CSE 259 Algorithms and Complexity

Fall 2006; Exam III (model)

1. Present an algorithm to compute $f(x) = (x + a_1)(x + a_2) \cdots (x + a_n)$ where $a_1, a_2, \ldots, a_n$ are scalars. The output should be the coefficients of $f(x)$. Your algorithm should run in time $O(n \log^2 n)$.

2. The *template matching* problem takes as input an image $I[0 : n - 1]$ and a template $T[0 : m - 1]$ of pixel values. Here $m \leq n$. The problem is to compute $C[0 : n - 1]$ where $C[i] = \sum_{q=0}^{m-1} I[(i + q) \bmod n] * T[q]$. Present an $O(n \log n)$ time algorithm for the template matching problem.

3. The array A is an array of n keys, where each key is an integer in the range $[1, n]$. The problem is to decide whether there are any repeated elements in A. Show how you do this in $O(1)$ time on an n-processor CRCW PRAM. Which version of the CRCW PRAM are you using?

4. Show how to multiply two given $n \times n$ matrices in $O(\log n)$ time using n^3 CREW PRAM processors.

5. Let DK be an algorithm for the 0/1 knapsack decision problem. Let r be the value of an optimal solution to the knapsack optimization problem. Show how to obtain a 0/1 assignment for the x_i, $1 \leq i \leq n$, such that $\sum p_i x_i = r$ and $\sum w_i x_i \leq m$ by making n applications of DK.

6. Prove or disprove the following statement:

   > If there exists a deterministic polynomial time algorithm that converts any given formula in CNF into an equivalent formula in DNF, then $\mathcal{P} = \mathcal{NP}$.