Problem Set 2 Solutions

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November 8, 2007

Section 3.1: Problems 18

Parse the list from the first to the last element. Set the the smallest element pointer to the first element. Afterwards parse the list and compare each element e with the first element. If element e is smaller than the first element, set the pointer to the newer smallest element.

```
int index = 0;

for (i=0;i<list.size;i++) {

  if(list[i] <= list[index]) {

    index = i;

  }

  return index, list[index];

}
```

Section 3.1: Problems 56

If the change is 15 cent, the greedy algorithm would return 12,1,1,1. The fewest number of coins would be 10 and 5 coins.

Section 3.2: Problems 20

a) O($n^3 \log n$)
b) O(6^n)
d) O($n^n n!$)

Section 3.2: Problems 38

By the definition of $\Theta()$, there exist constants c, d, e, and q such that:
1. $c \times g(x) \leq f(x) \leq d \times g(x)$
2. $e \times h(x) \leq g(x) \leq q \times h(x)$
Plugging 2 in 1 we get:
3. $ce \times h(x) \leq c \times g(x) \leq f(x) \leq d \times g(x) \leq dq \times h(x)$; In other words,
4. $ce \times h(x) \leq f(x) \leq dq \times h(x)$
Therefore $f(x) \in \Theta(h(x))$

Section 3.3: Problems 12

a) $n - 1$
b) 1
c) 1

Section 3.3: Problems 26

Since the units which we subtract from n can be considered as constants, the running time is linear with n.

Section 3.4: Problems 24

Let $n = (2k + 1)$, for some integer k.
Then, $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1$. Since k or $k + 1$ is even, we can write $k(k + 1) = 2m$ for some integer m.
Thus, $n^2 = 8m + 1$ and hence $8|(n^2 - 1)$ $\square$

Section 3.4: Problems 30

```
global m=9; \* m can be changed at will *\

function gen(x,curr) {

if curr = m; exit;

a=7;
c=4;

print (a * x + c mod 9);
gen(a * x + c mod 9,curr++)
}
```

Section 3.5: Problems 16

$2^7 - 1$ is a prime number
$2^9 - 1 = 7$ x 73
$2^{11} - 1 = 23$ x 89
$2^{13} - 1$ is a prime number

Section 3.5: Problems 32

a ≡ b(mod m), implies b=a+km for some integer k. Let us assume that d_1 is a common divisor of a and m. Then there will exist some integers x_1, y_1 so that b=$d_1 x_1$+k($d_1 y_1) = d_1(x_1 + ky_1)$. This shows that d_1 is a common divisor of b and m. Similarily, let us assume that d_2 is a common divisor of b and m. Then there will exit some integers x_2, y_2, so that a=$d_2 x_2$+k($d_2 y_2) = d_2(x_2 + ky_2)$. So d_2 is a common divisor of a and m. Therefore, every divisor of b and m is also a divisor of a and m, and vice-versa. As a result, it follows that gcd(a,m)=gcd(b,m).

Section 3.6: Problems 2

$\frac{321}{2} = 160$ R 1
$\frac{160}{2} = 80$ R 0
$\frac{80}{2} = 40$ R 0
$\frac{40}{2} = 20$ R 0
$\frac{20}{2} = 10$ R 0
$\frac{10}{2} = 5$ R 0
$\frac{5}{2} = 2$ R 1
$\frac{2}{2} = 1$ R 0
$\frac{1}{2} = 0$ R 1

$(321)_{10} = (101000001)_2$

$\frac{1023}{2} = 511$ R 1
$\frac{511}{2} = 255$ R 1
$\frac{255}{2} = 127$ R 1
$\frac{127}{2} = 63$ R 1
$\frac{63}{2} = 31$ R 1
$\frac{31}{2} = 15$ R 1
$\frac{15}{2} = 7$ R 1
$\frac{7}{2} = 3$ R 1
$\frac{3}{2} = 1$ R 1
$\frac{1}{2} = 0$ R 1

$(1023)_{10} = (1111111111)_2$

$\frac{100632}{2} = 50316$ R 0
$\frac{50316}{2} = 25158$ R 0
$\frac{25158}{2} = 12579$ R 0
$\frac{12579}{2} = 6288$ R 1
$\frac{6288}{2} = 3144$ R 0
$\frac{3144}{2} = 1572$ R 0
$\frac{1572}{2} = 786$ R 0
$\frac{786}{2} = 393$ R 0
$\frac{393}{2} = 196$ R 1
$\frac{196}{2} = 98$ R 1
$\frac{98}{2} = 49$ R 0
$\frac{49}{2} = 24$ R 1
$\frac{24}{2} = 12$ R 0
$\frac{12}{2} = 6$ R 0
$\frac{6}{2} = 3$ R 0
$\frac{3}{2} = 1$ R 1
$\frac{1}{2} = 0$ R 1

$(100632)_{10} = (11000101100001000)_2$

Section 3.6: Problems 26

8 divisions are required

Section 3.8: Problems 6

The matrix A is:

$$\begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

Section 3.8: Problems 14

Given two diagonal matrices A, B, their product is again a diagonal matrix where each element in the diagonal is the product of the two diagonal elements (in A,B) in the same row.