Name: ____

CSE 361 Complexity of Sequential and Parallel Algorithms

Spring 2008 Exam II

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. Let $P_1, P_2, \ldots, P_n$ be a set of n programs that are to be stored on a tape of length l. Program P_i requires a_i amount of tape. If $\sum a_i \leq l$, then clearly all the programs can be stored on the tape. So, assume $\sum a_i > l$. The problem is to select a maximum subset Q of the programs for storage on the tape. (A maximum subset is one with the maximum number of programs in it). A greedy algorithm for this problem would build the subset Q by including programs in nondecreasing order of a_i.

   (a) (14 points) Show that this strategy always finds a maximum subset Q such that $\sum_{P_i \in Q} a_i \leq l$.

   (b) (3 points) Let Q be the subset obtained using the above greedy strategy. How small can the tape utilization ratio $(\sum_{P_i \in Q} a_i)/l$ get?

2. Consider the integer knapsack problem obtained by replacing the 0/1 constraint by $x_i \geq 0$ and integer. Generalize $f_i(y)$ to this problem in the obvious way. (Note that $f_i(y)$ is the maximum profit to the problem KNAP$(1, i, y)$.)

   (a) (6 points) Obtain the dynamic programming recurrence relation corresponding to $f_i(y)$.

   (b) (6 points) Show how to solve this relation in $O(m^2 n)$ time assuming that the weights are integers.

   (c) (5 points) Show that the real knapsack problem with $x_i \geq 0$ can be solved in $O(n)$ time.

3. (17 points) The SubsetSum problem takes as input a set $X = \{k_1, k_2, \ldots, k_n\}$ of integers and another integer K. The problem is to check if there exists a subset X' of X whose elements sum to K. For example, if $X = \{5, 3, 11, 8, 2\}$ and $K = 16$ then the answer is YES since the subset $X' = \{5, 11\}$ has a sum of 16. Present a dynamic programming algorithm for SubsetSum whose run time is $O(nK)$.

4. (17 points) The string editing problem can be extended to three (or more) sequences as follows. We still consider three operations viz., INSERT, DELETE, and CHANGE. In each step, the cost is zero if the corresponding symbols in all the sequences are the same and 1 otherwise (even if two sequences match and only one insertion, deletion, or change is necessary). As an example, consider the sequences $aabb$, bbb, and cbb. One possible edit sequence is inserting a in front of bbb and cbb (which costs 1), and replacing a b in bbb & a c in cbb with an a (for a cost of 1). The total cost is 2. Present an $O(n^3)$ time algorithm to compute the minimum edit cost between three given sequences each of length n.

5. (16 points) Input is an undirected graph $G(V, E)$. Present an $O(|V|^2)$ time algorithm to compute the reflexive transitive closure matrix A^* of G. ($A^*[i, j] = 1$ iff there is a path from node i to node j in G.)

6. (16 points) An undirected graph $G(V, E)$ is said to be *bipartite* if V can be partitioned into disjoint sets V_1 and V_2 such that no two nodes in V_1 have an edge between them and no two nodes in V_2 have an edge connecting them. (I.e., the only edges in G are from nodes in V_1 to nodes in V_2.) Present an $O(|V| + |E|)$ time algorithm to check if G is bipartite.