Homework 4 Solutions

CSE 254 — Prof. S. Rajasekaran; TA: S. Berhe — Fall 2007

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Section 6.1: Problems 22

$\frac{33}{100}$

Section 6.1: Problems 32

There are $\binom{100}{3}$ ways of selecting 3 prize winners. Thus the probability of interest is $\frac{1}{\binom{100}{3}}$.

Section 6.2: Problems 8

a) $\frac{1}{2}$ b) $\frac{1}{2}$ c) $\frac{(n-1)!}{n!} = \frac{1}{n}$ d) $\frac{(n-4)!\binom{n}{4}\binom{4}{2}}{n!}$ e) $\frac{(n-3)!\binom{n}{3}2}{n!}$

Section 6.2: Problems 12

For any two events E and F from the same sample space, $p(E \cup F) \geq p(E)$ and hence $p(E \cup F) \geq 0.8$. Also, assume that the sample space has n points (each point being equally likely). This means that $|E| = 0.8n$ and $|\bar{E}| = 0.2n$. This in turn means that $|F - E| \leq 0.2n$. Therefore, $|E \cap F| \geq 0.4n$ and hence $p(E \cap F) \geq 0.4$.

Section 7.1: Problems 8

a) $5(-1)^n$ b) $1 + 3n$ c) $4 - \frac{n(n+1)}{2}$ d) $-2^n - 3[2^n - 1]$ e) $2 \times (n+1)!$ f) $3 \times 2^n \times n!$ g) If n is odd then $a_n = \frac{n-1}{2} - 7$ and if n is even, $a_n = 7 + \frac{n}{2}$.

Section 4a

Here $a = 12, b = 4$, and $f(n) = n^{1.5}$. Case 1 of Master theorem applies and hence $T(n) = \Theta(n^{\log_4 12})$.

Section 4b

Master theorem does not apply here. Thus we use repeated substitutions to infer: $T(n) = \Theta(\log n)$.