Stratification Effect Compensation for Improved Underwater Acoustic Ranging

Christian R. Berger, *Student Member, IEEE,* Shengli Zhou, *Member, IEEE,* Peter Willett, *Fellow, IEEE,* and Lanbo Liu

***Abstract*— Underwater acoustic localization usually relies on time of arrival (ToA) measurements, which are then converted into range estimates. However, the water medium is inhomogeneous and the sound speed varies depending on several parameters, e.g., the temperature, pressure and salinity. As a result, sound waves do not necessarily travel in straight lines. Ignoring this stratification effect could lead to considerable bias in the range estimates. We propose a depth-based approach to compensate the stratification effect for improved underwater ranging. We assume that the sound velocity profile (SVP) is only vertically stratified, the position of the sender is known, and the receiver has a noisy depth estimate via a depth sensor. We find a numerically simple range estimator, based on reconstructing the slanted path using Fermat's Principle and calculus of variations. This estimator removes the bias and is asymptotically efficient. We compare our solution to the simplistic linear estimator that assumes straight-line propagation in a shallow-water example where the sound speed decreases monotonically with depth. We find that the bias of the linear estimator increases with range and is non-negligible when the ToA measurements have a small variance, while our solution is bias-free and meets the Cramér-Rao lower bound (CRLB).**

***Index Terms*— Localization, ranging, sensor networks, underwater acoustic communication, stratification effect.**

I. INTRODUCTION

UNDERWATER communications are mostly based on acoustic transmission, due to the high absorption of radio waves in water. Acoustic waves in water have an about five times higher propagation speed than in air, but about five orders of magnitudes slower than the propagation speed of radio waves. Additionally, water is an inhomogeneous medium, featuring water layers with different temperatures, increasing pressure with depth and location-dependent salinity. All these factors lead to sound speed variations in water.

Localization using time of arrival (ToA) measurements in terrestrial sensor networks has been studied extensively in the signal processing literature [1], [2]; unbiased and efficient estimators for the whole network setup have been developed, whose performance can be accurately characterized in terms of the Cramér-Rao lower bound (CRLB). Since air can be well approximated as a homogeneous media, ToA measurements can be linearly converted to range estimates, which are the basic building blocks for localization in sensor networks. In the context of underwater autonomous networks and deep sea exploration, underwater localization is an important task; see e.g., [3] and references therein. To accomplish precise localization based on ranging via the measurement of sound wave ToA, the inhomogeneity of water as a medium should be considered.

The effects of inhomogeneous media on wave propagation are characterized by the corresponding physical differential equations, which are well understood, e.g, in computational ocean acoustics [4]. Both matched field processing (MFP) and ocean acoustic tomography (OAT) (see [5], [6], [7] and references therein) solve for the complete wave-field as a function of space and time. Solutions are determined by measurements and necessary boundary conditions, like characterization of the ocean floor or the sound velocity profile (SVP). Also, the measurements are usually assumed noiseless, as they are averaged over a time period, and require a vertical sensor array of a minimum sensor density to avoid "spatial aliasing". These methods estimate the whole wave-field. They are computationally intensive and not suited for simple, but unbiased range estimation in underwater sensor networks.

In this writeup, we propose a depth-based solution to compensate the stratification effect for improved underwater acoustic ranging. We adopt several assumptions: 1) the sound velocity is only depth dependent, and the profile is known (an example profile is shown in Fig. 1.(a)); 2) the position of the sender is fixed and known; 3) the receiver has a noisy depth measurement via a depth sensor. Using Fermat's Principle and calculus of variations, we apply the ray-based solution known in computational acoustics, to determine the distance between the sender and the receiver based on the propagation time over a *slanted* path. (The slanted paths corresponding to the SVP in Fig. 1.(a) are shown in Fig. 1.(b)). This way we fully obtain the geometry of the slanted path as a function of depth and using the additional depth measurement, the stratification effects can be evaluated and compensated.

