CSE259 Algorithms and Complexity. Fall 2006

Home work 2. Due on October 12 (11AM).

1. Show how to form a heap out of the elements $4, 11, 3, 5, 8, 23, 7, 9, 14, 15$ using the **Heapify** algorithm discussed in class. Show the individual steps.

2. (Exercise 11.2-5 from the text): Show that if $|U| > mn$, there is a subset of U of size n consisting of keys that all hash to the same slot, so that the worst case searching time for hashing with chaining is $\Theta(n)$.

3. Find an efficient data structure for representing a subset S of the integers from 1 to n. Operations we wish to perform on the set are

   - **INSERT**(i) to insert the integer i to the set S. If i is already in the set, this instruction must be ignored.
   - **DELETE** to delete an arbitrary member from the set.
   - **MEMBER**(i) to check whether i is a member of the set.

   Your data structure should enable each one of the above operations in constant time (irrespective of the cardinality of S).

4. Input is a sequence X of n keys with many duplications such that the number of distinct keys is d $(< n)$. Present an $O(n \log d)$-time sorting algorithm for this input. (For example, if $X = 5, 6, 1, 18,\ 6, 4, 4, 1,\ 5, 17$, the number of distinct keys in X is six.)

5. Solve the following recurrence relations:

   (a)
   $$T(n) = \begin{cases} 1 & n \leq 4 \\ 12T(n/4) + n^{1.5} & n > 4 \end{cases}$$

   (b)
   $$T(n) = \begin{cases} 1 & n \leq 4 \\ T(\sqrt{n}) + \log n & n > 4 \end{cases}$$

6. Given two sets A and B with m and n elements (respectively) from a linear order. These sets are not necessarily sorted. Also given that $m \leq n$. Show how to compute $A \cup B$ and $A \cap B$ in $O(n \log m)$ time.

7. $X_1, X_2, \ldots, X_\ell$ are sorted sequences such that $\sum_{i=1}^{\ell} |X_i| = n$. Show how to merge these ℓ sequences in time $O(n \log \ell)$.