

CAD Algorithms

Steiner Tree Problem

Mohammad Tehranipoor
ECE Department



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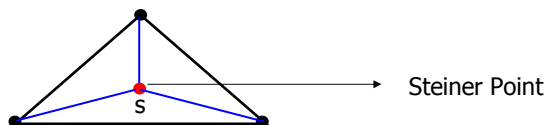
Definition

■ Definition:

- A Steiner tree (named after Jacob Steiner) is a tree in a distance graph which spans a given subset of vertices (Steiner Points) with the minimal total distance on its edges.

■ Steiner Point:

- A point that is not part of the input set of points.

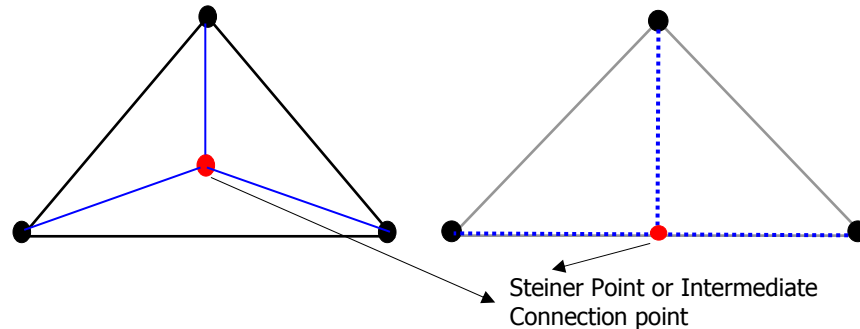


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Rectilinear Steiner Tree

- Rectilinear Steiner Tree over a set of points is a connected collection of vertical and horizontal lines which spans the points.



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Example

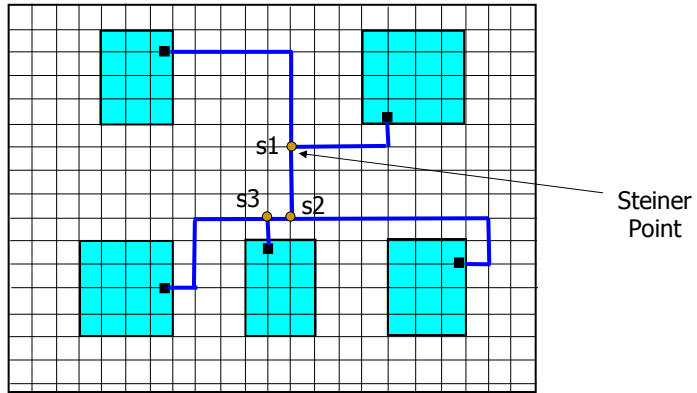
- Steiner tree arises in **network design** and **wiring layout** problems.
 - Suppose we are given a set of sites that must be connected by wires as cheaply as possible. The minimum Steiner tree describes the way to connect them using the smallest amount of wire.
 - Similar problems arise in designing networks of water pipes or heating ducts in buildings.
 - Same in VLSI circuit layout, where we seek to connect a set of terminals under constraints such as material cost, signal propagation time, or reducing capacitance.

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Example (1)

Three Steiner Points



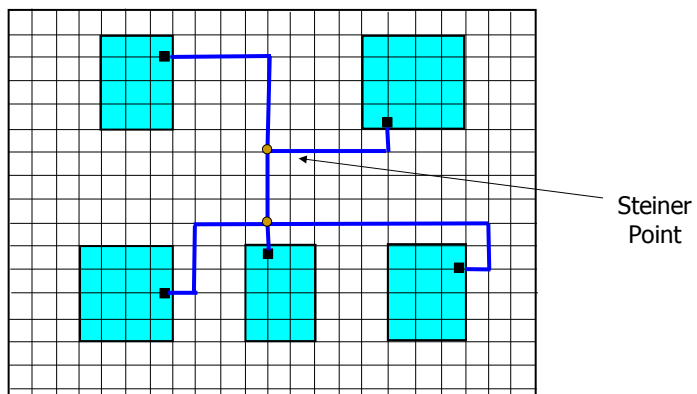
Total Length = 37

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Example (2)

Two Steiner Points



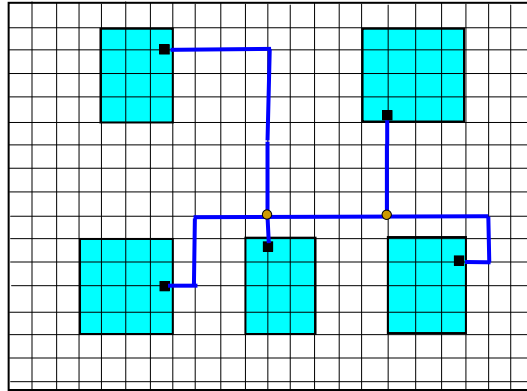
Total Length = 37

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Example (3)

