Recursive and Trellis-based Feedback Reduction for MIMO-OFDM with Transmit Beamforming

Shengli Zhou, Baosheng Li, and Peter Willett
Dept. of ECE, University of Connecticut, 371 Fairfield Road, CT, 06269

***Abstract*— We consider a MIMO-OFDM system with transmit beamforming applied on each OFDM subcarrier, where each beamforming vector is drawn from a codebook with finite size. Depending on the channel realization, the receiver decides the optimal beamforming vector on each subcarrier, and informs the transmitter through a rate-limited feedback link. Exploiting the fact that the channel responses across OFDM subcarriers are correlated, we propose two methods to reduce the amount of needed feedback. One is recursive feedback encoding that selects the optimal beamforming vectors sequentially across the subcarriers, and adopts a smaller-size time-varying codebook per subcarrier depending on prior decisions. The other is trellis-based feedback encoding that selects the optimal beamforming vectors for all subcarriers at once along a trellis structure via the Viterbi algorithm. The trellis-based feedback encoding outperforms the recursive feedback encoding at the expense of encoding complexity at the receiver. Simulation results demonstrate that our trellis-based approach outperforms an existing interpolation-based alternative, as the latter incurs diversity loss at high SNR.**

I. Introduction

Multi-antenna communications have attracted a tremendous amount of attention lately because of their promises of high transmission rate and much improved performance in fading channels. Adaptive multi-antenna transmissions can further improve system performance by matching transmission parameters to fading channels. Essential to adaptive transmissions is a feedback link from the receiver to the transmitter, which is usually bandwidth limited and susceptible to feedback error and delays.

Transceiver design with bandwidth limited feedback has been studied from different perspectives. Power control based on finite-rate feedback is investigated in [1] to reduce the outage probability that the mutual information falls below a certain rate. Finite-rate transmit beamforming has been investigated based on various criteria such as average signal to noise ratio (SNR) [11], [13], outage probability [12], and symbol error rate [20]. Subject to finite-rate feedback, optimal transmission is also pursued in [2], [8], [14] to maximize the average channel capacity, while adaptive modulation together with transmit beamforming has been pursued in [17] to enhance the transmission rate. Recently, the application of finite rate feedback in a precoded spatial multiplexing system has been addressed in [9], [10], [19], where various criteria on precoder selection and codebook construction have been proposed. These studies, however, focused on *flat fading* multi-input multi-output (MIMO) channels.

With high symbol rate in broadband wireless systems, the wireless channels become strongly frequency selective. Multi-carrier techniques, especially orthogonal frequency division multiplex (OFDM) modulation, have prevailed in recent broadband wireless standards, as they convert frequency selective channels into a set of parallel flat-fading subchannels, thus enabling low complexity equalization [15]. Wedding of MIMO and OFDM leads to an appealing system design, termed as MIMO-OFDM, for high rate applications. Adaptive MIMO-OFDM has been investigated in [18] based on statistical channel information, while finite-rate beamforming has been applied on each OFDM subcarrier in [4]. Exploiting the fact that the channel responses across OFDM subcarriers are correlated, an interpolation-based feedback reduction has been proposed in [4], where only a subset of optimal beamforming vectors are fed back and the rest rely on linear interpolation from those known at the transmitter.

As in [4], we in this paper consider MIMO-OFDM with finite-rate beamforming on each subcarrier. We link feedback reduction to a compression type of problem for correlated sources. Relying on tools from the vector quantization literature [7], we propose two methods to reduce the needed number of feedback bits. One is recursive feedback encoding that selects the optimal beamforming vectors sequentially across the subcarriers, and adopts a smaller-size time-varying codebook per subcarrier depending on prior decisions. The other is trellis-based feedback encoding that selects the optimal beamforming vectors for all subcarriers at once along a trellis structure via the Viterbi algorithm. Trellis-based feedback encoding outperforms recursive feedback encoding at the expense of complexity increase at the receiver. Simulations results demonstrate that our trellis-based approach outperforms the interpolation-based alternative in [4], especially in the high signal-to-noise-ratio (SNR) region. The bit error rate (BER) curves of the trellis-based approach keep the same slope as the BER curves with full feedback. This is not the case for both the recursive feedback encoding and the interpolation-based approach.

