of row  $n$  – feedback coefficients  $h_i$
  - Rest is identity matrix  $I$  – means a right shift
- **Near-exhaustive (maximal length) LFSR**
  - Cycles through  $2^n - 1$  states (excluding all-0)
  - 1 pattern of  $n$  1's, one of  $n-1$  consecutive 0's

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## Standard $n$ -Stage LFSR



- If  $h_j = 0$ , that XOR gate is deleted

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## Matrix Equation for Standard LFSR

$$\begin{bmatrix} X_0(t+1) \\ X_1(t+1) \\ \vdots \\ X_{n-3}(t+1) \\ X_{n-2}(t+1) \\ X_{n-1}(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 1 & h_1 & h_2 & \dots & h_{n-2} & h_{n-1} \end{bmatrix} \begin{bmatrix} X_0(t) \\ X_1(t) \\ \vdots \\ X_{n-3}(t) \\ X_{n-2}(t) \\ X_{n-1}(t) \end{bmatrix}$$

$$\mathbf{X}(t+1) = \mathbf{T}_s \mathbf{X}(t) \quad (\mathbf{T}_s \text{ is companion matrix})$$

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## LFSR Theory (contd.)

- Cannot initialize to all 0's – hangs
- If  $X$  is initial state, progresses through states  $X, T_S X, T_S^2 X, T_S^3 X, \dots$
- **Matrix period:**  
 Smallest  $k$  such that  $T_S^k = I$ 
  - $k \equiv$  LFSR cycle length
- Described by characteristic polynomial:  

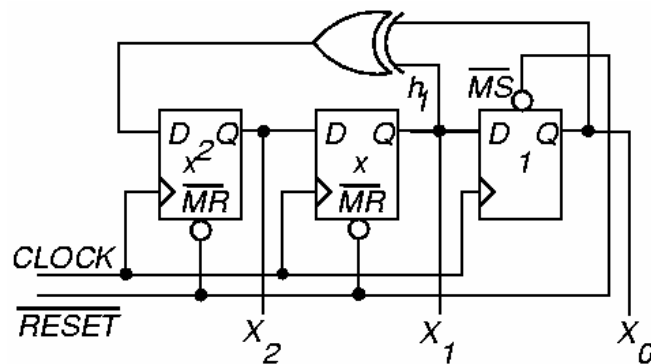
$$f(x) = |T_S - IX|$$

$$= 1 + h_1 x + h_2 x^2 + \dots + h_{n-1} x^{n-1} + x^n$$

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## Example External XOR LFSR



$$F(x) = 1 + x + x^3$$

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## Example: External XOR LFSR (contd.)

- Matrix equation:

$$\begin{bmatrix} X_0(t+1) \\ X_1(t+1) \\ X_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_0(t) \\ X_1(t) \\ X_2(t) \end{bmatrix}$$

- Companion matrix:

$$T_s = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- Characteristic polynomial:

- $f(x) = 1 + x + x^3$   
(read taps from right to left)

- Always have **1** and  $x^n$  terms in polynomial

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## External XOR LFSR

- Pattern sequence for example LFSR (earlier):

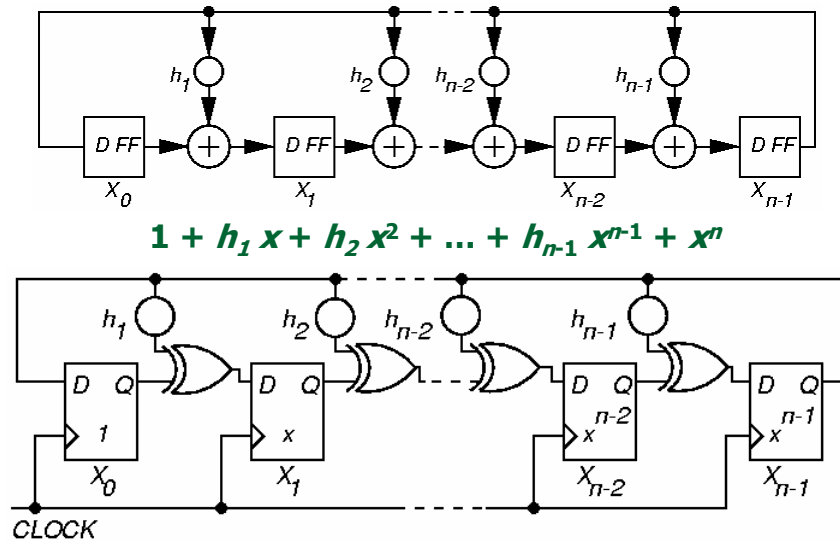
$$\begin{array}{l|cccccccc} X_0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ X_1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & \dots \\ X_2 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}$$