Our contribution is the following:

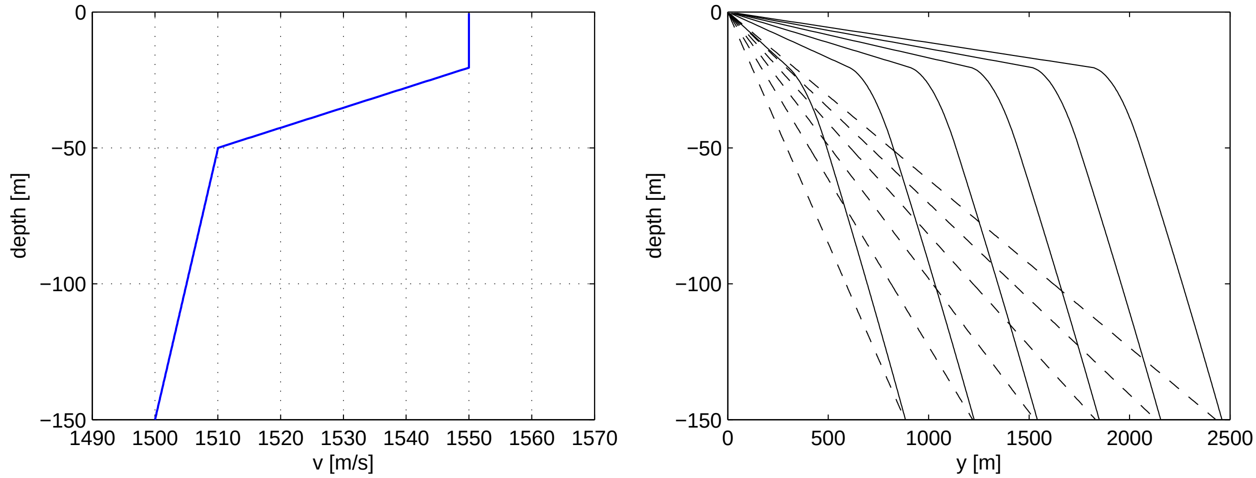
1) We are the first to consider stratification effects in ToA based ranging and bring together knowledge available in ocean acoustics and signal processing to find a novel

Manuscript received March 16, 2007; revised November 29, 2007. This work is supported by the ONR YIP grant N00014-07-1-0805, the ONR grant N00014-07-1-0055, and the NSF grant ECCS-0725562. The associate editor coordinating the review of this paper and approving if for publication was Dr. Rahim Leyman.

C. R. Berger and S. Zhou and P. Willett are with the Department of Electrical and Computer Engineering, University of Connecticut, 371 Fairfield Way U-2157, Storrs, Connecticut 06269, USA (email: crberger@engr.uconn.edu; shengli@engr.uconn.edu; willett@engr.uconn.edu).

L. Liu is with the Department of Civil and Environmental Engineering, University of Connecticut, 261 Glenbrook Road, U-2037, Storrs, CT 06269, USA (email: lanbo.liu@uconn.edu)

Digital Object Identifier 00.0000/TSP.2008



(a) Sound velocity profile (SVP)

(b) Sound wave propagation paths

Fig. 1. (a) Example of a sound velocity profile in shallow water; (b) Propagation of sound waves in an inhomogeneous medium; solid lines are the actual paths.

and simple solution based on a single acoustic sensor, an additional depth sensor and a known SVP.

2) In the presence of noisy ToA and depth measurements with Gaussian measurement noise, our solution is derived from the maximum likelihood (ML) criterion, leading to an unbiased and asymptotically efficient estimator.
3) We supply complete performance characterization of the suggested setup by developing the Cramér-Rao lower bound (CRLB).

In a simple shallow-water example, where the sound speed decreases monotonically with depth, we show that it is necessary to deal with the bias associated with the straight-line propagation assumption, even when the sound velocity profile is known. For a fixed depth we find increasing bias for larger range as the path becomes more slanted. Numerical results show that our proposed approach achieves bias-free range estimation, and meets the CRLB on the estimation performance for any unbiased estimator.

This paper has the following structure. We describe the problem in Section II and propose our solution in Section III. We then calculate the CRLB in Section IV and present numerical results in Section V. We conclude in Section VI.

II. Problem Description

We consider ranging in an underwater system based on ToA measurements of acoustic transmission between two nodes. We use cylindrical coordinates, (y, φ, z), but due to the axis-symmetric assumption of the medium physical parameters (equivalent to depth-dependent), it is sufficient to consider a two-dimensional plane that includes both the sender and the receiver. Suppose that the sender is at position $\mathbf{p}_s = (y_s, z_s)$ and the receiver is at position $\mathbf{p}_r = (y_r, z_r)$. We assume that:

A1) There is a direct propagation path between the sender and the receiver in the presence of potential dense multipath which is typical for underwater acoustic channels; and

A2) The receiver is able to pick up the first arrival to measure the ToA even in the presence of dense multipath via some advanced synchronization algorithms[1].