Notation: Bold upper and lower letters denote matrices and column vectors, respectively; $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^{\mathcal{H}}$ denote transpose, conjugate, and Hermitian transpose, respectively; $|\cdot|$ stands for the absolute value of a scalar; $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ stand for the trace and the determinant of a matrix, respectively. $\|\cdot\|$ denotes the two-norm of a vector or a matrix, while $\|\cdot\|_F$ is the Frobenius norm of a matrix. $\mathbf{I}_N$ is the $N \times N$ identity matrix; $\mathbf{0}_{M\times N}$ denotes an all-zero matrix of size $M \times N$.

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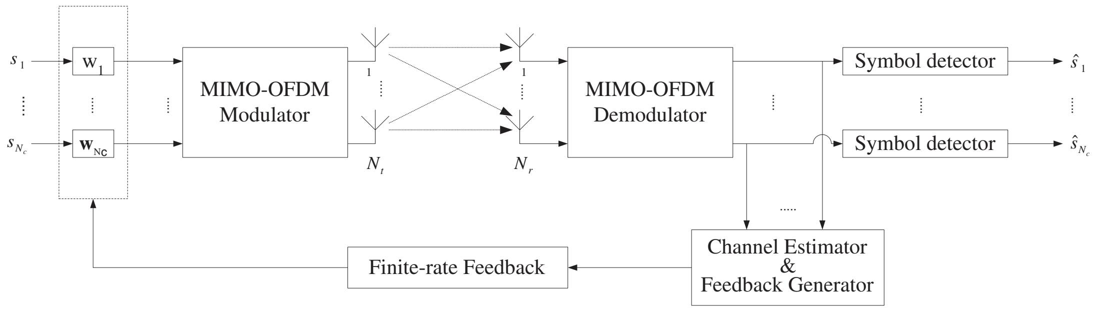

Fig. 1. The MIMO-OFDM system with transmit beamforming per subcarrier

II. System Model

We consider a multi-antenna wireless communication system with N_t transmit-antennas and N_r receive-antennas, where OFDM utilizing N_c subcarriers is employed per antenna transmission. The fading channel between the μth transmit-antenna and the νth receive-antenna is assumed to be frequency-selective but time-flat, and is described by the discrete-time baseband equivalent impulse response vector $\mathbf{h}_{\nu\mu} := [h_{\nu\mu}(0), \ldots, h_{\nu\mu}(L)]^T$, where L is the channel order. The channel impulse response includes the effects of transmit-receive filters, physical multipath, and relative delays among antennas.

As depicted in Fig. 1, we consider transmit beamforming on each sub-carrier p. Specifically, the information symbol $s[p]$ is multiplied by a $N_t \times 1$ beamforming vector $\mathbf{w}[p]$ and transmitted through the pth subcarrier of the OFDM system. (We omit the details on OFDM modulation, which includes an IFFT and the cyclic prefix (CP) insertion at the transmitter, together with an FFT and CP removal at the receiver; see e.g., the tutorial paper [15].) The input-output relationship on the pth subcarrier can be expressed as

$$\mathbf{y}[p] = \mathbf{H}[p]\mathbf{w}[p]s[p] + \mathbf{v}[p], \tag{1}$$

where $\mathbf{y}[p]$ is the $N_r \times 1$ received vector, and $\mathbf{v}[p]$ is additive white Gaussian noise (AWGN) with each entry having variance N_0, and $\mathbf{H}[p]$ is the $N_r \times N_t$ channel matrix with the (ν, μ)th entry as

$$H_{\nu\mu}[p] = \sum_{l=0}^{L} h_{\nu\mu}(l) e^{-j2\pi pl/N_c}, \quad p = 0, \ldots, N_c - 1. \tag{2}$$

