Name: _____

CSE 259 Algorithms and Complexity

Exam II (Model), Fall 2006

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (16 points) $A[1:n]$ is an array of distinct real numbers such that the difference between any two numbers in A is at least $\frac{1}{n^5}$ and at most n^5. Present an $O(n)$ time algorithm to sort A.

2. (16 points) Find an optimal solution to the (real) knapsack instance $n = 9, m = 45$, $(p_1, p_2, \ldots, p_9) = (10, 20, 12, 25, 30, 20, 15, 10, 15)$, and $(w_1, w_2, \ldots, w_9) = (5, 12, 8, 15, 20, 8, 8, 7, 10)$.

3. (16 points) Find a minimum spanning tree for the following graph $G(V, E)$ either using Prim's algorithm or using Kruskal's algorithm: $V = \{1, 2, 3, 4, 5, 6, 7\}$. The edge weights are: $W(1,2) = 8; W(1,3) = 5; W(1,4) = 7; W(1,6) = 4; W(1,7) = 5$; $W(2,4) = 4; W(3,4) = 2; W(3,5) = 3$; $W(3,6) = 6; W(4,5) = 2; W(4,7) = 4$; $W(5,6) = 8; W(5,7) = 11; W(6,7) = 5$.

4. (16 points) Let T be a minimum cost spanning tree (MCST) of a weighted undirected graph $G(V, E)$. If the weight on every edge is now increased by the same value c, will T continue to be a MCST of G? If yes, prove your answer; if not how will you modify T into a MSCT of G?

5. (18 points) Use Dijkstra's algorithm to solve the single source shortest path problem on the directed graph $G(V, E)$: $V = \{s, 1, 2, 3, 4, 5\}$. Edge weights are: $W(s,1) = 5; W(s,2) = 21$; $W(s,3) = 11; W(1,2) = 4; W(1,4) = 8$; $W(2,3) = 3; W(2,4) = 3$; $W(2,5) = 12; W(3,4) = 5$; $W(4,5) = 5; W(5,2) = 3$.

6. (18 points) Solve the following instance of the 0/1 knapsack problem using dynamic programming:

Weight	1	2	2	3
Profit	20	10	15	25

The capacity of the knapsack $m = 5$.