Let $T(\mathbf{p}_s, \mathbf{p}_r, v)$ denote the true travel time of the direct path from the sender to the receiver, which is parametrized by the SVP v. The noisy TOA measurement is

$$\hat{T} = T(\mathbf{p}_s, \mathbf{p}_r, v) + n \tag{1}$$

where we assume that the measurement noise n is zero-mean Gaussian with variance σ_T^2.

Now we convert the ToA estimate $\hat{T}$ to a range estimate $\hat{R}$. The conventional approach ignores the stratification effect. By assuming that the sound travels on a straight line from the sender to the receiver, it amounts to a linear estimator as

$$\hat{R} = c\hat{T} \tag{2}$$

where c is a nominal sound speed. Such an estimator introduces a bias

$$\mu = cT - R, \tag{3}$$

and the estimation variance is

$$\text{var}(\hat{R}) = c^2 \sigma_T^2. \tag{4}$$

The mean square error (MSE) of the estimator is

$$E\left[|R - \hat{R}|^2\right] = \mu^2 + \text{var}(\hat{R}). \tag{5}$$

Note that the variance depends only on the measurement accuracy and the nominal speed used in the conversion, but not on the locations $\mathbf{p}_s$ and $\mathbf{p}_r$. On the contrary, the bias term μ is location dependent, since T has a non-linear relationship with respect to $\mathbf{p}_s$ and $\mathbf{p}_r$ which is affected by the sound speed profile.

[1]This assumption is justified as ranging in dense multipath indoor radio environments has been studied extensively; see synchronization algorithms that localize the "first arrival" in the presence of dense multipath in e.g., [8], [9], [2] and references therein.

For precise ranging where the measurement noise is small, the bias term will dominate the MSE. The question is then: *how to remove the estimation bias due to the stratification effect?*

III. Proposed Solution

We propose a depth-based approach for stratification effect compensation. For presentation convenience, we assume that the sender position is known a priori[2], and its coordinate is $(0, z_s)$. The receiver has a depth sensor that gives a noisy depth estimate as

$$\hat{d}_0 = z_r + w, \tag{6}$$

where we assume the measurement noise w Gaussian distributed with zero mean and variance σ_d^2. We next derive an unbiased range estimate based on $\hat{T}$ and $\hat{d}_0$, and the knowledge of the sound velocity profile.

A. Maximum Likelihood Formulation

In addition to A1), A2), and the Gaussian assumption on $\hat{T}$ and $\hat{d}_0$, we further assume:

A3) The water field of interest is a vertically stratified media, i.e., the sound velocity is only depth dependent, and the SVP v is known[3].

Since $\mathbf{p}_s$ is known, we denote the travel time $T(\mathbf{p}_s, \mathbf{p}_r, v)$ by $T(y_r, z_r, v)$. Assuming the Gaussian measurement noises are uncorrelated, the likelihood function is

$$\Lambda(y_r, z_r) = \frac{1}{\sqrt{2\pi}\sigma_T} \exp\left(-\frac{1}{2\sigma_T^2}\left[\hat{T} - T(y_r, z_r, v)\right]^2\right) \times \frac{1}{\sqrt{2\pi}\sigma_d} \exp\left(-\frac{(\hat{d}_0 - z_r)^2}{2\sigma_d^2}\right). \tag{7}$$

The ML solution of (y_r, z_r) is then defined as

$$(\hat{y_r}, \hat{z_r}) = \arg\max_{(y,z)} \Lambda(y, z). \tag{8}$$

Since the measurement errors in $\hat{d}_0$ and $\hat{T}$ are uncorrelated, the ML solution $(\hat{y}_r, \hat{z}_r)$ will satisfy

$$\hat{z}_r = \hat{d}_0, \tag{9}$$

$$T(\hat{y_r}, \hat{d}_0, v) = \hat{T}, \tag{10}$$

if a solution to (10) exists. If it does, then the exponents in (7) are zero and neither σ_T^2 or σ_d^2 matter.