Denote E_s as the average energy per symbol $s[p]$. With a maximum ratio combining (MRC) receiver $\hat{s}[p] = \mathbf{w}^{\mathcal{H}}[p]\mathbf{H}^{\mathcal{H}}[p]\mathbf{y}$, the signal to noise ratio (SNR) on each sub-channel is

$$\gamma[p] = \frac{E_s}{N_0} \big\| \mathbf{H}[p]\mathbf{w}[p] \big\|^2. \tag{3}$$

Based on feedback, the transmitter seeks to match the beamforming vector $\mathbf{w}[p]$ to the channel $\mathbf{H}[p]$, to improve the system performance. We will detail beamformer selection based on finite rate feedback in Section II-B. Before that, we present the system BER performance when $\{\mathbf{w}[p]\}_{p=0}^{N_c-1}$ are given, and the channel realizations are $\{\mathbf{H}[p]\}_{p=0}^{N_c-1}$.

A. BER performance

Let $\mathrm{BER}(\gamma)$ denote the relationship between BER and SNR γ in an AWGN channel. We consider the square quadrature-amplitude-modulation (QAM) with size M. The closed-form expression for $\mathrm{BER}(\gamma)$ is [3]:

$$\begin{aligned}\mathrm{BER}(\gamma) = \frac{1}{\log_2 \sqrt{M}} \sum_{k=1}^{\log_2 \sqrt{M}} \frac{1}{\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \\ \times \left\{ (-1)^{\left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} \right\rfloor} \left(2^{k-1} - \left\lfloor \frac{i \cdot 2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right) \right. \\ \left. \cdot\, 2Q\left((2i+1)\sqrt{\frac{3}{M-1}\gamma} \right) \right\},\end{aligned} \tag{4}$$

where $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ is the Gaussian-Q function. For pulse-amplitude-modulation (PAM) and rectangular QAMs, closed-form BER expressions are also available [3]. For 4-QAM, eq. (4) is simply:

$$\mathrm{BER}(\gamma) = Q(\sqrt{\gamma}). \tag{5}$$

With the SNR expressed in (3) for each subcarrier, the BER performance for the MIMO-OFDM system is:

$$\overline{\mathrm{BER}} = \frac{1}{N_c} \sum_{p=0}^{N_c-1} \mathrm{BER}(\gamma[p]). \tag{6}$$

We will use $\overline{\mathrm{BER}}$ in (6) as the performance criterion for our trellis-based approach in Section III-B.

B. Per subcarrier feedback

If the transmitter has perfect knowledge of $\mathbf{H}[p]$, the optimal beamforming vector will be the eigenvector of $\mathbf{H}^{\mathcal{H}}[p]\mathbf{H}[p]$, corresponding to the largest eigenvalue, to maximize $\gamma[p]$. However, the feedback bandwidth is usually limited in practice, hence cannot convey $\mathbf{H}[p]$ exactly.

Transmit beamforming with finite rate feedback has been adequately addressed for flat fading channels [11], [12], [20]. We first apply the results in [11], [12], [20] on each OFDM subcarrier separately. Assuming that B_1 feedback bits are available per subcarrier, the transceiver will need a codebook $\mathcal{W}$ of size 2^{B_1}, which is a collection of beamforming vectors $\{\mathbf{w}_1, \ldots, \mathbf{w}_{2^{B_1}}\}$. Here we assume that the codebook $\mathcal{W}$ is the same across subcarriers. The beamforming vector is chosen at the receiver to maximize $\gamma[p]$ at the pth subcarrier as

$$\mathbf{w}^{\rm opt}[p] = \arg\max_{\mathbf{w}\in\mathcal{W}} \|\mathbf{H}[p]\mathbf{w}\|^2. \quad (7)$$

The index of $\mathbf{w}^{\rm opt}[p]$ in the codebook will be fed back to the transmitter via B_1 feedback bits. The transmitter then switches to $\mathbf{w}^{\rm opt}[p]$ after looking up in the codebook based on the feedback. To quantify the impact of feedback rate constraint, we assume for convenience that the feedback link is error-free and delay-free [11], [12], [20].

