Problem Set 1 Solutions

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Section 1.1 Problem 10

a) $r \wedge \neg q$
b) $p \wedge q \wedge r$
c) $p \rightarrow r$
d) $p \wedge \neg q \wedge r$
e) $p \wedge q \rightarrow r$
f) $r \leftrightarrow q \vee p$

Section 1.1 Problem 32

p	q	r	$(p \vee q) \vee r$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

p	q	r	$(p \vee q) \wedge r$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

p	q	r	$(p \wedge q) \vee r$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

p	q	r	$(p \wedge q) \wedge r$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

p	q	r	$(p \vee q) \wedge \neg r$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

p	q	r	$(p \wedge q) \vee \neg r$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Section 1.2 Problem 10

p	q	$[\neg p \land (p \lor q)]$	$[\neg p \land (p \lor q)] \to q$
0	0	0	1
0	1	1	1
1	0	0	1
1	1	0	1

p	q	r	$[(p \to q) \land (q \to r)]$	$[(p \to q) \land (q \to r)] \to (p \to r)$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

p	q	$[p \land (p \to q)]$	$[p \land (p \to q)] \to q$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	1

p	q	r	$[(p \lor q) \land (p \to r) \land (q \to r)]$	$[(p \lor q) \land (p \to r) \land (q \to r)] \to r$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	0	1
1	1	1	1	1

Section 1.2 Problem 32

For the assignment p=1 q=0 and r=0 both statements have different results. The first statement equals to 1 and the second statement equals to 0.

Section 1.3 Problem 10

C(x) = "x has a cat"
D(x) = "x has a dog"
F(x) = "x has a ferret"
a) $\exists$ x $C(x) \land D(x) \land F(x)$
b) $\forall$ x $C(x) \lor D(x) \lor F(x)$
c) $\exists$ x $C(x) \land F(x) \land \neg D(x)$
d) $\forall$ x $\neg(C(x) \land D(x) \land F(x))$

e) $\exists$ x $C(x) \wedge \exists y D(y) \wedge \exists z F(z)$

Section 1.3 Problem 42

a)
Let domain consist of all users.
A(x) = "user x has access to an electronic mailbox"
$\forall$ x A(x)

b)
Let domain consist of all users in the group.
A(x) = "user x has access to the system mailbox"
B = "File system is locked"
$B \rightarrow \forall x A(x)$

c)
A = "The firewall is in a diagnostic state"
B = "The proxy server is in diagnostic state"
$A \rightarrow B$

d)
A(x) = "Router x is functioning normally"
B = "Throughput is between 100kbps and 500kbps"
C = "The proxy server is not in a diagnostic mode"
$(B \wedge C) \rightarrow \exists x \ A(x)$

Section 1.4 Problem 4

a) There is at least one student who has taken a CS class.
b) There is a student who has taken all the CS classes.
c) Every student has taken at least one CS class.
d) There is at lease one CS class which has been taken by all the students.
e) For every CS class there is at least one student who has taken it.
f) All the students have taken all the CS classes.

Section 1.4 Problem 26

Q(x,y) = "x + y = x - y"
a) false
b) true
c) false
d) false
e) true
f) true (y=0)

g) true (y=0)
h) false
i) false

Section 1.5 Problem 12

Using exercise 11, it suffices to prove that the premises: (1) $(p \wedge t) \rightarrow (r \vee s)$, (2) $q \rightarrow (u \wedge t)$, (3) $u \rightarrow p$, (4) $\neg s$, and (5) q and conclusion r is valid. From (3) and (2) we get: (6) $q \rightarrow (p \wedge t)$. From (1) and (6) we get: (7) $q \rightarrow (r \vee s)$. From (5) and (7) we get (8): $r \vee s$. From (8) and (4) we get (9): r which is the conclusion.

Section 1.5 Problem 20

a) No, not if a is a negative real number
b) Yes the arguments are valid.

Section 1.6 Problem 18

Proof by contraposition:
$\neg$ (n is even) $\rightarrow \neg$ ($3n + 2$ is even) If n is odd then $3n$ is odd and $3n + 2$ is odd as well.

Proof by contradiction:
Assume that $3n + 2$ is odd and n is even. Since n is even, $n = 2k$ for some integer k. This means that $3n + 2 = 6k + 2 = 2(3k + 1)$. This in turn implies that $3n + 2$ is even, which is a contradiction.

Section 1.6 Problem 32

We'll prove this by showing that (i) $\rightarrow$ (ii), (ii) $\rightarrow$ (iii), and (iii) $\rightarrow$ (i).

Proof that (i) $\rightarrow$ (ii): Since x is rational we can find integers a and b such that $x = a/b$. Then, $\frac{x}{2} = \frac{a}{2b}$. Therefore, $x/2$ is also a rational number.

Proof that (ii) $\rightarrow$ (iii): If $x/2$ is rational, then we can find integers a and b such that $\frac{x}{2} = \frac{a}{b}$. Then, $x = \frac{2a}{b}$ and $3x - 1 = \frac{6a-b}{b}$ and hence $3x - 1$ is also rational.

