

Name: _____

CSE 259 Algorithms and Complexity

Fall 2006; Exam I; 10-12-2006

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. Prove or disprove:

- (8 points) $n^n = O\left(2^{2^{\log^2 n}}\right)$.
- (8 points) If $f(n) = \Theta(n^3)$ and $g(n) = \Theta(n^3)$, then $f(n) - g(n) = \Theta(n^3)$.

2. (20 points) Input is an array $a[1 : n]$ of arbitrary real numbers. The array could only be of one of the following two types: 1) **Type I:** All the elements in the array are distinct; or 2) **Type II:** The array has $n^{2/3}$ copies of one element, the other elements being distinct. Present a Monte Carlo algorithm that determines the type of the array in $O(n^{2/3} \log n)$ time. Show that the output of your algorithm will be correct with high probability.

3. (16 points) Present a data structure that supports the following operations:

- $\text{INSERT}(x)$: Insert the element x into the data structure.
- $\text{MIN}()$: Find the smallest element in the data structure.

Each operation should take $O(1)$ time.

4. (16 points) Show how you'd use **Heapify** to form a max-heap out of the following elements: 23, 12, 5, 6, 11, 17, 14, 8, 2, 34, 21. Show the tree that results after each application of **Heapify**.

5. (16 points) \mathcal{A} and \mathcal{B} are two different divide-and-conquer algorithms for solving the same problem P . \mathcal{A} reduces a problem instance of size n into 8 subproblems each of size $n/2$ and takes n^2 time for the partition and combine steps. \mathcal{B} reduces any problem instance of size n into 64 subproblems each of size $n/8$ and takes $n^{2.8}$ time for the partition and combine steps. Which algorithm would you use to solve P ? Why?

6. (16 points) An array has the following elements: $k_1, k_2, \dots, k_\ell, q_1, q_2, \dots, q_m$, where the sequence k_1, k_2, \dots, k_ℓ is monotonically increasing and the sequence $k_\ell, q_1, q_2, \dots, q_m$ is monotonically decreasing. Also, $\ell + m = n$ and ℓ is unknown. Present an $O(\log n)$ time algorithm to determine ℓ . For example, if the input is 2, 5, 11, 24, 8, 1, the answer is 4.