We term the selection in (7) as per-subcarrier feedback, as the beamforming vector selection is done independently across subcarriers. The needed feedback in this case is $N_c B_1$ bits, which is a significant amount as N_c is usually large.

Notice that the channel responses across subcarriers are correlated. Indeed, the frequency responses evaluated at N_c subcarriers (see (2)), are parameterized by $(L+1)$ channel taps in the time domain. Per subcarrier feedback does not take into account this inherent correlation structure. Transmitting the codeword indices obtained in (7) per subcarrier will lead to inefficient usage of the precious feedback bandwidth.

Choi and Heath proposed to reduce the amount of feedback via interpolation [4]. The idea is to split N_c subcarriers into N_g groups with N_c/N_g consecutive subcarriers per group. Only $\{\mathbf{w}_{\rm opt}[lN_g]\}_{l=0}^{N_c/N_g-1}$ will be fed back to the transmitter, and the rest subcarriers relies on the following interpolation

$$\mathbf{w}[lN_g+k] = \frac{(1-k/N_g)\mathbf{w}^{\rm opt}[lN_g] + e^{j\theta_l}(k/N_g)\mathbf{w}^{\rm opt}[(l+1)N_g]}{\|(1-k/N_g)\mathbf{w}^{\rm opt}[lN_g] + e^{j\theta_l}(k/N_g)\mathbf{w}^{\rm opt}[(l+1)N_g]\|}, \quad (8)$$

where θ_l is chosen from a finite set $\{e^{jn2\pi/P}\}_{n=0}^{P-1}$. Feedback of $\{\mathbf{w}^{\rm opt}[lN_g], \theta_l\}$ requires $(N_c/N_g)(B_1 + \log_2 P)$ bits. A significant portion of reduction could be achieved by careful choices of N_g and P.

III. Recursive and Trellis-based Feedback Reduction

Feedback reduction is possible since channel responses across OFDM subcarriers are correlated. Hence, this problem is essentially a *compression* problem. Tools from the source coding or vector quantization literature can be used to solve our problem at hand. We here propose two approaches, one recursive feedback encoding and the other trellis-based feedback encoding. They correspond to recursive vector quantization (VQ) and trellis coded quantization, respectively [7].

A. Recursive feedback encoding

Recursive vector quantizer is a vector quantizer with *memory*, where the quantizer output depends not only on current input, but also on prior inputs [7]. Using the state variables to summarize the influence of the past on the current operation of the quantizer, recursive VQ can be effectively described by state transition and state-dependent encoding [7]. A finite state vector quantizer (FSVQ) is simply a recursive VQ with a finite number of states.

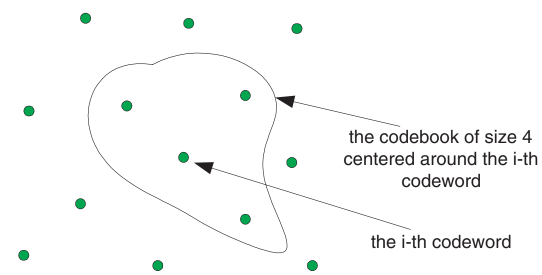

Fig. 2. Neighbouring codewords form a small-size state-dependent codebook

To apply the concept of FSVQ in our problem, we need to introduce the time evolution. We view the subcarrier index p as the *virtual* time index, and pursue the beamforming vectors *sequentially* across the subcarriers from $p=0$ to $p=N_c-1$.