- Never repeat an LFSR pattern more than **1** time –Repeats same error vector, cancels fault effect

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## Generic Modular (Internal XOR) LFSR



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## Modular Internal XOR LFSR

- Described by *companion matrix*  $T_m = T_s^T$
- Internal XOR LFSR – XOR gates in between D flip-flops
- Equivalent to standard External XOR LFSR
  - With a different state assignment
  - Faster – usually does not matter
  - Same amount of hardware
- $X(t+1) = T_m \times X(t)$
- $f(x) = |T_m - IX|$   
 $= 1 + h_1 x + h_2 x^2 + \dots + h_{n-1} x^{n-1} + x^n$
- Right shift – equivalent to multiplying by  $x$ , and then dividing by characteristic polynomial and storing the remainder

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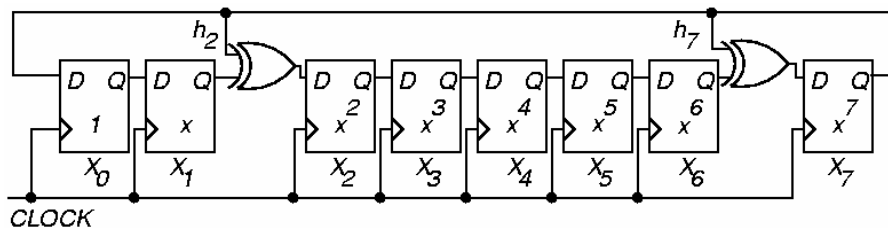
## Modular LFSR Matrix

$$\begin{bmatrix} X_0(t+1) \\ X_1(t+1) \\ X_2(t+1) \\ \vdots \\ X_{n-3}(t+1) \\ X_{n-2}(t+1) \\ X_{n-1}(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 1 \\ 1 & 0 & 0 & \dots & 0 & 0 & h_1 \\ 0 & 1 & 0 & \dots & 0 & 0 & h_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & h_{n-3} \\ 0 & 0 & 0 & \dots & 1 & 0 & h_{n-2} \\ 0 & 0 & 0 & \dots & 0 & 1 & h_{n-1} \end{bmatrix} \begin{bmatrix} X_0(t) \\ X_1(t) \\ X_2(t) \\ \vdots \\ X_{n-3}(t) \\ X_{n-2}(t) \\ X_{n-1}(t) \end{bmatrix}$$

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## Example Modular LFSR



- $f(x) = 1 + x^2 + x^7 + x^8$
- Read LFSR tap coefficients from left to right

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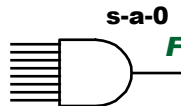
## Primitive Polynomials

- Want LFSR to generate all possible  $2^n - 1$  patterns (except the all-0 pattern)
- Conditions for this – must have a *primitive polynomial*:
  - *Monic* – coefficient of  $x^n$  term must be 1
    - Modular LFSR – all D FF's must right shift through XOR's from  $X_0$  through  $X_1, \dots$ , through  $X_{n-1}$ , which must feed back directly to  $X_0$
    - Standard LFSR – all D FF's must right shift directly from  $X_{n-1}$  through  $X_{n-2}, \dots$ , through  $X_0$ , which must feed back into  $X_{n-1}$  through XORing feedback network

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## Weighted Pseudo-Random Pattern Generation



- If  $p(1)$  at all PIs is 0.5,  $p_F(1) = 0.5^8 = \frac{1}{256}$ 

$$p_F(0) = 1 - \frac{1}{256} = \frac{255}{256}$$
- Will need enormous # of random patterns to test a stuck-at 0 fault on  $F$ -- LFSR  $p(1) = 0.5$ 
  - We must not use an ordinary LFSR to test this
- IBM – holds patents on weighted pseudo-random pattern generator in ATE

