CSE 259 Algorithms and Complexity

Homework 1: Due on September 28, 2006, 11AM

1. What is the run time of the following algorithm?

```
Algorithm Sum();
    Result := 0;
    for i := 1 to n do
        for j := 1 to i do
            for k := 1 to j do
                Result++;
    Output Result;
```

2. Show that: (a) $14n^3 \log n + 5n^2 = \Theta(n^3 \log n)$; (b) $\log(n!) = \Theta(n \log n)$; (c) $(\sqrt{n})^{\sqrt{n}} = o\left(2^{n^{0.6}}\right)$.

3. Prove or disprove:

   a) If $f(n) = O(g(n))$ then $2^{f(n)} = O(2^{g(n)})$.

   b) $\max\{f(n),\ g(n)\} = \Theta(f(n) + g(n))$.

   c) For any real constants a and b, $(n + a)^b = \Theta(n^b)$.

4. Input are an array $a[1 : n]$ of arbitray real numbers and another real number u. The problem is to check if the array has two elements x and y such that $x + y = u$. Present an algorithm to solve this problem. What is the run time of your algorithm?

5. Input is an array $a[1 : n]$ of arbitrary real numbers. It is given that the array has two elements such that each of these elements is repeated $\frac{n}{4}$ times. The other elements are unique. (In other words the array has $\frac{n}{2} + 2$ distinct elements). Present a Las Vegas algorithm to identify the two repeated elements. The run time of your algorithm should be $\widetilde{O}(\log n)$. Prove the run time of your algorithm.