Proof that (iii) $\rightarrow$ (i): Since $3x - 1$ is rational, we can find integers a and b such that $3x - 1 = \frac{a}{b}$. This means that $3x = 1 + \frac{a}{b}$ and $x = \frac{a+b}{3b}$. Thus, x is also rational. $\square$

Section 1.7 Problem 4

There are the following cases
a) $a > b > c$ min(b,c)=c min(a,b)=b, therefore min(a,min(b,c)) = min(min(a,b),c) = c
b) $a > c > b$ min(b,c)=b min(a,b)=b, therefore min(a,min(b,c)) = min(min(a,b),c) = b
c) $b > a > c$ min(b,c)=c min(a,b)=a, therefore min(a,min(b,c)) = min(min(a,b),c) = c
d) $b > c > a$ min(b,c)=c min(a,b)=a, therefore min(a,min(b,c)) = min(min(a,b),c) = a

e) $c > a > b$ min(b,c)=b min(a,b)=b, therefore min(a,min(b,c)) = min(min(a,b),c) = b
f) $c > b > a$ min(b,c)=b min(a,b)=a, therefore min(a,min(b,c)) = min(min(a,b),c) = a
For a=b or b=c or a=c, choose one of both. Then the solution is not unique anymore. And for a=b=c choose any of them.

Section 1.7 Problem 32

We will prove this by contradiction. Assume third the third root of 2 is rational. Then there exist integers p and q so that the third root of 2 is equal to $\frac{p}{q}$ and $gcd(q,p) = 1$. This means that $2 \times q^3$ $= p^3$. This in turn means that p^3 (and also p) is even. As a result, $p = 2r$ for some integer r. Now, $2 \times q^3 = (2r)^3$, which means that $q^3 = 4r^3$. Therefore, q^3 (and hence q also) should be even. We have concluded that both p and q are even. This is a contradiction to the assumption that $gcd(q,p) = 1$. □

Section 2.1 Problem 8

a) true
b) true
c) false
d) true
e) true
f) true
g) true

Section 2.1 Problem 28

A = {a,b,c}, B={x,y}, C={0.1}
a) A x B x C = {(a,x,0),(a,x,1),(a,y,0),(a,y,1),
(b,x,0),(b,x,1),(b,y,0),(b,y,1),
(c,x,0),(c,x,1),(c,y,0),(,y,1)}

b) C x B x A = {(0,x,a),(0,x,b),(0,x,c),(0,y,a),(0,y,b),(0,y,c),
(1,x,a),(1,x,b),(1,x,c),(1,y,a),(1,y,b),(1,y,c)}

c) C x A x B = {(0,a,x),(0,a,y),(0,b,x),(0,b,y), (0,c,x),(0,c,y),
(1,a,x), (1,a,y),(1,b,x), (1,b,y),(1,c,x),(1,c,y)}

d) B x B x B = {(x,x,x),(x,x,y),(x,y,x),(x,y,y),
(y,x,x),(y,x,y),(y,y,x),(y,y,y)}

Section 2.2 Problem 36

You can show this with the help of Venn diagrams. Draw tow diagrams A and B that intersect. Then you can see that A$\otimes$B = (A-B) $\bigcup$ (B-A).

Section 2.2 Problem 48

a)
$\bigcup_{i=1}^{\infty} A_i$ = {1,2,3, ...}
$\bigcap_{i=1}^{\infty} A_i = \emptyset$

b)
$\bigcup_{i=1}^{\infty} A_i$ = {0,1,2, ...}
$\bigcap_{i=1}^{\infty} A_i$ = {0}

c)
$\bigcup_{i=1}^{\infty} A_i$ = the set of real numbers > 0
$\bigcap_{i=1}^{\infty} A_i = (0, 1)$

d)
$\bigcup_{i=1}^{\infty} A_i$ = the set of real numbers > 1
$\bigcap_{i=1}^{\infty} A_i = \emptyset$

Section 2.3 Problem 18

a) yes (injective and sujective)
b) no (injective and not surjective)
c) yes (injective and surjective)
d) yes (injective and surjective)

Section 2.3 Problem 32

a) f(g(x)) = f(x+2) = $(x+2)^2$+1
b) g(f(x)) = g(x^2+1) = x^2+3

Section 2.4 Problem 10

a) $f(i) = 3 + \sum_{j=1}^{i-1}(2j-1)$ (123,146,171)
b) $f(i) = 3 + 4i$ (47,51,54)
c) $f(i_{10}) = i_2$ (1100,1101,1110)
d) To form this sequence we use the following integers: $1, 2, 3, 5, 8, \ldots$ (where the next integer in the sequence is the sum of the previous two integers). 1 is repeated once, 2 is repeated 3 times, and so on. (8,8,8)

e) $f(0) = 0$ and $f(i) = \sum_{j=0}^{i-1} 2 * 3^j$, for $i = 1, 2, \ldots$
f) $f(1) = 1$ and $f(i) = (2i - 1) * f(i - 1)$ for $i = 2, 3, \ldots$
g) $f(i) = (n | \frac{n(n-1)}{2} < i \leq \frac{n(n+1)}{2}) \bmod 2$
h) $f(i) = 2^{2^{i-1}}$

Section 2.4 Problem 14

a) 16
b) 84
c) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{176}{105}$
d) 4