We next solve (10) to determine $\hat{y}_r$. Based on $\hat{y}_r$ and $\hat{z}_r$, we can find the range estimate $\hat{R} = \sqrt{\hat{y}_r^2 + (\hat{z}_r - z_s)^2}$.

[2]This is reasonable, for example, the sender could be below a fixed ship or a surface buoy equipped with GPS. On the other hand, the inclusion of uncertainty in the sender position can be done similarly, but notationally more cumbersome.

[3]In practice, the SVP needs to be obtained by measurements.

B. Ray Solution in Vertically Stratified Media

We now solve (10), but for convenience with notation changed to $T(y_r, z_r, v) = T$, where z_r and T are observed in noise and y_r is the unknown to be resolved. The solution to this problem can be found, using what is known in computational acoustics as the ray-based approach [4, pp. 177-179] or in similar form in seismic wave propagation [10, pp. 16-20]. We will rederive the solution for the reader, to this end we will go through the following steps:

- Using Fermat's Principle that sound waves travel along the fastest path, we find the correct path by minimizing the travel time over all possible paths.
- We express all possible paths as a function of depth $f(z)$ and solve for this function using a variational approach to find an analytic solution for each candidate range.
- Last we have to use a simple bisectional search to match the measured travel time to the right candidate range.

Fermat's Principle states that the travel time for a ray path is stationary [11]. This can be interpreted as (locally) minimizing the travel time over different ray paths. We calculate the travel time of a ray from $\mathbf{p}_s$ to $\mathbf{p}_r$ as

$$T = \int_{\mathcal{S}} \frac{1}{v(s)}\, ds \tag{11}$$

where $\mathcal{S}$ is a line curve which depicts a possible path between the given end points and v is the velocity which can be varying along the path s. Assuming a two-dimensional problem, we have

$$ds = \sqrt{dz^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dz}\right)^2}\, dz. \tag{12}$$

Now by defining $y = f(z)$ to describe the propagation path, and assuming that $f'(z) = dy/dz$ is well defined[4], we have

$$ds = \sqrt{1 + f'(z)^2}\, dz \tag{13}$$

and accordingly

$$T = \int_{z_s}^{z_r} \frac{\sqrt{1 + f'(z)^2}}{v\,(f(z), z)}\, dz = \int_{z_s}^{z_r} L(f, f', z)\, dz. \tag{14}$$

Each possible line curve s is defined via a candidate function f. According to Fermat's Principle, we find the true travel path of the sound wave by minimizing T with respect to f, which can be accomplished using calculus of variations. To solve for the functional $f(z)$, we use the Euler-Lagrange equation [12],

$$\frac{\partial}{\partial f} L - \frac{d}{dz}\frac{\partial}{\partial f'} L = 0. \tag{15}$$

As stated before, we assume the velocity of sound only changes with depth. This simplifies this most general formulation, as the z-axis is the depth. We get $v(f(z), z) = v(z)$,

[4]This formulation does not allow for bottom-reflected or refracted rays, since then f' would not be well defined. In this case we would have to integrate along the z-axis in separate parts, i.e., forwards and backwards in an alternating fashion.

which makes L independent of $f(z)$. We find the following relationship

$$-\frac{d}{dz}\left(\frac{\partial}{\partial f'}\frac{\sqrt{1+f'(z)^2}}{v(z)}\right) = -\frac{d}{dz}\left(\frac{f'(z)}{v(z)\sqrt{1+f'(z)^2}}\right) = 0. \quad (16)$$

Integrating both sides this leads to

$$\frac{f'(z)}{v(z)\sqrt{1+f'(z)^2}} = C, \quad (17)$$

where C is an integration constant. Hence, we obtain

$$f'(z) = \frac{Cv(z)}{\sqrt{1-[Cv(z)]^2}} = \tan[\theta(z)] \quad (18)$$

where we defined $\theta(z) := \arctan[dy/dz]$ as the angle of the ray path at depth z, c.f. $\theta_r = \theta(z_r)$ in Fig 2. Inserting (18) back into (17) we obtain

$$C = \frac{\tan\theta}{v(z)\sqrt{1+\tan^2[\theta(z)]}} = \frac{\sin[\theta(z)]}{v(z)}. \quad (19)$$

Therefore, the constant C is a generalization of the constant defined by Snell's Law, when assuming an arbitrary SVP is formed by infinitely many thin homogeneous layers in the limit.

