Name: ______________________________

# CSE 254 Introduction to Discrete Systems

## Fall 2007; Exam I; 10-9-2007

**Note:** Read the questions carefully before attempting to solve them.

1. (10 points) Construct truth tables for each of the following propositions: (i) $((p \wedge q) \vee r) \rightarrow (q \wedge r)$ and (ii) $(p \wedge q) \oplus (q \wedge r) \oplus (r \wedge p)$

2. (10 points) Determine whether $((p \rightarrow q) \wedge (q \rightarrow r) \wedge p) \rightarrow r$ is a tautology.

3. (10 points) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives: (i) Not everyone in Connecticut is rich; (ii) Some people in Connecticut own a Ferrari; (iii) Even though John is not rich he owns a Ferrari; and (iv) My other car is a Ferrari.

4. (10 points) Let $P(x, y)$ stand for the predicate "Student $x$ is enrolled in class $y$", where the domain of $x$ is the set of all students in your college and the domain of $y$ is the set of all classes offered in your college. Express each of these statements by a simple English sentence: (i) $P$(Paul Newman, CSE 259); (ii) $\exists x\ P(x,\text{CSE259}) \wedge P(x,\text{CSE 254})$; (iii) $\forall x\ (P(x,\text{CSE 254}) \rightarrow P(x,\text{CSE 259}))$; and (iv) $\exists x \exists y \forall z((x \neq y) \wedge (P(x, z) \leftrightarrow P(y, z)))$

5. (10 points) Show that the premises $p \to q$, $q \to r$, $s \to \neg r$, $s$ and the conclusion $\neg p$ form a valid argument.

6. (10 points) Prove the following statement: If $m$ balls are thrown into $n$ boxes then at least one box will end up with at least $\left\lceil \frac{m}{n} \right\rceil$ balls.

7. (10 points) Prove that there are infinitely many solutions in positive integers $x, y, z$ to the equation: $x^2 + y^2 = z^2$.

8. (10 points) If $A, B$, and $C$ are any sets then prove or disprove: $(A - (B \cup C)) \cup (B - C) \cup C = A \cup B \cup C$. (*Hint:* Use Venn diagram).

9. (10 points) (i) Is the function $f : Z \rightarrow Z$ defined as $f(n) = 5n^2 + 7$ one-to-one? (ii) Is the function $f : Z \rightarrow Z$ defined as $f(n) = 3n^3$ one-to-one? (iii) Given an example of a function from $N$ to $N$ that is one-to-one but not onto; (iv) Give an example of a function from $N$ to $N$ that is onto but not one-to-one.

10. (10 points) (i) Provide a simple formula of an integer sequence that begins with this list: $3, 10, 35, 132, 517, 2054$; (ii) What is $\sum_{i=1}^{n}(3i^2 + 4i + 5)$? (*Hint:* Use summation formulas).