CSE 259 Algorithms and Complexity

Homework 4: Due on December 8, 2006, 2PM

1. Present an $O(n)$ time algorithm to compute the coefficients of the polynomial $(1+x)^n$. How much time is needed if you use the FFT algorithm to solve this problem?

2. An $n \times n$ Toeplitz matrix is a matrix A with the property that $A[i,j] = A[i-1, j-1]$, $2 \leq i, j \leq n$. Give an $O(n \log n)$ time algorithm to multiply a Toeplitz matrix with an arbitrary $(n \times 1)$ column vector.

3. The string matching problem takes as input a text t and a pattern p, where t and p are strings from an alphabet Σ. The problem is to determine all the occurrences of p in t. Present an $O(1)$ time PRAM algorithm for string matching. Which PRAM are you using and what is the processor bound of your algorithm?

4. The inputs are an array A of n elements and an element x. The goal is to rearrange the elements of A such that all the elements of A that are less than or equal to x appear first (in successive cells) followed by the rest of the elements. Give an $O(\log n)$-time $\frac{n}{\log n}$-processor CREW PRAM algorithm for this problem.

5. Let π_2 be a problem for which there exists a deterministic algorithm that runs in time $2^{\sqrt{n}}$ (where n is the input size). Prove or disprove:

   If π_1 is another problem such that π_1 is polynomially reducible to π_2, then π_1 can be solved in deterministic $O(2^{\sqrt{n}})$ time on any input of size n.

6. Assume that there is a polynomial time algorithm CLQ to solve the CLIQUE decision problem. Show how to use CLQ to determine the maximum clique size of a given graph in polynomial time.