Substituting (18) into (14), we find the minimum travel time associated with one path

$$T = \int_{z_s}^{z_r} \frac{1}{v(z)}\frac{1}{\sqrt{1-[Cv(z)]^2}}\,dz = \int_{z_s}^{z_r}\frac{1}{v(z)}\frac{1}{\cos[\theta(z)]}\,dz. \quad (20)$$

With z_s known, and the measured T and z_r, we can determine C numerically. For a positive C within the valid range $0 < C < \min_z(1/v(z))$, T is a monotonically increasing function with respect to C. Thus efficient numerical search can be used. After finding C, we will have

$$y_r = f(z_r) = \int_{z_s}^{z_r} f'(z)\,dz = \int_{z_s}^{z_r}\frac{Cv(z)}{\sqrt{1-[Cv(z)]^2}}\,dz. \quad (21)$$

In addition, by letting z_r in (21) vary, we can determine the full geometry of the slanted path $f(z)$ that the sound wave has traveled along.

IV. Cramér-Rao Bound for Range Estimation

We now explore the theoretical lower bound on the range estimation accuracy for any unbiased estimator. With the ray solution in Section III-B, we will first derive the Fisher information matrix of estimating C and z_r. Then the Fisher information of y_r and z_r (and finally R) can be calculated as for functions of parameters [13].

Replacing $T(y_r, z_r, v)$ by $T(C, z_r, v)$ in (7), we obtain the Fisher information matrix as [14]

$$\mathbf{J}_1(C, z_r) = E\left[[\nabla \log\Lambda(C,z_r)]\,[\nabla\log\Lambda(C,z_r)]^T\right] \quad (22)$$

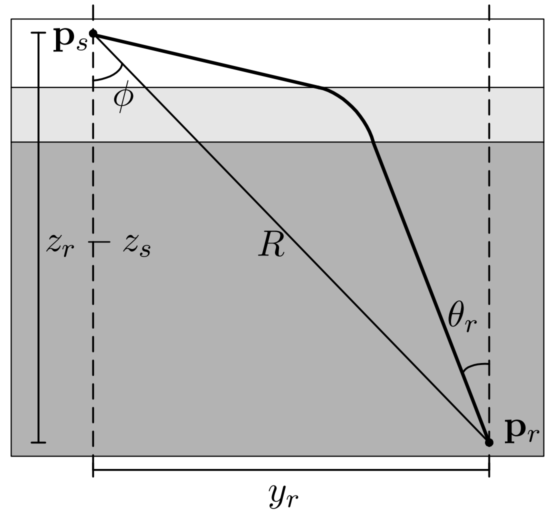


Fig. 2. Illustration of the relationship between (y_r, z_r) and (R, ϕ)

Since only the mean of the likelihood function $\Lambda(C, z_r)$ is dependent on the parameters, we simplify (22) to:

$$\mathbf{J}_1(C, z_r) = \frac{1}{\sigma_T^2}\begin{bmatrix}\frac{\partial T}{\partial C}\\ \frac{\partial T}{\partial z_r}\end{bmatrix}\begin{bmatrix}\frac{\partial T}{\partial C} & \frac{\partial T}{\partial z_r}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & \sigma_d^{-2}\end{bmatrix}. \quad (23)$$

The necessary partial derivatives $\frac{\partial T}{\partial C}$ and $\frac{\partial T}{\partial z_r}$ are

$$\frac{\partial T}{\partial C} = \int_{z_s}^{z_r}\frac{1}{v(z)}\frac{\partial}{\partial C}\left(\frac{1}{\sqrt{1-[Cv(z)]^2}}\right)dz = \int_{z_s}^{z_r}\frac{Cv(z)}{(1-[Cv(z)]^2)^{\frac{3}{2}}}\,dz, \quad (24)$$

$$\frac{\partial T}{\partial z_r} = \frac{1}{v(z_r)}\frac{1}{\sqrt{1-[Cv(z_r)]^2}}. \quad (25)$$

