Name: _____

CSE 361 Complexity of Sequential and Parallel Algorithms

Spring 2008 Exam I

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (18 points) $\mathcal{A}$ is a Las Vegas algorithm that solves the problem π. The expected run time of $\mathcal{A}$ is T_n on any input of size n. How will you make use of $\mathcal{A}$ to devise a Las Vegas algorithm for solving π in $\widetilde{O}(T_n \log n)$ time? (**Hint:** You may want to use Markov's inequality that states: If X is any non-negative random variable, then $P[X \geq a\mu] \leq \frac{1}{a}$ where $\mu = E[X]$ and a is any real number ≥ 1.)

2. (16 points) A department has to keep records of its employees such that the following operations can be performed on the records:

   - Find_Name(SSN): Return the name of the person whose social security number is SSN; and
   - Find_SSN($Name$): Return the social security number of the person whose name is $Name$.

   Present a data structure for keeping the records that will take $O(\log n)$ time to perform each of the above operations, n being the number of persons in the department. You can use $O(n)$ space.

3. (17 points) Consider a group of n persons named $1, 2, \ldots, n$. Let R be a set of tuples with $|R| = m$. If $(i, j) \in R$ then it means that persons i and j are related. Write an algorithm that will take as input R, p_1, and p_2 (where p_1 and p_2 are persons) and decide if p_1 and p_2 are related or not. What is the run time of your algorithm? (Note that if a and b are related and if a or b is related to c then all the three are related).

4. (16 points) Two different divide-and-conquer algorithms $\mathcal{A}$ and $\mathcal{B}$ have been designed for solving the problem π. $\mathcal{A}$ partitions π into 5 subproblems each of size $\frac{n}{3}$. Here n is the input size for π. It takes a total of $\Theta(n^2)$ time for the partition and combine steps. $\mathcal{B}$ partitions π into 10 subproblems each of size $\frac{n}{4}$. It takes a total of $\Theta(n^{1.8})$ time for the partition and combine steps. Which algorithm is preferable? Why?

5. (16 points) Input are an array $a[\,]$ of n elements and an element x. We have to check if there are three elements in $a[\,]$ whose sum is x. Assume that the array elements are distinct. Present an $O(n^2 \log n)$ time algorithm for this problem.

6. (17 points) Input is a sequence of n keys $k_1, k_2, \ldots, k_n$. For each key k_i $(1 \leq i \leq n)$, its position in sorted order differs from i by at most d. Present an $O(n \log d)$ worst case time algorithm to sort this sequence.