1. a) (8 points) Prove or disprove: $(\log n)^{\log \log n} = O\left((\log \log n)^{\sqrt{\log n}}\right)$.

b) (8 points) Prove or disprove: $n^{10}2^n = O((2.01)^n)$.
2. (16 points) What is the run time of the following algorithm:

\begin{algorithm}
\textbf{Test}(n)
\begin{align*}
\text{Result} & := 0; \\
\text{for } i & := 1 \text{ to } 3n \text{ do} \\
& \quad \text{for } j := 1 \text{ to } 5n \text{ do} \\
& \quad \quad \text{for } k := 1 \text{ to } 7j^2 + 5j \text{ do} \\
& \quad \quad \quad \text{Result}++; \\
\end{align*}
\end{algorithm}
3. (17 points) Input is an array $A$. The array can only be one of two types:

- **Type I**: $A$ contains $n$ zeros
- **Type II**: $A$ contains $\frac{n}{2}$ zeros and $\frac{n}{2}$ ones.

Present an $O(\log n)$ time Monte Carlo algorithm to determine the type of a given array $A$. Show that the output of your algorithm will be correct with high probability.
4. (17 points) Present an implementation of a data structure that supports the following operations:

- **INSERT**\(_{(x)}\): insert the element \(x\) into the data structure.
- **SEARCH**\(_{(x)}\): search for the element \(x\) in the data structure.
- **DELETE**\(_{(x)}\): delete the element \(x\) from the data structure.
- **ADD\_ALL**\(_{(y)}\): add the value \(y\) to each key in the data structure.

Each operation should take \(O(\log n)\) time, where \(n\) is the maximum number of elements that the data structure will ever have.
5. (18 points) Input are $m$ sorted sets $S_1, S_2, \ldots, S_m$ with $\sum_{i=1}^{m} |S_i| = n$. Present an $O(n)$ time algorithm to compute the intersection of these $m$ sets.
6. (16 points) A and B are two different divide-and-conquer algorithms for solving the same problem P. A reduces a problem instance of size n into 27 subproblems each of size \(n/3\) and takes \(n^3\) time for the partition and combine steps. B reduces any problem instance of size n into 225 subproblems each of size \(n/15\) and takes \(n^{2.5}\) time for the partition and combine steps. Which algorithm would you use to solve P? Why?