The Fisher information matrix with respect to estimating y_r and z_r can be calculated as [13]

$$\mathbf{J}_2^{-1}(y_r, z_r) = \mathbf{H}^T\mathbf{J}_1^{-1}(C, z_r)\mathbf{H}, \quad (26)$$

where the matrix $\mathbf{H}$ is the Jacobian of the variable transform $(C, z_r) \to (y, z_r)$

$$\mathbf{H} = \frac{\partial(y_r, z_r)}{\partial(C, z_r)} = \begin{bmatrix}\frac{\partial y_r}{\partial C} & \frac{\partial y_r}{\partial z_r}\\ 0 & 1\end{bmatrix}. \quad (27)$$

The second row is simply a row of the identity matrix, since in this mapping z_r is only a function of itself. The derivatives $\frac{\partial y_r}{\partial C}$ and $\frac{\partial y_r}{\partial z_r}$ are

$$\frac{\partial y_r}{\partial C} = \int_0^{z_r}\frac{\partial}{\partial C}\left(\frac{Cv(z)}{\sqrt{1-[Cv(z)]^2}}\right)dz = \int_0^{z_r}\frac{v(z)}{(1-[Cv(z)]^2)^{\frac{3}{2}}}\,dz = \frac{1}{C}\frac{\partial T}{\partial C} \quad (28)$$

$$\frac{\partial y}{\partial z_r} = \frac{Cv(z_r)}{\sqrt{1-[Cv(z_r)]^2}}. \quad (29)$$

After straightforward manipulation, we obtain:

$$\mathbf{J}_2^{-1}(y_r, z_r) = \sigma_T^2 \begin{pmatrix} \frac{1}{C^2} & 0 \\ 0 & 0 \end{pmatrix} + \sigma_d^2 \begin{pmatrix} \frac{(1-[Cv_r]^2)}{[Cv_r]^2} & -\frac{\sqrt{1-[Cv_r]^2}}{Cv_r} \\ -\frac{\sqrt{1-[Cv_r]^2}}{Cv_r} & 1 \end{pmatrix}. \quad (30)$$

where we abbreviate $v_r := v(z_r)$.

Applying a variable change from (y_r, z_r) to (R, ϕ) satisfying $y_r = R\sin\phi$ and $z_r - z_s = R\cos\phi$, as depicted in Fig. 2, we obtain the CRLB on the range estimate as

$$\begin{aligned} J_3^{-1}(R,\phi) &= \begin{bmatrix} \sin\phi & \cos\phi \end{bmatrix} \mathbf{J}_2^{-1}(y_r, z_r) \begin{bmatrix} \sin\phi \\ \cos\phi \end{bmatrix} \\ &= \sigma_T^2 \left(\frac{\sin\phi}{C}\right)^2 + \sigma_d^2 \left(\sin\phi \frac{\sqrt{1-[Cv_r]^2}}{Cv_r} - \cos\phi\right)^2 \\ &= \sigma_T^2 v_r^2 \left(\frac{\sin\phi}{\sin\theta_r}\right)^2 + \sigma_d^2 \cos^2\theta_r \left(\frac{\sin\phi}{\sin\theta_r} - \frac{\cos\phi}{\cos\theta_r}\right)^2 \\ &= \sigma_T^2 v_r^2 \left(\frac{\sin\phi}{\sin\theta_r}\right)^2 + \sigma_d^2 \left(\frac{\sin[\phi-\theta_r]}{\sin\theta_r}\right)^2 \end{aligned} \quad (31)$$

where $\theta_r := \theta(z_r)$ is abbreviated similar to v_r (c.f. Fig. 2), and we have used (19) to substitute C. For any unbiased estimator, we have

$$\mathrm{var}(\hat{R}) \geq J_3^{-1}(R,\phi). \quad (32)$$

Remark 1 The CRLB in (31) clearly isolates the effects from the ToA measurement accuracy and the depth measurement accuracy. When $\sigma_T v_r$ and σ_d are on the same order, the impact of σ_d is much smaller than that of $\sigma_T v_r$, as $|\sin(\phi-\theta_r)|$ is usually smaller than $|\sin(\phi)|$. This is encouraging in that the depth sensor does not need to be highly accurate to reduce the stratification effect. See also numerical results with noisy depth estimates.