Denote $\zeta[p]$ as the quantizer state at time p. We assume that $\zeta[p]$ can take values from a finite set of states denoted as $\{\zeta_i\}_{i=1}^N$. Given the previous state $\zeta[p-1]$ and the current channel input $\mathbf{H}[p]$, we denote the state transition as

$$\zeta[p] = \text{nextstate}(\zeta[p-1], \mathbf{H}[p]), \quad (9)$$

where nextstate$(\cdot)$ is a function to be specified. To perform a state dependent encoding, we associate each state ζ_i with a codebook $\mathcal{W}_i$ which contains 2^{B_2} codewords. Similar to (7), the optimal beamforming vector at time p is then

$$\mathbf{w}^{\rm opt}[p] = \arg\max_{\mathbf{w}\in\mathcal{W}[p-1]} \|\mathbf{H}[p]\mathbf{w}\|^2, \quad (10)$$

where $\mathcal{W}[p-1]$ stands for the current codebook associated with the state $\zeta[p-1]$ known at time p. Specifying $\mathbf{w}^{\rm opt}[p]$ only requires B_2 bits, when $\mathcal{W}[p-1]$ is available.

Designing the states $\{\zeta_i\}$ and the state-dependent codebooks $\{\mathcal{W}_i\}$ is an interesting problem. The optimal design may require the channel correlation information. We next specify one simple design based on heuristics.

- We construct the same number of states as the codebook size of $\mathcal{W}$; hence $N = 2^{B_1}$. Each state ζ_i is characterized by one beamforming vector $\mathbf{w}_i$.
- We initialize $\zeta[0]$ based on (7), that requires B_1 feedback bits.
- We design each new codebook $\mathcal{W}_i$ as a subset of $\mathcal{W}$ as follows:

$$\mathcal{W}_i = \text{collection of } 2^{B_2} \text{ codewords from } \mathcal{W} \text{ that are closest to } \mathbf{w}_i, \quad (11)$$

  where the distance measure is the chordal distance $d(\mathbf{w}_i, \mathbf{w}_j) = \sqrt{1-|\mathbf{w}_i^{\mathcal{H}}\mathbf{w}_j|^2}$ [6]. Notice that $\mathcal{W}_i$ is centered around $\mathbf{w}_i$ and includes $\mathbf{w}_i$ itself, as illustrated in Fig. 2. Selecting the optimal beamforming vector as in (10) requires B_2 bits per subcarrier.
- We define the state transition as

$$\zeta[p] = \zeta_j, \quad \text{if } \mathbf{w}^{\rm opt}[p] = \mathbf{w}_j. \quad (12)$$

  In such a way, the codebook $\mathcal{W}[p]$ will be centered around the most recent beamforming vector $\mathbf{w}^{\rm opt}[p]$.

The receiver needs to feed back the initial state $\zeta[0]$ and the state-dependent codeword index back to the transmitter. The transmitter starts from $\zeta[0]$, decides $\mathbf{w}[p]$ and $\zeta[p]$ ([c.f. (12)]) based on the knowledge of $\zeta[p-1]$ and the state-dependent codeword index. Follow the state transition, the transmitter outputs all beamforming vectors for N_c subcarriers. The total feedback required in this scheme is:

$$B_1 + (N_c - 1)B_2. \tag{13}$$

Our simple choice here is somewhat related to DPCM (differential pulse coded modulation). Instead of coding the codeword index itself per subcarrier, we now quantize the relative difference with respect to the previous codeword by only searching its neighbor (a total of 2^{B_2} nearest neighbors). If indeed the channel response changes slowly from subcarrier to subcarrier, the optimal codeword $\mathbf{w}[p]$ specified in (7) could be right in neighbourhood of $\mathbf{w}[p-1]$. In this case, the same performance is achieved with reduced feedback. However, if some abrupt change happens between adjacent subcarriers, the state transition may lose track, and the system performance will deteriorate considerably.

B. Trellis-based feedback encoding

The drawback of the recursive feedback encoding is that the state transition may lose track from time to time. Notice that the decision on $\mathbf{w}[p]$ has only relied on prior channels inputs $\mathbf{H}[p-k]$, $k > 0$. Hence, the correlation across subcarriers is only utilized in a "causal" fashion. This causality is not necessary in MIMO-OFDM as the feedback is done on a block basis. If we follow the state transition from $p = 0$ to $p = N_c - 1$, the decision shall be made at time $p = N_c - 1$, to specify the optimal codeword indexes for all subcarriers at once. This is along the principle of tree or trellis based vector quantization [7].

