Homework 3 Solutions

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Section 4.1: Problem 12

Basic Step:

$$n = 0 \rightarrow (-\frac{1}{2})^0 = 1 = \frac{2^1 + (-1)^0}{3 \times 2^0} \quad (1)$$

Inductive Step: Assuming equation is correct for k=n, show that it is also correct for k+1.

$$\sum_{j=0}^{k+1} (-\frac{1}{2})^j = \frac{2^{k+2} + (-1)^(k+1)}{3 \times 2^{k+1}} \quad (2)$$

under the assumption that:

$$\sum_{i=0}^{k} (-\frac{1}{2})^j = (\frac{2^{k+1} + (-1)^n}{3 \times 2^k}) \quad (3)$$

it can be concluded that:

$$\sum_{j=0}^{k+1} (-\frac{1}{2})^j = \sum_{j=0}^{k} (-\frac{1}{2})^j + (-\frac{1}{2})^{k+1} \quad (4)$$

$$\Leftrightarrow \sum_{j=0}^{k+1} (-\frac{1}{2})^j = (\frac{2^{k+1} + (-1)^n}{3 \times 2^k}) + (-\frac{1}{2})^{k+1} \quad (5)$$

$$\Leftrightarrow \sum_{j=0}^{k+1} (-\frac{1}{2})^j = \frac{2^{k+2} + (-1)^(k+1)}{3 \times 2^{k+1}}.\square \quad (6)$$

Section 4.1: Problem 44

Basic Step:
$n = 1 \rightarrow A_1 - B = A_1 - B$
Inductive Step:
Assuming equation is correct for n=k, such that $(A_1 - B) \cup (A_2 - B) ... \cup (A_k - B) = (A_1 \cup A_2 ... \cup A_k) - B$, it must be shown that it is also true for $k+1$. $(A_1 - B) \cup (A_2 - B) ... \cup (A_k - B) \cup (A_{k+1} - B)$ $= ((A_1 \cup A_2 ... \cup A_k) - B) \cup (A_{k+1} - B) = ((A_1 \cup A_2 ... \cup A_k \cup A_{k+1}) - B)$. $\square$

Section 4.2: Problem 12

Basic Step:
P(1) = $2^0 = 1$
Inductive Step:
Assuming this is true for $n = k$, it must be shown for $k + 1$. If $k + 1$ is even, it will be a multiple of 2 and $k + 1 = 2 * a$, where a is an integer. Therefore, it can be written as $a + a$. Since $a \leq k$ it is a sum of powers of 2. Thus $k + 1$ is also a sum of powers of 2. If $k + 1$ is odd it must be equal to $a + a + 2^0$, for some integer $a \leq k$. So by induction hypothesis it follows that $k + 1$ is a sum of power of two. □

Section 4.2: Problem 26

a) $P(n)$ is true for $n = 2k$ $(\forall k \in N_0)$.
b) $P(n)$ is true for $n = 3k$ $(\forall k \in N_0)$.
c) $P(n)$ is true for all $n \in N_0$.
c) $P(n)$ is true for all $n \in N_0 - \{1\}$.

Section 4.3: Problem 4

a)
f(2)=0
f(3)= -1
f(4)= -1
f(5)= 0

b)
f(2)= 1
f(3)= 1
f(4)= 1
f(5)= 1

c)
f(2)= 2
f(3)= 5
f(4)= 33
f(5)= 1214

d)
f(2)= 1
f(3)= 1
f(4)= 1
f(5)= 1

Section 4.3: Problem 8

a) $a_{n+1} = a_n + 4$ and $a_1 = 2$
b) $a_{n+1} = (a_n + 2) \bmod 4$ and $a_1 = 0$
c) $a_{n+1} = a_n + 2n$ and $a_1 = 2$
d) $a_{n+1} = (a_n + 1)^2$ and $a_1 = 1$

Section 4.4: Problem 8

```
function sum(n) {
  if (n=1) {
     return 1
  } else {
     return n + sum(n-1)
  }
}
```

Section 4.4: Problem 44

L = 4,3,2,5,1,8,7,6
L = 4,3,2,5 | 1,8,7,6
L = 4,3 | 2,5 | 1,8,7,6
L = 4,3 | 2,5 | 1,8 | 7,6
L = 3,4 | 2,5 | 1,8 | 6,7
L = 2,3,4,5 | 1,8 | 6,7
L = 2,3,4,5 | 1,6,7,8
L = 1,2,3,4,5,6,7,8

Section 5.1: Problem 26

$2 \times 10^3 \times 26^3$

Section 5.1: Problem 30

a) 26^8 b) $\frac{26!}{18!}$ c) 26^7 d) $\frac{25!}{18!}$ e) 26^6 f) 26^6 g) 26^4 h) $2 * (26^6)$

Section 5.2: Problem 8

We can prove this by contradiction. Assume, for all elements $e_i, e_j \in S(i \neq j) : f(e_i) \neq f(e_j)$, then $|S| \leq |T|$, which is a contradiction to the assumption. Therefore, two different elements in S are mapped to the same element in T.

Section 5.2: Problem 22

In class we proved the following theorem: "Every sequence of n^2+1 distinct real numbers contains a sequence of length $n+1$ that is either strictly increasing or decreasing". 101=10^2+1. For 101 poeple, it can be concluded that there are $n+1$ (10+1) people standing that are strictly decreasing or increasing.

Section 5.3: Problem 18

a) 2^8 b) $\binom{8}{3}$ c) 2^8 - ($\binom{8}{2}+\binom{8}{1}+\binom{8}{0}$) d) $\binom{8}{4}$

Section 5.3: Problem 34

$\binom{15}{6}\binom{10}{0}+\binom{15}{5}\binom{10}{1}+\binom{15}{4}\binom{10}{2}$

Section 5.4: Problem 8

Using the binomial theorem, the coefficient of x^8y^9 is: $\binom{17}{9}3^8 2^9 = 81662929920$.

Section 5.4: Problem 24

$\binom{p}{k} = \frac{p!}{k!(p-k)!} = \frac{p(p-1)!}{k!(p-k)!}$. It is given that $1 \leq k \leq p-1$ and that p is a prime. Thus it follows that p does not divide $k!$ and p does not divide $(p-k)!$. In other words, $k!(p-k)!$ divides $(p-1)!$. Therefore, p divides $\binom{p}{k}$.