CSE 5500 Advanced Sequential and Parallel Algorithms
Exam III; May 3, 2011

Note: You are supposed to give proofs to the time and processor bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (16 points) Input are a sorted sequence $X = k_1, k_2, \ldots, k_n$ of distinct elements and another element $x$. The problem is to check if $x \in X$. Present an algorithm for this problem that uses $p$ CREW PRAM processors (where $p \leq n$) and runs in time $O\left(\frac{\log n}{\log p}\right)$.
2. (16 points) Input is an undirected graph $G(V, E)$ (with $|V| = n$) in the form of an adjacency matrix. The problem is to check if $G$ has a triangle as a subgraph. Present an $O(1)$ time algorithm for this problem that uses up to $n^3$ common CRCW PRAM processors.
3. (16 points) Input is a (not necessarily sorted) sequence $X$ of $n$ arbitrary real numbers. Present a Monte Carlo algorithm to output an element of $X$ whose rank in $X$ is $\geq 0.9n$. Your algorithm should run in $O(\log \log n)$ time using up to $\log \frac{n}{\log \log n}$ CREW PRAM processors. Show that the output of your algorithm will be correct with high probability. (Recall that the rank of an element $x$ in a sequence $X$ is defined to be $|\{q \in X : q < x\}| + 1$.)
4. (17 points) Input is a sorted sequence $X$ of $n$ arbitrary elements. The problem is to find one of the modes of $X$. A mode of any sequence is one of the most frequently occurring elements in $X$. Present an $O(\log n)$ time algorithm for this problem that uses at most $\frac{n}{\log n}$ CREW PRAM processors. (For example, if $X = 1, 2, 2, 3, 3, 3, 3, 3, 5, 5, 6, 7, 4, 7, 4, 7, 4, 7, 4, 8, 8$, then $X$ has only one mode, namely, 3. If $X$ has an additional two 8’s then 8 will also be a mode.)
5. (18 points) Present an $O(\sqrt{n})$ time algorithm for the selection problem. You can use up to $\sqrt{n}$ CREW PRAM processors. Recall that the selection problem takes as input a sequence $X$ of $n$ arbitrary elements and an integer $i$ (with $1 \leq i \leq n$) and outputs the $i$th smallest element of $X$. 
6. (17 points) Present an $O(n)$ time algorithm to sort $n$ given arbitrary real numbers. You can use up to $\log n$ CREW PRAM processors.