V. Numerical Results

We use the sound velocity profile in Fig. 1.(a) for numerical testing. For the linear estimator, the nominal sound speed c could take different values, for example:

1) *Sender local speed*: $c = v(z_s)$. The speed is measured locally at the sender (surface) and assumed constant throughout the propagation path. Due to the SVP in Fig. 1.(a), the sound speed is over-estimated, leading to a distance estimate systematically larger than the true distance.
2) *Receiver local speed*: $c = v(z_r)$. Measuring the sound speed at the receiver (underwater) leads to a too small sound speed in this scenario, therefore under-estimating the distance.
3) *Arithmetic mean speed*: $c = \bar{v}_a$. If the SVP and the depth are available, one could use the mean sound speed, averaged over the depth, for range estimation:
$$\bar{v}_a = \frac{1}{z_r - z_s}\int_{z_s}^{z_r} v(z)\,dz.$$
4) *Geometric mean:* $c = \bar{v}_g$. Since travel time is inversely proportional to the sound speed, the geometric mean could be another reasonable choice when the SVP and the depth are available:
$$\bar{v}_g = \frac{z_r - z_s}{\int_{z_s}^{z_r} \frac{1}{v(z)}\,dz}$$

We consider ranging between a buoy at the surface ($z_s = 0$ m) and an underwater vehicle or sensor at a depth $z_r = 150$ m and a horizontal range of y_r between 0 m and $3,500$ m.

Fig. 3.(a) depicts the location-dependent bias μ defined in (3) for the linear estimator with different nominal sound speed values. Using the two local sound speeds, the distance is systematically over- or under-estimated, respectively. When both the SVP and the depth are available, the linear estimator with the arithmetic or geometric mean of sound speed is rather accurate until $y_r \approx 750$ m, which corresponds to the situation when the rays are only slightly slanted (c.f. Fig. 1.(b)). Afterwards the bias steadily increases as y_r increases, meaning that even if the precise SVP is available the assumption of straight line propagation leads to non-negligible bias.

The root mean square error (RMSE) of our proposed approach with noisy $\hat{T}$ and $\hat{d}_0$ is shown in Fig. 3, where $\sigma_d = 10$ m and $\sigma_T = 10$ ms. As comparison we also plot the CRLB and the RMSE performance for the linear estimator. We observe that the RMSE performance of the proposed approach meets the CRLB. The variance of the linear estimator introduced by the noisy ToA measurements is $c\sigma_T \approx 15$ m. Comparing Fig. 3.(b) to Fig. 3.(a), we see that the RMSE of the linear estimator is dominated by the bias once it rises above the variance.

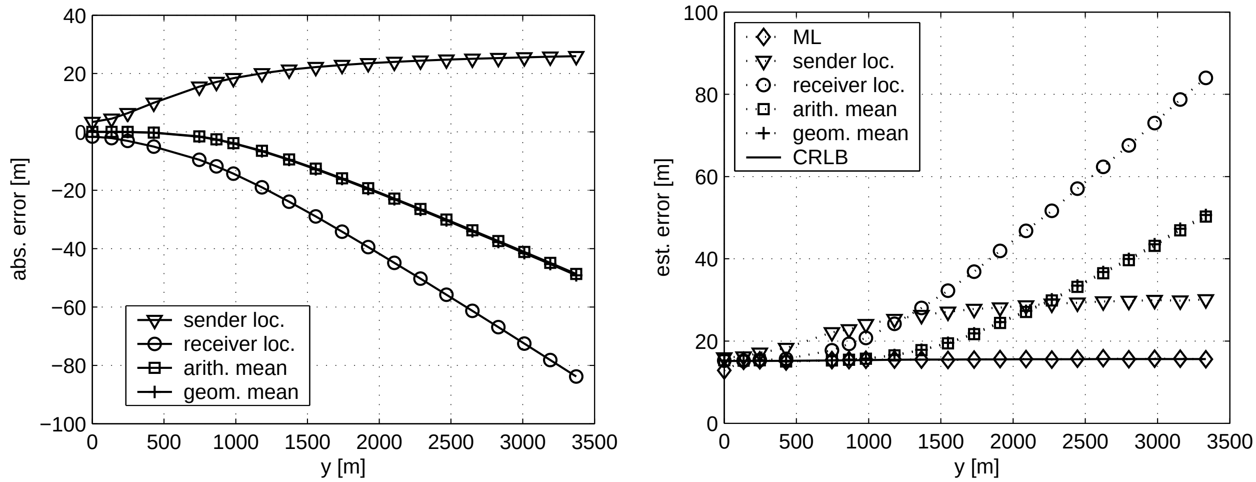
According to (31), the variance introduced by the noisy ToA measurement is the dominant part in the CRLB. As $|\sin(\phi - \theta_1)|/|\sin(\phi)|$ does not exceed 0.2 in this scenario, σ_d would have to be about five times of $c\sigma_T$ to make a similar impact.