We define N states as $\{\zeta_i\}_{i=1}^{N}$ as before. To specify a trellis, we assume that each state is connected to 2^{B_2} next states. Hence, each state has 2^{B_2} outgoing branches, and we number them with an integer $j = 0, 1, \ldots, 2^{B_2}-1$. Denote the state at the virtual time p as $\zeta[p]$. The trellis transition corresponding to the jth branch of state $\zeta[p]$ can be described by

$$\zeta[p] = \text{nextstate}(\zeta[p-1], j), \tag{14}$$

where nextstate$(\cdot)$ is a function to be designed. Correspondingly, we denote the output of the jth branch of state $\zeta[p]$ as

$$\mathbf{w}[p] = \text{output}(\zeta[p-1], j), \tag{15}$$

where output$(\cdot)$ is to be specified. An evolution path along the trellis will hence lead to beamforming vectors for all subcarriers.

The design of the trellis (14) and the output mapping (15) is an interesting problem. The optimal design shall depend on the channel correlation characteristics. We here specify a simple design following the same nearest neighbor rule as in Section III-A.

- We construct the same number of states as the codebook size of $\mathcal{W}$; hence $N = 2^{B_1}$. Each state ζ_i is characterized by one beamforming vector $\mathbf{w}_i$.
- We initialize $\zeta[0]$ based on (7), that requires B_1 feedback bits.
- For each state $\zeta[p]$, we define 2^{B_2} neighbor states, denoted by neighbor(ζ_i, j), for $j = 0, \ldots, 2^{B_2}-1$. The neighbor is defined based on the chordal distance. For each state ζ_i, we arrange the codewords in $\mathcal{W}_i$ of (11) in descending order of the chordal distances relative to $\mathbf{w}_i$. The states corresponding to the codewords in $\mathcal{W}_i$ are the neighboring states of ζ_i. Obviously, neighbor$(\zeta_i, 0) = \zeta_i$, as we include ζ_i itself as its closest neighbor. The nextstate function is then simplified as

$$\zeta[p] = \text{neighbor}(\zeta[p-1], j), \quad j = 0, 1, \ldots, 2^{B_2} - 1. \tag{16}$$

- We define the output $\mathbf{w}[p]$ as:

$$\mathbf{w}[p] = \mathbf{w}_j, \quad \text{if } \zeta[p] = \zeta_j. \tag{17}$$

- We define the branch metric

$$\begin{aligned}&\text{Metric}(\zeta[p-1], \zeta[p]) \\ &\quad = \frac{1}{N_c}\text{BER}(\|\mathbf{H}[p] \cdot \text{output}(\zeta[p-1], j)\|).\end{aligned} \tag{18}$$

For each path following the trellis, the resulting average BER of the system is:

$$\overline{\text{BER}} = \sum_{p=0}^{N_c-1} \text{Metric}(\zeta[p-1], \zeta[p]). \tag{19}$$

The best path that minimizes $\overline{\text{BER}}$ is what we are looking for. The search is easily done by the Viterbi algorithm.

To recover the optimal path at the transmitter, the receiver needs to feedback the initial state $\zeta[0]$ and the input branches from $p = 1$ to $p = N_c - 1$. Hence, the feedback amount is $B_1 + (N_c - 1)B_2$, the same as the recursive coding.

With 2^{B_1} states, 2^{B_2} branches per state, and N_c subcarriers, the trellis-based approach checks a total of $2^{B_1}(2^{B_2})^{N_c-1}$ different paths. The Viterbi complexity is at the order of $(N_c - 1)2^{B_1+B_2} + 2^{B_1}$. However, we should point out that branch metrics in (18) only depend on the next state $\zeta[p]$. Hence, only a total of $N_c 2^{B_1}$ different metrics are actually computed, where all incoming branches to one state share the same metric. If a look-up table is used in implementing the BER$(\cdot)$, then the complexity of branch computations would be the *same* as the per subcarrier feedback case. The complexity increase is at the *add-compare-select* part of the Viterbi algorithm. Notice that the recursive coding actually follows one valid path in the trellis. Hence, the trellis-based approach outperforms recursive encoding at the expense of complexity increase at the receiver.

