CSE 254 Introduction to Discrete Systems

Fall 2007; Exam I; Solutions

1. (i)

p	q	r	$p \wedge q$	$(p \wedge q) \vee r = I$	$q \wedge r = II$	$I \rightarrow II$
0	0	0	0	0	0	1
0	0	1	0	1	0	0
0	1	0	0	0	0	1
0	1	1	0	1	1	1
1	0	0	0	0	0	1
1	0	1	0	1	0	0
1	1	0	1	1	0	0
1	1	1	1	1	1	1

(ii)

p	q	r	$p \wedge q = I$	$q \wedge r = II$	$r \wedge p = III$	$I \oplus II$	$I \oplus II \oplus III$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	0
0	1	1	0	1	0	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	1	0	1
1	1	0	1	0	0	1	1
1	1	1	1	1	1	0	1

2.

p	q	r	$p \rightarrow q = I$	$q \rightarrow r = II$	$I \wedge II \wedge p = III$	$III \rightarrow r$
0	0	0	1	1	0	1
0	0	1	1	1	0	1
0	1	0	1	0	0	1
0	1	1	1	1	0	1
1	0	0	0	1	0	1
1	0	1	0	1	0	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

From the above truth table we realize that $((p \to q) \wedge (q \to r) \wedge p) \to r$ is a tautology.

3. Let $R(x)$ stand for the predicate "Person x is rich" where the domain of x is the set of residents of CT. Let $O(x, y)$ stand for the predicate "Person x owns car y" where the domain of x is the set of all residents of CT and the domain of y is the set of all cars. Then we can translate the given statements as follows: (i) $\neg(\forall x \; R(x))$ or $\exists x \; (\neg R(x))$; (ii) $\exists x \; O(x, \text{Ferrari})$; (iii) $(\neg R(\text{John})) \wedge O(\text{John}, \text{Ferrari})$; and (iv) $\exists y \; O(I, y) \wedge O(I, \text{Ferrari})$.

4. (i) Paul Newman is enrolled in CSE 259; (ii) There is at least one student who is enrolled in CSE 259 and CSE 254; (iii) If a student is enrolled in CSE 254 then (s)he is enrolled in CSE 259 also; and (iv) There is at least one pair of students who are enrolled in exactly the same set of classes.

5. The given premises are: (1) $p \to q$; (2) $q \to r$; (3) $s \to \neg r$; and (4) s. From (1) and (2) and the rule of hypothetical syllogism we infer: (5) $p \to r$. From (3) and (4) and the rule of modus ponens we infer: (6) $\neg r$. From (5) and (6) and the rule of modus tollens, we infer: $\neg p$ which is the conclusion.

6. We can prove this statement by contradiction. Assume that the number of balls ending up in each box is less than $\left\lceil \frac{m}{n} \right\rceil$. This means that the number of balls ending up in any box is $\leq \left\lceil \frac{m}{n} \right\rceil - 1$. This in turn means that the total number of balls is $\leq n \left(\left\lceil \frac{m}{n} \right\rceil - 1 \right) < n \left(\frac{m}{n} + 1 - 1 \right) = m$. This is a contradiction to the fact that there are a total of m balls.

7. Note that $x = 3a$, $y = 4a$, and $z = 5a$ is a solution to the equation: $x^2 + y^2 = z^2$ for every positive integer a.

8. Using Venn diagram we can see that $(A - (B \cup C)) \cup (B - C) \cup C = A \cup B \cup C$.

9. (i) The function $f : Z \to Z$ defined as $f(n) = 5n^2 + 7$ is not one-to-one since, for example, $f(2) = f(-2)$; (ii) The function $f : Z \to Z$ defined as $f(n) = 3n^3$ is one-to-one since if $f(n) = f(m)$ then $n = m$ (when both the domain and the codomain are Z); (iii) The function $f : N \to N$ defined as $f(n) = n^3$ is one-to-one but not onto; and (iv) The function $f : N \to N$ defined as $f(n) = \left\lceil \frac{n}{5} \right\rceil$ is onto but not one-to-one.

10. (i) $f(i) = i + 2^{2i-1}$ for $i = 1, 2, \ldots$; and (ii) $\sum_{i=1}^{n} (3i^2 + 4i + 5) = 3 \sum_{i=1}^{n} i^2 + 4 \sum_{i=1}^{n} i + 5 \sum_{i=1}^{n} 1 = 3 \frac{n(n+1)(2n+1)}{6} + 4 \frac{n(n+1)}{2} + 5n = \frac{2n^3 + 7n^2 + 15n}{2}$.