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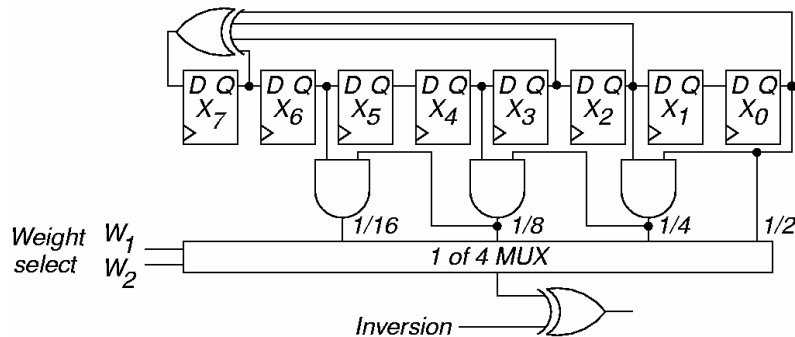
## Weighted Pseudo-Random Pattern Generator

- **LFSR  $p(1) = 0.5$**
- **Solution:**
  - Add programmable weight selection and complement LFSR bits to get  $p(1)$ 's other than **0.5**
- **Need 2-3 weight sets for a typical circuit**
- **Weighted pattern generator drastically shortens pattern length for pseudo-random patterns**

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## Weighted Pattern Gen.



$w_1$	$w_2$	Inv.	$p$ (output)	$w_1$	$w_2$	Inv.	$p$ (output)
0	0	0	$\frac{1}{2}$	1	0	0	$\frac{1}{8}$
0	0	1	$\frac{1}{2}$	1	0	1	$\frac{7}{8}$
0	1	0	$\frac{1}{4}$	1	1	0	$\frac{1}{16}$
0	1	1	$\frac{3}{4}$	1	1	1	$\frac{15}{16}$

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## Test Pattern Augmentation

- **Secondary ROM – to get LFSR to 100% SAF coverage**
  - Add a small ROM with missing test patterns
  - Add extra circuit mode to *Input MUX* – shift to ROM patterns after LFSR done
  - Important to compact extra test patterns
- **Use *diffracter*:**
  - Generates cluster of patterns in neighborhood of stored ROM pattern
- **Transform LFSR patterns into new vector set**
- **Put LFSR and transformation hardware in full-scan chain**

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## Response Compaction

- **Severe amounts of data in CUT response to LFSR patterns – example:**
  - Generate 5 million random patterns
  - CUT has 200 outputs
  - Leads to: **5 million x 200 = 1 billion bits** response
- **Uneconomical to store and check all of these responses on chip**
- **Responses must be compacted**

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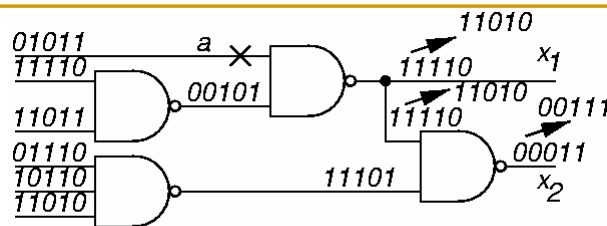
## Definitions

- **Aliasing** – Due to information loss, signatures of good and some bad machines match
- **Compaction** – Drastically reduce # bits in original circuit response – lose information
- **Compression** – Reduce # bits in original *circuit response* – *no information loss* – *fully invertible* (can get back original response)
- **Signature analysis** – Compact good machine response into *good machine signature*. Actual signature generated during testing, and compared with good machine signature
- **Transition Count Response Compaction** – Count # transitions from 0 → 1 and 1 → 0 as a signature

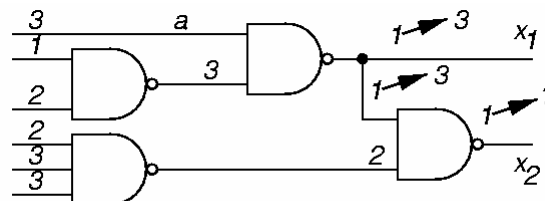
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## Transition Counting



(a) Logic simulation of good machine and fault a stuck-at-1.



(b) Transition counts of good and failing machines.

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## Transition Counting Details

- **Transition count:**

$$C(R) = \sum_{i=1}^m (r_i \oplus r_{i-1}) \text{ for all } m \text{ primary outputs}$$

- **To maximize fault coverage:**
  - **Make  $C(R0)$  – good machine transition count – as large or as small as possible**

