

Name: _____

CSE 254 Introduction to Discrete Systems

Fall 2007; Exam II; 11-8-2007

Note: Read the questions carefully before attempting to solve them.

1. (12 points) Input are an array $a[1 : n]$ of arbitrary real numbers and another real number x . The goal is to check if there are two elements in the array whose sum is x . Present an algorithm for solving this problem. What is the time complexity of your algorithm?

2. (9 points) Use the bubble sort to sort the sequence 17, 8, 12, 5, 3, 9 showing the lists obtained at each step.

3. a) (5 points) Show that $\log(7n^2 + 5n)$ is $\Theta(\log n)$; b) (5 points) Show that $5n^3 + 7n^2 - 125n = \Theta(n^3)$.

4. (12 points) Let $f(n)$ and $g(n)$ be any non-negative integer functions of n with a domain of positive integers. Show that $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.

5. (9 points) Consider the following algorithm:

```
Algorithm IsThereALargeElement(a[1:n]:real)
  for i:=1 to n do
    if (a[i]>1000) then return i and quit;
  Return "No";
```

What is the worst case number of comparisons made by the above algorithm? What is the best case number of comparisons made by this algorithm?

6. (10 points) Show that if a, b, c , and m are integers such that $m \geq 2$, $c > 0$, and $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{mc}$.

7. (9 points) Check if 10531 is a prime.

8. (9 points) Find the GCD of 3456 and 113 using Euclid's algorithm. Show the details of every step. What is the LCM of these numbers?

9. (9 points) Convert $(17156)_{10}$ to its binary representation.

10. (11 points) Find a matrix A such that

$$\begin{bmatrix} 5 & 7 \\ 1 & 3 \end{bmatrix} A = \begin{bmatrix} 11 & -22 \\ 7 & -14 \end{bmatrix}$$