Two Steiner Points



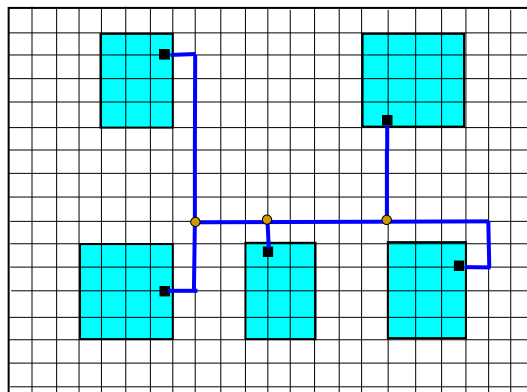
Total Length = 35

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Example (4)

Three Steiner Points



Total Length = 32

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MST vs. Steiner Tree

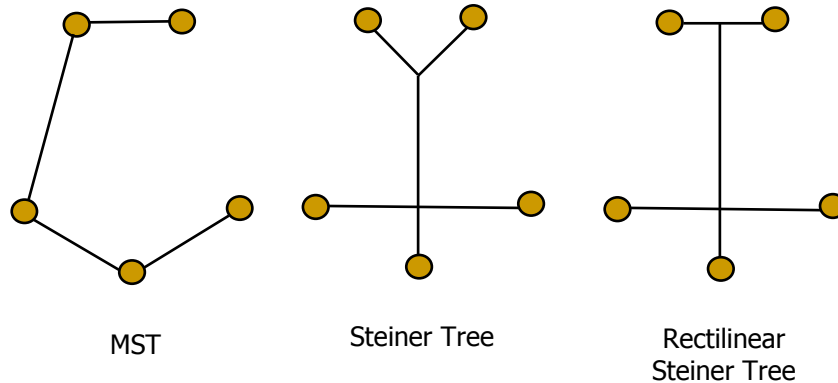
- The Steiner tree problem is distinguished from the minimum spanning tree problem in that we are permitted to construct or select intermediate connection points to reduce the cost of the tree.
- Steiner tree problem is a minimum interconnection problem.

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Example

- MST vs. Steiner



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Algorithms

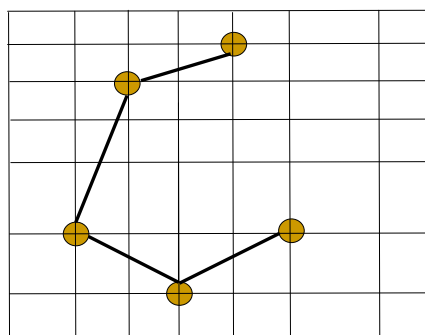
- Steiner tree problem is NP-Complete.
- Certain heuristic algorithms have been designed to approximate the result within polynomial time.
- There are many algorithms that use MST.
- Problem:
 - **Input:** an undirected graph $G=(V,E,d)$ and a set of Steiner Points **S** subset of **V**
 - **Output:** a Steiner Tree **T** for **G** and **S**
(**V** must exist in the final tree, but that is not a required condition for Steiner points (S))

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Using MST

All the points beside the five vertices are considered Steiner points.

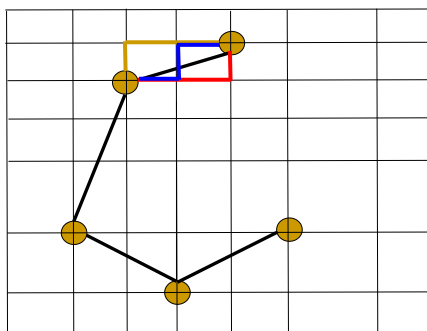


MST

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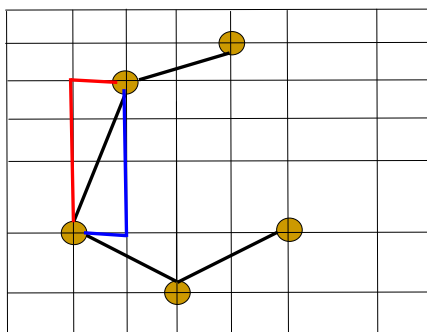
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Steiner Tree



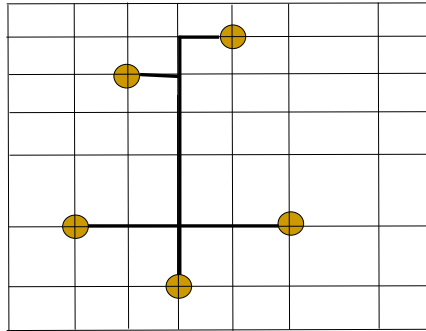
MST

Steiner Tree



MST

Rectilinear Steiner Tree



*****See also example 4.13 (page 114)*****