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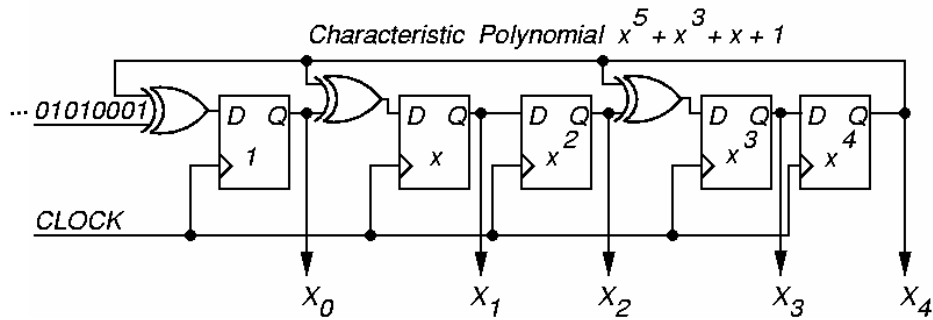
## LFSR for Response Compaction

- Use *cyclic redundancy check code* (CRCC) generator (LFSR) for response compacter
- Treat data bits from circuit POs to be compacted as a decreasing order coefficient polynomial
- CRCC divides the PO polynomial by its characteristic polynomial
  - Leaves remainder of division in LFSR
  - Must initialize LFSR to *seed value* (usually 0) before testing
- After testing – compare signature in LFSR to known good machine signature
- **Critical: Must compute good machine signature**

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## Example Modular LFSR Response Compacter



- LFSR seed value is "00000"

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## Polynomial Division

	<b>Inputs</b>	$x^0$	$x^1$	$x^2$	$x^3$	$x^4$
	<b>Initial State</b>	0	0	0	0	0
	1	1	0	0	0	0
	0	0	1	0	0	0
<b>Logic</b>	0	0	0	1	0	0
<b>Simulation:</b>	0	0	0	0	1	0
	1	1	0	0	0	1
	0	1	0	0	1	0
	1	1	1	0	0	1
	0	1	0	1	1	0

Logic simulation: **Remainder** =  $1 + x^2 + x^3$

0 1 0 1 0 0 0 1

$0 \cdot x^0 + 1 \cdot x^1 + 0 \cdot x^2 + 1 \cdot x^3 + 0 \cdot x^4 + 0 \cdot x^5 + 0$

$x^6 + 1 \cdot x^7$

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## Symbolic Polynomial Division

$$\begin{array}{r}
 x^5 + x^3 + x + 1 \quad \overline{) \quad x^7 \quad + \quad x^3 \quad + \quad x} \\
 \underline{x^7 + x^5 + x^3 + x^2} \phantom{+ x} \\
 x^5 \phantom{+ x^3} + x^2 + x \\
 \underline{x^5 + x^3 \phantom{+ x^2} + x + 1} \\
 x^3 + x^2 + 1
 \end{array}$$

**remainder** →

**Remainder matches that from logic simulation of the response compacter!**

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## Multiple-Input Signature Register (MISR)

- **Problem with ordinary LFSR response compacter:**
  - Too much hardware if one of these is put on each *primary output (PO)*
- **Solution: MISR – compacts all outputs into one LFSR**
  - Works because LFSR is linear – obeys *superposition principle*
  - Superimpose all responses in one LFSR – final remainder is XOR sum of remainders of polynomial divisions of each PO by the characteristic polynomial

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## MISR Matrix Equation

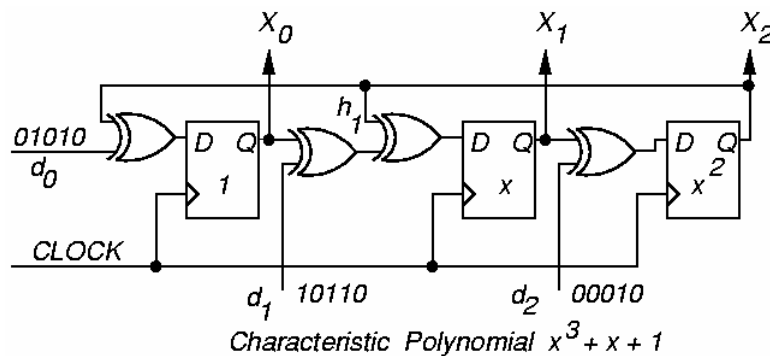
- $d_i(t)$  – output response on  $PO_i$  at time  $t$

$$\begin{bmatrix} X_0(t+1) \\ X_1(t+1) \\ \vdots \\ X_{n-3}(t+1) \\ X_{n-2}(t+1) \\ X_{n-1}(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \\ 1 & h_1 & \dots & h_{n-2} & h_{n-1} \end{bmatrix} \begin{bmatrix} X_0(t) \\ X_1(t) \\ \vdots \\ X_{n-3}(t) \\ X_{n-2}(t) \\ X_{n-1}(t) \end{bmatrix} + \begin{bmatrix} d_0(t) \\ d_1(t) \\ \vdots \\ d_{n-3}(t) \\ d_{n-2}(t) \\ d_{n-1}(t) \end{bmatrix}$$