One final note is that the beamforming vectors in our proposed approaches are always drawn from the codebook $\mathcal{W}$. This is not the case for the interpolation-based method, as evidenced by (8).

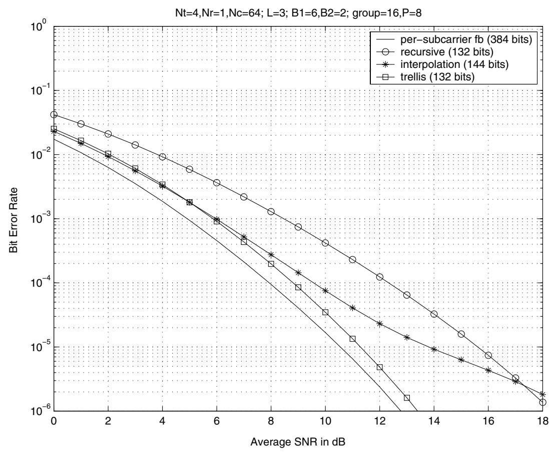

Fig. 3. Comparison with the interpolation-based method, $L = 3$.

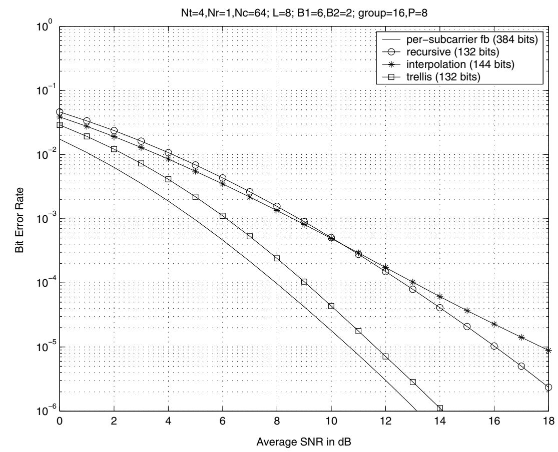

Fig. 4. Comparison with the interpolation-based method, $L = 8$.

IV. NUMERICAL RESULTS

We consider a system with $N_t = 4, N_r = 1$ and $N_c = 64$. We test two cases with $L = 3$ and $L = 8$, where all the channel taps are independent and identically distributed (i.i.d.) with zero mean and variance $1/(L+1)$. We adopt the beamforming codebook with size 64 from [16], hence $B_1 = 6$. Per-subcarrier feedback will require $N_c B_1 = 384$ bits.

We compare our proposed methods with the interpolation based approach. We set $N_g = 16$ and $P = 8$, so that the number of feedback bits needed for the interpolation-based method is 144. For our proposed recursive and trellis-based methods, we use $B_2 = 2$, so that the number of feedback bits is 132.

Figs. 3 and 4 depict the performance with $L = 3$ and $L = 8$, respectively. The recursive feedback reduction suffers from large performance loss in both cases. When L is small, the interpolation-based method has good performance at low to medium SNR. But at high SNR, the BER curve of the interpolation-based method will level off, indicating "diversity loss"; this observation has already been pointed out in [4]. When L is large, the interpolation-based method suffers from considerable performance loss across the SNR range. On the other hand, the BER curve of the proposed trellis-based method differs from the benchmark per-subcarrier feedback case by only a constant amount across the SNR range, which is also insensitive to the channel order L. Figs 3 and 4 demonstrate that the trellis-based feedback reduction outperforms the interpolation-based method in both cases.

V. CONCLUSIONS

In this paper, we proposed two methods to reduce the amount of feedback in MIMO-OFDM with transmit beamforming, one recursive feedback encoding, and the other trellis-based feedback encoding. The trellis based approach achieves considerable feedback reduction with graceful performance degradation. It outperforms an existing interpolation-based method, especially at high SNR as it does not suffer from diversity loss.

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