CSE 5500 Advanced Sequential and Parallel Algorithms
Exam III; May 4, 2010

Note: You are supposed to give proofs to the time and processor bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (16 points) Input is an undirected graph $G(V,E)$ in the form of an adjacency matrix. The problem is to check if $G$ is complete or not. Present an $O(1)$ time algorithm for this problem that uses at most $|V|^2$ common CRCW PRAM processors.
2. (16 points) Input is an array $a[1 : n]$ of elements. The problem is to identify the largest value of $i$ such that $a[i] = n$. Present an $O(1)$ time algorithm for this problem that uses $n$ CRCW PRAM processors.
3. (18 points) Let $k_1, k_2, \ldots, k_n$ be a sequence of arbitrary real numbers. The *prefix minima* problem is the same as the prefix computation problem where the associative operation of interest is minimum. In particular, the output should be $x_1, x_2, \ldots, x_n$ where $x_i$ is the minimum of $k_1, k_2, \ldots, k_i$ (for $1 \leq i \leq n$). Show how to solve the prefix minima problem in $O(\log \log n)$ time using $\frac{n}{\log \log n}$ CRCW PRAM processors.
4. (16 points) Input are two (not necessarily sorted) sets $A$ and $B$ with $n$ and $m$ elements, respectively. Assume that $n > m$. Present an algorithm for computing $A \cap B$ in $\tilde{O}(\log m)$ time using at most $n$ CRCW PRAM processors.
5. (17 points) In this problem we are interested in sorting $n$ integers where each integer has $c$ bits ($c$ being a constant). Present an $O(\log n)$ time algorithm for this problem. You can use $\frac{n}{\log n}$ CREW PRAM processors.
6. (17 points) Use the fact that we can merge two sorted sequences of length $m$ each in $O(\log \log m)$ time using $m$ CRCW PRAM processors to show that we can sort a sequence of $n$ arbitrary keys in $O(\log n \log \log n)$ time using $n$ CRCW PRAM processors.