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## Modular MISR Example



$$\begin{bmatrix} X_0(t+1) \\ X_1(t+1) \\ X_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_0(t) \\ X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} d_0(t) \\ d_1(t) \\ d_2(t) \end{bmatrix}$$

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## Multiple Signature Checking

- **Use 2 different testing epochs:**
  - 1<sup>st</sup> with MISR with 1 polynomial
  - 2<sup>nd</sup> with MISR with different polynomial
- **Reduces probability of aliasing –**
  - Very unlikely that both polynomials will alias for the same fault
- **Low hardware cost:**
  - A few XOR gates for the 2<sup>nd</sup> MISR polynomial
  - A 2-1 MUX to select between two feedback polynomials

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## Aliasing Probability

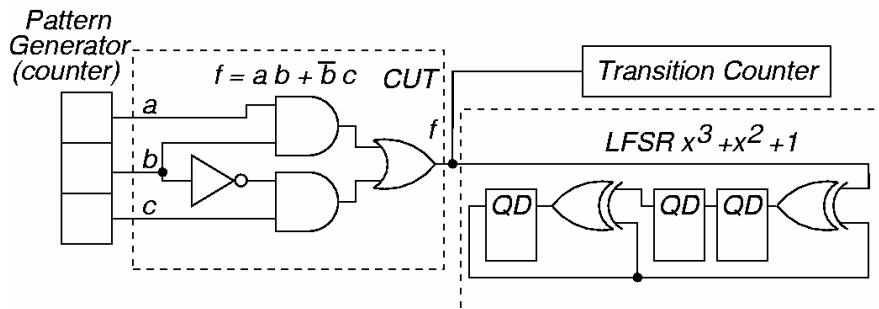
- **Aliasing – when bad machine signature equals good machine signature**
- **Aliasing:  $1/2^n$**
- **Consider error vector  $e(n)$  at POs**
  - Set to a 1 when good and faulty machines differ at the PO at time  $t$
- **$P_{al} \equiv$  aliasing probability**
- **$p \equiv$  probability of 1 in  $e(n)$**
- **Aliasing limits:**
  - $0 < p \leq 1/2, p^k \leq P_{al} \leq (1-p)^k$
  - $1/2 \leq p \leq 1, (1-p)^k \leq P_{al} \leq p^k$

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## Experiment Hardware



- **3 bit exhaustive binary counter for pattern generator**

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## Transition Counting vs. LFSR

- LFSR aliases for *f sa1*, transition counter for *a sa1*

<b>Pattern</b>	<b>Responses</b>			
	<b>Good</b>	<b>a sa1</b>	<b>f sa1</b>	<b>b sa1</b>
<b>000</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>
<b>001</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>0</b>
<b>010</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>0</b>
<b>011</b>	<b>0</b>	<b>1</b>	<b>1</b>	<b>0</b>
<b>100</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>
<b>101</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
<b>110</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
<b>111</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
<b>Signatures</b>				
<b>Transition Count</b>	<b>3</b>	<b>3</b>	<b>0</b>	<b>1</b>
<b>LFSR</b>	<b>001</b>	<b>101</b>	<b>001</b>	<b>010</b>

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## Summary

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- **LFSR pattern generator and MISR response compacter – preferred BIST methods**
  - **BIST has overheads: test controller, extra circuit delay, Input MUX, pattern generator, response compacter, DFT to initialize circuit & test the test hardware**
  - **BIST benefits:**
    - **At-speed testing for delay & stuck-at faults**
    - **Drastic ATE cost reduction**
    - **Field test capability**
    - **Faster diagnosis during system test**
    - **Less effort to design testing process**
    - **Shorter test application times**
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## Appendix

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## LFSR Fault Coverage Projection

- **Fault detection probability by a random number**  
 $p(x) dx$  = fraction of detectable faults with detection probability between  $x$  and  $x + dx$ 
  - $p(x) dx \geq 0$  when  $0 \leq x \leq 1$
  - $\int_0^1 p(x) dx = 1$
- Exist  $p(x) dx$  faults with detection probability  $x$
- Mean coverage of those faults is  $x p(x) dx$
- Mean fault coverage  $y_n$  of 1<sup>st</sup>  $n$  vectors:  

$$I(n) = \int_0^1 (1-x)^n p(x) dx$$

$$y_n = \frac{1}{n} \int_0^1 (1-x)^n p(x) dx + \frac{1}{n} \quad (15.6)$$

**total faults**

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## LFSR Fault Coverage & Vector Length Estimation