VI. Conclusion

We analyzed the bias in underwater acoustic ranging when converting ToA measurements to range estimates based on a straight-line propagation assumption. To remove this bias, we used Fermat's Principle (minimum-time path) to trace the slanted path associated with the shortest travel time, this allows us to determine the exact distance between the sender and the receiver if the sound velocity profile (SVP) and the depth are available. Assuming that the ToA and depth estimates are noisy, we presented our solution starting from a maximum-likelihood criterion and derived the Cramér-Rao performance bound. For an example shallow water SVP, we found that the bias of the linear estimator is non-negligible for ToA measurements with low measurement noise, while our approach is bias free and meets the performance bound.

Acknowledgement

The authors would like to thank Dr. Thomas Torgersen from the Department of Marine Science at the University of Connecticut for directing their attention to this research problem. The authors would also like to thank Dr. Robert



(a) Bias

(b) RMSE

Fig. 3. (a) Bias μ for the linear estimator with four different nominal sound speeds. (b) The RMSE performance of different schemes together with the CRLB. The depth uncertainty $\sigma_d = 10$ m and the ToA uncertainty $\sigma_T = 10$ ms.

Headrick from ONR for helpful comments, especially in pointing out the reference [4].

REFERENCES

[1] N. Patwari, J. Ash, S. Kyperountas, A. O. Hero III, R. Moses, and N. Correal, "Locating the nodes: cooperative localization in wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54–69, Jul. 2005.

[2] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. Molisch, V. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios - a look at positioning aspects of future sensor networks," *IEEE Trans. Signal Processing*, vol. 22, no. 4, pp. 70–84, Jul. 2005.

[3] J.-H. Cui, J. Kong, M. Gerla, and S. Zhou, "The challenges of building mobile underwater wireless networks for aquatic applications," *IEEE Network, Special Issue on Wireless Sensor Networking*, vol. 20, no. 3, pp. 12–18, May/June 2006.

[4] F. B. Jensen, W. A. Kuperman, M. B. Portor, and H. Schmidt, *Computational Ocean Acoustics*, 1st ed. New York: AIP Press, 1994.

[5] J. Candy and E. Sullivan, "Model-based ocean acoustic passive localization," in *Proc. of OCEANS '94*, vol. 2, Sept. 1994, pp. II/615 – II/620.

[6] J. Tabrikian and J. Krolik, "Barankin bounds for source localization in an uncertain ocean environment," *IEEE Trans. Signal Processing*, vol. 47, no. 11, pp. 2917–2927, Nov. 1999.

[7] P. Elisseeff, H. Schmidt, and W. Xu, "Ocean acoustic tomography as a data assimilation problem," *IEEE Journal of Oceanic Engineering*, vol. 2, no. 27, pp. 275–282, Apr. 2002.

[8] C. R. Berger, S. Zhou, Z. Tian, and P. Willett, "Precise timing for multiband OFDM in a UWB system,," in *Proc. of ICUWB 2006*, Waltham, Sep. 2006, pp. 269–274.

[9] ——, "Performance analysis on an MAP fine timing algorithm in UWB multiband OFDM," *IEEE Trans. Commun.*, 2008, to be published.

[10] C. Chapman, *Fundamentals of Seismic Wave Propagation*, 1st ed. Cambridge, UK: Cambridge University Press, 2004.

[11] E. W. Weisstein, "Fermat's principle," *World of Science–A Wolfram Web Resource*, http://scienceworld.wolfram.com/physics/FermatsPrinciple.html.

[12] ——, "Euler-lagrange differential equation," *MathWorld–A Wolfram Web Resource*, http://mathworld.wolfram.com/Euler-LagrangeDifferentialEquation.html.

[13] L. Scharf, *Statistical Signal Processing*, 1st ed. Addison-Wesley, 1991.

[14] H. Van Trees, *Detection, Estimation, and Modulation Theory*, 1st ed. New York: John Wiley & Sons, Inc., 1968.