1. There are $\sqrt{n}$ copies of an element in the array $c$. Every other element of $c$ occurs exactly once. If the algorithm RepeatedElement is used to identify the repeated element of $c$, will the run time still be $\tilde{O}(\log n)$? If so, why? If not, what is the new run time?

2. Let $A$ be a Monte Carlo algorithm that solves a decision problem $\pi$ in time $T$. The output of $A$ is correct with probability $c$, $c$ being a constant greater than $1/2$. Show how you can modify $A$ so that its answer is correct with high probability. The modified version can take $O(T \log n)$ time.

3. In an infinite array, the first $n$ cells contain integers in sorted order and the rest of the cells are filled with $\infty$. Present an algorithm that takes $x$ as input and finds the position of $x$ in the array in $O(\log n)$ time. You are not given the value of $n$.

4. Find an efficient data structure for representing a subset $S$ of the integers from 1 to $n$. Operations we wish to perform on the set are

   - INSERT($i$) to insert the integer $i$ to the set $S$. If $i$ is already in the set, this instruction must be ignored.
   - DELETE to delete an arbitrary member from the set.
   - MEMBER($i$) to check whether $i$ is a member of the set.

Your data structure should enable each one of the above operations in constant time (irrespective of the cardinality of $S$).

5. Input is a sequence $X$ of $n$ keys with many duplications such that the number of distinct keys is $d$ ($< n$). Present an $O(n \log d)$-time sorting algorithm for this input. (For example, if $X = 5, 6, 1, 18, 6, 4, 4, 1, 5, 17$, the number of distinct keys in $X$ is six.)

6. Input is an array of $n$ arbitrary real numbers (where $n$ is odd). The array has $(n + 1)/2$ distinct numbers such that each number has exactly two copies excepting for one number. Present an $O(n)$ time algorithm to identify the unique number.

7. Input is a (not necessarily sorted) sequence $S = k_1, k_2, \ldots, k_n$ of $n$ arbitrary numbers. Consider the collection $C$ of $n^2$ numbers of the form $\min\{k_i, k_j\}$, for $1 \leq i, j \leq n$. Present an $O(n)$-time and $O(n)$-space algorithm to find the median of $C$.

8. Two sets $A$ and $B$ have $n$ elements each. Assume that each element is an integer in the range $[0, n^{100}]$. These sets are not necessarily sorted. Show how to check whether these two sets are disjoint in $O(n)$ time. Your algorithm should use $O(n)$ space.