- **Random-fault-detection (RFD) variable:**
  - Vector # at which fault first detected
  - $w_i \equiv$  # faults with RFD variable  $i$
- So  $p(x) = \frac{1}{n_s} \sum_{i=1}^N w_i p_i(x)$
- $n_s$  size of sample simulated:  $\leq N$  # test vectors
- $w_i \leq n_s - \sum_{j=1}^N w_j$
- **Method:**  $i=1$ 
  - Estimate random first detect variables  $w_i$  from fault simulator using fault sampling
  - Estimate  $I(n)$  using book Equation 15.8
  - Obtain test length by inverting Equation 15.6 & solving numerically

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## Primitive Polynomials

- Want LFSR to generate all possible  $2^n - 1$  patterns (except the all-0 pattern)
- Conditions for this – must have a *primitive polynomial*:
  - *Monic* – coefficient of  $x^n$  term must be 1
    - Modular LFSR – all D FF's must right shift through XOR's from  $X_0$  through  $X_1, \dots$ , through  $X_{n-1}$ , which must feed back directly to  $X_0$
    - Standard LFSR – all D FF's must right shift directly from  $X_{n-1}$  through  $X_{n-2}, \dots$ , through  $X_0$ , which must feed back into  $X_{n-1}$  through XORing feedback network

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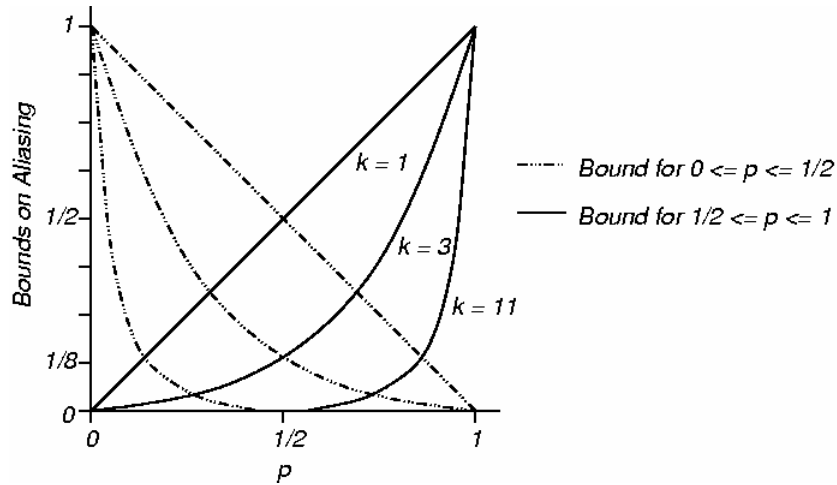
## Primitive Polynomials (continued)

- Characteristic polynomial must divide the polynomial  $1 + x^k$  for  $k = 2^n - 1$ , but not for any smaller  $k$  value
- See Appendix B of book for tables of primitive polynomials
- Following is related to aliasing:
  - If  $p(\text{error}) = 0.5$ , no difference between behavior of primitive & non-primitive polynomial
  - But  $p(\text{error})$  is rarely = 0.5 In that case, non-primitive polynomial LFSR takes much longer to stabilize with random properties than primitive polynomial LFSR

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## Aliasing Probability Graph



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## Additional MISR Aliasing

- **MISR has more aliasing than LFSR on single PO**
  - **Error in CUT output  $d_j$  at  $t_j$ , followed by error in output  $d_{j+h}$  at  $t_{j+h}$ , eliminates any signature error if no feedback tap in MISR between bits  $Q_j$  and  $Q_{j+h}$ .**

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## Aliasing Theorems

- **Theorem 15.1:** Assuming that each circuit PO  $d_{ij}$  has probability  $p$  of being in error, and that all outputs  $d_{ij}$  are independent, in a  $k$ -bit MISR,  $P_{al} = 1/(2^k)$ , regardless of initial condition of MISR. Not exactly true – true in practice.
- **Theorem 15.2:** Assuming that each PO  $d_{ij}$  has probability  $p_j$  of being in error, where the  $p_j$  probabilities are independent, and that all outputs  $d_{ij}$  are independent, in a  $k$ -bit MISR,  $P_{al} = 1/(2^k)$ , regardless of the initial condition.

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